EARTH'S GRAVITY FIELD

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Contents

Summary

The problem of the determination of the shape of the Earth without any preliminary hypothesis on the distribution of the masses within the Earth body, started to be investigated as early as in the 19th century. Already at this time, it was known, the exact determination of the orthometric heights was not possible by means of the classic methods. In the fortieths of the 20th century, Molodenskii found a solution. Instead of the physically defined level surface geoid, a new non-level surface quasigeoid was introduced. The sum of the new type of heights, the normal heights, and the quasigeoid heights equals the sum of the orthometric heights and geoid heights, and it is physically defined.

Thus the newly introduced quantities, the normal heights, and the quasigeoid heights, by itself, which have two preferences: Firstly, they can be determined with an arbitrary accuracy from observational data on the Earth´s surface, without any hypothesis on the structure of the Earth body. Secondly, their sum is physically defined quantity, namely the distance of the point under discussion from the reference body, measured along the outer normal to it. The rotational ellipsoid is usually chosen as the normal reference body. It is limited by the surface which is level of the gravity potential. The theory of such a body was developed by Pizzetti. Such a body can be defined by four parameters. The choice of the parameters is not unique. During the general assembly of the International Association of Geodesy in Luzern 1967, the following quantities were adopted for these parameters:

a = the large semi-major axis of the rotational ellipsoid,
J_2 = Stokes coefficient,
ω= the angular velocity of the rotation of the Earth,
GM = the product of the gravitation constant G and of the Earth mass M.

Recently, it has been shown that it would be more acceptable to select the potential value W_o on the mean sea level instead of the large semi-major axes. The gravity potential of the reference body is considered for the normal potential. Its derivative in the direction of the outer normal to the reference body is considered for the normal gravity. The difference of the actual gravity and normal gravity is called gravity anomaly and it is usually measured in the units of the acceleration. In practice, the actual gravity and the normal gravity are not applied to the same point, it is the question of the mixed gravity anomaly. The problem of determining the normal gravity potential and its derivatives from the known data on the Earth's surface was firstly solved by Pizzetti. The problem was worked in the system of the space curvilinear coordinates as the first (Dirichlet) boundary value problem. The Dirichlet problem can possess one solution only. If we get more solutions, for example because we have solved the work in another coordinate system, all the results must be identical. The crucial role in the solution plays the so-called coefficients Lamé. These values make possible to transform the changes of the curvilinear coordinates into the changes in the Cartesian coordinates. With a useful selection of the coefficients Lamé we have got various types of the curvilinear coordinates. We can find out the economically most useful case for the numerical calculus.

1. Introduction

Helmert and Pizzetti, outstanding geodesists of the past, were aware that the figure of the geoid and orthometric heights couldn't be determined accurately without detailed knowledge of the distribution of masses within Earth. The effort to solve the fundamental problem of geodesy—determining the gravity field and figure of Earth—using Stokes' theory and computation of orthometric heights proved to be contrary to the increasing demands on accuracy of observed data. The discovery of the Molodenskii theory helped in overcoming the crisis which had arisen in geodesy. The Molodenskii 1945, 1948, 1960 theory enabled the external gravity field of Earth and the figure of Earth's surface to be determined from gravimetric and geodetic observations on this surface. The theory provides a joint solution for determining the field and the heights. The Molodenskii theory included a new type of problem of mathematical physics. In problems studied earlier, it was assumed that the figure of the surface was known. The field could be determined from the values of the potential, of its derivatives in any direction, or from any combination thereof, determined on Earth's surface. The Molodenskii theory determines the surface from the values of the potential and of gravity. To be more precise, the potential is assumed to have been measured with respect to some point which has been chosen as the origin of heights, usually close to sea level. Several origins of calculating the heights may be used. It is then sufficient to know the differences in potential to be able to solve the boundary value problem. Their total values can be determined by comparison with the results of measurements in astronomical-geodetic nets, or with satellite data.

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Biographical Sketch

Milos Pick, was born in 1923 in Luže, Czech Republic. In 1950, he graduated from the Czech Technical University, Prague. In 1951–1953 he was a research fellow at the Military Topographic Institute in the department of mathematical cartography, Dobruška. From 1959 to 1963 he was a researcher at the Geophysical Institute, Czech Academy of Sciences, Prague, and received the degree Csc (candidate of science; PhD equivalent) with the thesis "Investigations of the Earth's Gravity Field in Mountains." In 1963 he received the degree of DrSc (doctor of sciences) with the thesis "Geoid and Outer Gravity Field." From 1960 to 1970 he was the director of the Geophysical Institute, Czechoslovak Academy of Sciences, Prague, Czechoslovakia. In 1967 he was invited to give a series of lectures on Newtonian Potential Theory at Charles University in Prague, Czechoslovakia.

Since 1991 Pick has been a consultant for the Geophysical Institute, Czech Academy of Sciences, Prague. In 1994 he became professor of Advanced Geodesy, Czech Technical University, Prague. He is co-author of the monograph "Theory of the Earth Gravity Field" (co-authors Pícha J. and Vyskočil V.; published by Elsevier, Amsterdam in 1973, and also in Czech as a textbook). Pick is the author of the twelfth chapter "The Geoid and Tectonic Forces" of "Geoid and its Geophysical Interpretation," edited by Vaníček P. and Christou N.T. (published by CRC Press in 1994). Pick is also author of the monograph "Advanced Physical Geodesy and Gravimetry," (Ministry of Defense, Prague, 2000), seven undergraduate textbooks, and about 200 original scientific papers. He is included in some "Who´s Who" publications, for example, Marquis' "Who's Who in the World." Pick currently serves as a member of the Editorial Board of the journal Studia Geophysica and Geodaetica. For many years he was also chairman of the commission for awarding candidate of science (PhD equivalent) and doctor of science in geodesy and geophysics. Pick has also organized two international symposia for the International Association of Geodesy (IAG). He is also a member of the IAG's Special Study Group "Test Areas," and headed the Subgroup "Testing Areas Artificial Models."