SPATIO-TEMPORAL METHODS IN CLIMATOLOGY

Christopher K. Wikle

Department of Statistics, University of Missouri –Columbia, USA

Keywords: Data assimilation, Empirical orthogonal function, Principal oscillation pattern, Canonical correlation

Contents

Summary

This article contains a comprehensive discussion of descriptive spatio-temporal statistical methods that have been applied extensively in the climatological sciences. The climate system is composed of many processes that exhibit complicated variability over a vast range of spatial and temporal scales. Data sets of measurements collected on this system are typically very large by statistical standards, and their analysis typically requires dimension reduction in space and / or time. Scientists have developed or borrowed and refined many descriptive statistical techniques that aid in the summary

and interpretation of these data. The focus here is on a subset of some the most useful methodologies: empirical orthogonal function (EOF) analysis, principal oscillation pattern (POP) analysis, spatio-temporal canonical correlation analysis (CCA), and space time spectral analysis. These methods are described in detail along with physical motivation, discussion of estimation issues, and practical considerations.

1. Introduction

The climate system can be described as the superposition of a set of deterministic, multivariate, and nonlinear interactions over an enormous range of spatial and temporal scales. In order to understand this system, scientists must observe, summarize, make inference, and ultimately predict its behavior at each scale of variability, as well as the interaction between these scales. Unfortunately, although the system is deterministic in principle, the collective knowledge is incomplete at each of the observation, summarization, and inference stages, and thus ultimate understanding is clouded by uncertainty. Consequently, by the time one considers the prediction phase, this lack of certainty, combined with the nonlinear dynamics of the system, contributes to what is now known as dynamical chaos. Although one is always faced with the inherent chaotic nature of the climate system, many of the relevant scientific questions can be approached from a probabilistic viewpoint, which allows useful inference to be made in the presence of uncertainty, at least for relatively large spatial scales and relatively short temporal scales. Furthermore, one is then able to look for possible associations within and between variables in the system, which may ultimately extend the still incomplete physical theory.

Central to the observation, summarization, inference, and prediction of the atmosphere/ocean system is *data.* Unfortunately, all data come bundled with error. This is an inescapable fact of scientific life. In particular, along with the obvious errors associated with the measuring, manipulating, and archiving of data, there are errors due to the discrete spatial and temporal sampling of an inherently continuous system. Consequently, there are always scales of variability that are unresolvable, and which will surely contaminate the observations. In atmospheric science, this is considered a form of "turbulence", and corresponds to the well known aliasing problem in time–series analysis and the "nugget effect" in geostatistics. Furthermore, atmospheric and oceanic data are rarely sampled at spatial or temporal locations that are optimal for the solution of a specific scientific problem. For instance, there is an obvious bias in data coverage towards areas where population density is large and, due to the cost of obtaining observations, towards a country whose Gross Domestic Product is relatively large. Thus, the location of measuring site and its temporal sampling frequency may have very little to do with science. To gain scientific insight, these uncertainties must be considered when framing scientific questions, choosing analysis techniques, and interpreting results. This task is complicated further since atmospheric and oceanic data are nearly always correlated in space and time. In this case most of the traditional statistical methods taught in introductory statistics courses (which assume independent and identically distributed errors) do not apply.

Because the physically–based deterministic models used for weather and climate prediction require "gridded" initial conditions, scientists have long been interested in

methods to estimate atmospheric variables on regular grids. This problem is exactly the spatial prediction problem addressed by "kriging". In fact, L.S. Gandin developed a complete theory of spatial best linear unbiased prediction in the context of the spatial prediction (gridding) problem in meteorology, which he called optimum interpolation (at the same time that Matheron was developing the foundations of kriging.) The meteorological spatial prediction problem has some unique features that make its application different from those in traditional geostatistics. First, the data come in space and time, effectively providing replications (although correlated in time) from which to deduce potentially nonstationary covariance structures. Furthermore, given that the physical phenomena governing the climate processes are relatively well-known (at some scales); spatial prediction must adhere to certain dynamical constraints. Thus, a substantial component of the spatial prediction problem in this discipline is devoted to including appropriate dynamic constraints. The procedure for generating dynamically consistent initial conditions for deterministic geophysical models is known as data assimilation. Current work in the area focuses on performing the spatio-temporal prediction in three spatial dimensions and time, so–called four-dimensional data assimilation. The most promising methods use variational approaches or Kalman filters. Although these efforts have traditionally been tied to operational numerical weather prediction, they are also used to develop large dynamically consistent "data" sets for climate analysis and prediction.

A key challenge in climate research is the extraction of information from the huge spatio–temporal datasets generated by the data assimilation process, deterministic model output, and raw observation networks. These data sets comprise observations of extremely complicated multivariate processes. Thus, methods of analysis must be able to account for multiscale dynamical variability across different dynamical variables in space and time, account for various sources of error, and provide efficient dimension reduction. There are several key methodologies that have proven to be essential to this task, namely, empirical orthogonal function (EOF) analysis, principal oscillation pattern (POP) analysis, spatio-temporal canonical correlation analysis (CCA), and space–time spectral analysis. In the remainder of this article, these methods are described in detail. The interested reader should note that there are several excellent reference books that have been written on statistics in the climatological sciences. A partial list is included in the Bibliography.

2. Descriptive Statistical Methods

2.1. Empirical Orthogonal Function (EOF) Analysis

EOF analysis is the geophysicist's manifestation of the classic eigenvalue / eigenvector decomposition of a correlation (or covariance) matrix. In its discrete formulation, EOF analysis is simply Principal Component Analysis (PCA), while in the continuous frame work, it is simply a Karhunen–Loeve(K-L) expansion. Depending on the application, EOFs are usually used(1) in a diagnostic mode to find principal (in terms of explanation of variance) spatial structures, along with the corresponding time variation of these structures, and (2) to reduce the dimension (spatially) in large geophysical data sets while simultaneously reducing noise.

One finds in the meteorological literature, extensive use of EOFs since their introduction by Lorenz in the mid 1950s. For example, they have been used for describing climate, comparing simulations of general circulation models, developing regression forecast techniques, weather classification, map typing, the interpretation of geophysical fields, and the simulation of random fields, particularly non-homogenous processes. In addition, as in the psychometric literature for PCAs, orthogonal and oblique rotation of EOFs often aids in the interpretation of meteorological data. Furthermore, because EOFs have difficulty resolving traveling wave disturbances, complex EOF analysis was introduced in the early 1970s and has proven to be very useful in applications to climatological analysis.

2.1.1. Continuous K-L Formulation

Consider a continuous spatial process measured at discrete time intervals. The goal is to find an optimal and separable orthogonal decomposition of a spatio-temporal process $Z(\mathbf{s};t)$, where $\mathbf{s}$ denotes a spatial location in some spatial domain in Euclidean space, and $t \in \{1, 2, ..., T\}$ denotes some time. That is, consider

$$Z(\mathbf{s};t) = \sum_{k=1}^{\infty} a_k(t)\phi(\mathbf{s})$$

such that $\mathrm{var}[a_1(t)] > \mathrm{var}[a_2(t)] > ...,$ and $\mathrm{cov}[a_i(t), a_k(t)] = 0$ for all $i \neq k$. A well known solution to this problem is obtained through a Karhunen-Loève (K-L) expansion. Suppose $\mathrm{E}[Z(\mathbf{s};t)] = 0$, and define the covariance function as $\mathrm{E}[Z(\mathbf{s};t)Z(\mathbf{r};t)] \equiv c_0^Z(\mathbf{s},\mathbf{r})$, which need not be stationary in space, but is assumed to be invariant in time. The K-L expansion then allows the covariance function to be decomposed as follows:

$$c_0^Z(\mathbf{s},\mathbf{r}) = \sum_{k=1}^{\infty} \lambda_k \phi_k(\mathbf{s})\phi_k(\mathbf{r}),$$

where $\{\phi_k(\cdot) : k = 1, ..., \infty\}$ are the eigenfunctions and $\{\lambda_k : k = 1, ..., \infty\}$ are the associated eigenvalues of the Fredholm integral equation.

$$\int_D c_0^Z(\mathbf{s},\mathbf{r})\phi_k(\mathbf{s})d\mathbf{s} = \lambda_k \phi_k(\mathbf{r}), \tag{1}$$

where

$$\int_D \phi_k(\mathbf{s})\phi_1(\mathbf{s})d\mathbf{s} = \begin{cases} 1 \text{ for } k = l \\ 0 \text{ otherwise.} \end{cases}$$

Assuming completeness of the eigenfunctions, one can expand $Z(\mathbf{s};t)$ according to

$$Z(\mathbf{s};t) = \sum_{k=1}^{\infty} a_k(t)\phi_k(\mathbf{s}), \tag{2}$$

where $\{\phi_k(\mathbf{s}) : \mathbf{s} \in D\}$ is known as the *k*-th EOF and the associated time series $a_k(t)$ is the *k*-th principal component time series, or "amplitude" time series. This time series is derived from the projection of the *Z* process onto the EOF basis,

$$a_k(t) = \int_D Z(\mathbf{s};t)\phi_k(\mathbf{s})d\mathbf{s}.$$

It is easy to verify that these time series are uncorrelated, with variance equal to the corresponding eigenvalues; that is, $\mathrm{E}[a_i(t)a_k(t)] = \delta_{ik}\lambda_k$ where δ_{ik} equals one when $i = k$ and zero otherwise.

If the expansion (2) is truncated at *K*, yielding

$$Z_K(\mathbf{s};t) \equiv \sum_{k=1}^{K} a_k(t)\phi_k(\mathbf{s}),$$

then it can be shown that the finite EOF decomposition minimizes the variance of the truncation error, $\mathrm{E}\{[Z(\mathbf{s};t) - Z_K(\mathbf{s};t)]^2\}$, and is thus optimal in this regard when compared to all other basis sets.

Since data are always discrete, in practice one must solve numerically the Fredholm integral equation (1) to obtain the EOF basis functions. For example, numerical quadrature approaches for discretizing the integral equation succeed in that they give estimates for the eigenfunctions and eigenvalues that are weighted according to the spatial distribution of the data locations, but only for the eigenfunctions at locations $\{\mathbf{s}_1,...,\mathbf{s}_n\}$ for which there are data. Alternatively, one can discretize the K-L integral equation and interpolate the eigenfunctions to locations where data are not available.

2.1.2. Discrete EOF Analysis

Although the continuous K-L representation of EOFs is the most realistic from a physical point-of–view, it is only rarely considered in applications. This is due simply to the discrete nature of data observations and the added difficulty of solving the K-L integral equation. Consider a discrete EOF analysis by using the PCA formulation as given in standard multivariate statistics textbooks, but according to the spatio-temporal notation introduced here. In that case, let $\mathbf{Z}(t) \equiv (Z(\mathbf{s}_1;t),...,Z(\mathbf{s}_n;t))'$ and define the *k*-th EOF $(k = 1,...,n)$ to be $\boldsymbol{\psi}_k \equiv (\psi_k(\mathbf{s}_1),...,\psi_k(\mathbf{s}_n))'$, where $\boldsymbol{\psi}_k$ is the vector in the linear combination: $a_k(t) = \boldsymbol{\psi}_k'\mathbf{Z}(t)$. Furthermore, $\boldsymbol{\psi}_1$ is the vector that allows $\mathrm{var}[a_1(t)]$ to be maximized subject to the constraint $\boldsymbol{\psi}'_1 \boldsymbol{\psi}_1 = 1$. Then $\boldsymbol{\psi}_2$ is the

vector that maximizes $\text{var}[a_2(t)]$ subject to the constraint $\psi'_2\psi_2 = 1$ and $\text{cov}[a_1(t), a_2(t)] = 0$. Thus, ψ_k is the vector that maximizes $\text{var}[a_k(t)]$ subject to the orthogonality constraint $\psi'_k\psi_k = 1$ and $\text{cov}[a_k(t), a_j(t)] = 0$ for all $k \neq j$. This is equivalent to solving the eigensystem $\mathbf{C}_0^Z\mathbf{\Psi} = \mathbf{\Psi}\mathbf{\Lambda}$, where $\mathbf{C}_0^Z \equiv \mathbf{E}[\mathbf{Z}(t)\mathbf{Z}(t)']$, $\mathbf{\Psi} = (\psi_1, ..., \psi_n)$, $\mathbf{\Lambda} = \text{diag}(\lambda_1 \cdots \lambda_n)$, and $\text{var}[a_i(t)] = \lambda_i$, $i = 1, ..., n$. The solution is obtained by a symmetric decomposition of $\mathbf{C}_0^Z$, given by $\mathbf{C}_0^Z = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}'$.

It is straightforward to show that if a discretization of the K-L integral equation assumes equal areas of influence for each observation location, then such a discretization is equivalent to the PCA formulation of EOFs. Conversely, an EOF decomposition of irregularly spaced data without consideration of the relative area associated with each observation location leads to improper weighting of the significance of each element of the covariance matrix $\mathbf{C}_0^Z$. This can give erroneous results in the EOF analysis. The distinction between EOFs on a regular grid and on an irregular gird is the source of many incorrect applications of the technique in the literature.

2.1.3. Estimation of EOFs

Since the EOF analysis depends on the decomposition of a covariance matrix, we must estimate this matrix in practice. The traditional approach is based on the method of moments (MOM) estimation procedure. For example, in the discrete case with equally spaced observations, one needs an estimate of $\mathbf{C}_0^Z$. The MOM estimator for an element of $\mathbf{C}_0^Z$, is given by

$$\hat{c}_0^Z\left(\mathbf{s}_i; \mathbf{s}_j\right) \equiv (1/T)\sum_{t=1}^{T}\left[Z(\mathbf{s}_i; t) - \hat{\mu}_Z\left(\mathbf{s}_i; t\right)\right]\left[Z\left(\mathbf{s}_j; t\right) - \hat{\mu}_Z\left(\mathbf{s}_j; t\right)\right],$$

where $\hat{\mu}_Z(\mathbf{s}_i; t)$ is an estimate of the mean of $Z(\mathbf{s}_i; t)$, for $i = 1, ... n$. This mean correction must be included since data usually show a nonzero mean. Typically, investigators use the estimated "time mean",

$$\hat{\mu}_Z\left(\mathbf{s}_i\right) \equiv \left(1/T\right)\sum_{t=1}^{T}Z\left(\mathbf{s}_i; t\right),$$

although other estimators can be used, depending on the application.

Given an estimate $\hat{\mathbf{C}}_0^Z$ of $\mathbf{C}_0^Z$ that is symmetric and non-negative definite (so that all eigenvalues are greater than or equal to zero), an estimate of its eigenvectors and

eigenvalues can be obtained through the diagonalization $\hat{\mathbf{C}}_0^Z = \hat{\mathbf{\Psi}}\hat{\mathbf{\Lambda}}\hat{\mathbf{\Psi}}'$. Approximate formulas for the bias and variance of the standard eigenvalue estimator can be obtained. In general, the sample eigenvalue $\hat{\lambda}_k$ is a biased estimator of λ_k and the bias is positive for the larger λ_k 's and negative for the smaller λ_k 's. Unbiased estimators can be constructed, but the decrease in bias is accompanied by an increase in the variance of the estimator. Furthermore, the sampling error associated with the estimated EOFs leads to numerical instability in the eigenvectors. This has led to sampling–based selection strategies for the truncation level, K.

2.1.4. Complex EOF Analysis

Consider a spatio-temporal process consisting of a sinusoid in one spatial dimension that is invariant in time, $Z(s;t)=B\sin(ls)$, where s is some location in one-dimensional space, t is a time index, B is an amplitude coefficient, and l is the spatial wave number, which is related to the wavelength L such that $l = 2\pi/L$. Now, consider the same sinusoid but allow it to have a temporal phase component (i.e., it can be considered as a wave in space which propagates in time):

$$Z(s;t) = B\sin(ls + \omega t) = B\cos(\omega t)\sin(ls) - B\sin(\omega t)\cos(ls), \quad (3)$$

where ω is the temporal frequency. In order to characterize the phase propagation of such a sinusoid, information is needed regarding the coefficients of the *two* components, sin *(ls)* and cos *(ls)*, which are a quarter of a cycle out of phase. In time series analysis, this is analogous to the need for both the quadrature and co-spectrum between two time series in order to determine their spectral coherence and phase relationships.

One advantage of the EOF approach described previously is its ability to compress the complicated variability of the original data set onto a relatively small set of eigenvectors. Unfortunately, such an EOF analysis only detects spatial structures that do not change position in time. To extend the EOF analysis to the study of spatial structures that can propagate in time, one can perform a complex principal component analysis in the frequency domain. The technique involves the computation of complex eigenvectors from cross-spectral matrices. The limitation of this technique is that it only gives the decomposition for individual (i.e., very narrow) frequency bands. Consequently, If the power of a phenomenon is spread over a wide frequency band (as is generally the case with physical phenomena), then several EOF spatial maps (one for each spectral estimate) are needed to evaluate the phenomenon. This complicates the physical interpretation.

Complex empirical orthogonal function (CEOF) analysis in the time domain was developed as an alternative to the frequency–domain approach described above. This method differs from the frequency–domain approach in that Hilbert transforms (see below) are used to shift the time series of the data at each location by a quarter cycle. Analogous to (3), the original data and its Hilbert transform allow the examination of propagating disturbances.

Consider $\{Z(\mathbf{s}_j;t); j = 1,...,n\}$ as described previously. Under certain regularity conditions, $Z(\mathbf{s}_j;t)$ has a Fourier representation of the form

$$Z\left(\mathbf{s}_j;t\right) = \sum_{\omega} \alpha_j(\omega)\cos(\omega t) + \beta_j(\omega)\sin(\omega t)$$

where $\alpha_j(\omega)$ and $\beta_j(\omega)$ are the Fourier coefficients, and ω is the frequency $(-\pi \leq \omega \leq \pi)$. Since the description of propagating features requires phase information, it is convenient to use the complex representation:

$$Z^f\left(\mathbf{s}_j;t\right) = \sum_{\omega} g_j(\omega)e^{-i\omega t}, \tag{4}$$

where $g_j(\omega) = \alpha_j(\omega) + i\beta_j(\omega)$. Expanding (4) gives

$$Z^f(\mathbf{s}_j;t) = Z(\mathbf{s}_j;t) + i\tilde{Z}(\mathbf{s}_j;t),$$

where

$$Z\left(\mathbf{s}_j;t\right) = \alpha_j(\omega)\cos(\omega t) + \beta_j(\omega)\sin(\omega t),$$

and,

$$\tilde{Z}\left(\mathbf{s}_j;t\right) = \beta_j(\omega)\cos(\omega t) - \alpha_j(\omega)\sin(\omega t).$$

The real part $Z(\mathbf{s}_j;t)$ is the original process and the imaginary part $\tilde{Z}(\mathbf{s}_j;t)$ is the Hilbert transform of the original process, which is just the original process with its phase shifted in time by $\pi/2$.

Now, the covariance matrix of $Z^f\left(\mathbf{s}_j;t\right)$ can be written as:

$$\mathbf{C}_0^{Zf} = \left[c_0^{Zf}\left(\mathbf{s}_j;\mathbf{s}_k\right) \right]_{j,k=1,...,n}$$

where $c_0^{Zf}(\mathbf{s}_j,\mathbf{s}_k) \equiv \mathbf{E}[Z^{f*}(\mathbf{s}_j;t)Z^f(\mathbf{s}_k;t)]$ (assuming zero mean),and where* denotes the complex conjugate. Note that $\mathbf{C}_0^{Zf}$ is essentially the cross–spectral matrix averaged over all frequencies $(-\pi \leq \omega \leq \pi)$, and thus leads to an average depiction of the propagating disturbances present in the data. If interest concerns phenomena occurring over a certain spectral frequency range of ω, then the original

process $Z(\cdot\,;t)$ and its Hilbert transform $\tilde{Z}(\cdot\,;t)$ can be filtered accordingly before the CEOF analysis.

Since $\mathbf{C}_0^{Zf}$ is Hermitian, it possesses real eigenvalues $\{\lambda_k\}$ and complex eigenvectors, $\boldsymbol{\gamma}_k \equiv (\gamma_k(\mathbf{s}_1),...,\gamma_k(\mathbf{s}_n))'$ where $k=1,...,n$. The EOF representation of $Z^f(\cdot;t)$, which optimally account for the variance of $Z(\cdot\,;t)$ in the frequency band of interest, is:

$$Z^f(\mathbf{s}_i;t) = \sum_{k=1}^{n} a_k(t)\gamma_k^*(\mathbf{s}_i),$$

where the complex time-dependent principal components are given by:

$$a_k(t) = \sum_{i=1}^{n} Z^f(\mathbf{s}_i;t)\gamma_k(\mathbf{s}_i).$$

Four measures are generally used to examine the structure of the CEOFs.

- *Spatial Phase Function*. The spatial phase function is given by :

$$\theta_k(\mathbf{s}_i) = \arctan\left[\frac{\mathrm{Im}(\gamma_k(\mathbf{s}_i))}{\mathrm{Re}(\gamma_k(\mathbf{s}_i))}\right].$$

This function can take any value between $-\pi$ and π. In the case of the simple sinusoid with temporal phase(3), this corresponds to *ls*. In that case the spatial phase will go through one complete cycle (2π) over the distance $(2\pi)/l$. Note that for data fields that include many different scales of variability, the spatial phase plot can be very difficult to interpret. Prefiltering generally improves interpretability.

- *Spatial Amplitude Function* The spatial amplitude function is given by:

$$S_k(\mathbf{s}_i) = \left[\gamma_k(\mathbf{s}_i)\gamma_k^*(\mathbf{s}_i)\right]^{1/2}.$$

This function is interpreted in the same way as the eigenfunctions in traditional EOF analysis.

- *Temporal Phase Function.* The temporal phase function is given by:

$$\xi_k\left(t\right) = \arctan\left[\frac{\text{Im}\left(a_k\left(t\right)\right)}{\text{Re}\left(a_k\left(t\right)\right)}\right].$$

Consider the simplest sinusoid example in (3). For a fixed frequency ω_0, this temporal phase function would give $\omega_0 t$ (i.e. a linear relationship in time). In practice, this provides information as to the frequency of the dominant component of a particular eigenvector at a given time.

- *Temporal Amplitude Function*. The temporal amplitude function is given by:

$$R_k\left(t\right) = \left[a_k\left(t\right)a_k^*\left(t\right)\right]^{1/2}.$$

This function corresponds to the amplitude time series as given in traditional EOF analysis.

2.1.5. Multivariate EOF Analysis

Often, one may be interested in the simultaneous analysis of two or more processes. Consider two fields observed over time at the same spatial locations; that is, consider $Z(\mathbf{s}_i;t)$ and $X(\mathbf{s}_i;t)$, where $i=1,...,n;\ t=1,...,T$. Then, write $\mathbf{W}(t) \equiv [\mathbf{Z}(t)'\ \mathbf{X}(t)']'$ where $\mathbf{X}(t) \equiv (X(\mathbf{s}_1;t),...,X(\mathbf{s}_n;t))'$, $\mathbf{Z}(t)$ is defined as before, and the covariance matrix of $\mathbf{W}(t)$ is given by $\mathbf{C}_0^W$. This matrix includes off-diagonal submatrices that represent the covariance between $\mathbf{Z}(t)$ and $\mathbf{X}(t)$. One can then obtain the EOF solution in the conventional manner by diagonalizing the $\mathbf{C}_0^W$ matrix; that is $\mathbf{C}_0^W = \mathbf{\Psi}_W \mathbf{\Lambda}_W \mathbf{\Psi}_W'$, where the columns of $\mathbf{\Psi}_W$ are the eigenvectors (i.,e ., EOF s) and $\mathbf{\Lambda}_W$ is a diagonal matrix containing the eigenvalues of $\mathbf{C}_0^W$. Then, the first n elements of the k-th eigenvector correspond to the portion of the k-th EOF for the Z process, and the last n elements correspond to the portion representative of the k-th EOF of the X process. Theoretically, there is no limit to the number of processes that can be considered simultaneously. However, there is a practical limitation to this procedure since the covariance matrix can easily become very large if the number of observation locations or variables increases. Comparisons of this approach to other multivariate methods such as CCA suggests that in some cases the multivariate EOF approach has large biases and does not perform well in small signal-to-noise ratio situations.

2.1.6. Extended EOF Analysis

Extended EOFs are simply multivariate EOFs in which the additional variables are lagged versions of the same process. For example, we could let $\mathbf{W}(t) = [\mathbf{Z}(t)'\ \mathbf{Z}(t-1)']'$. In this case, if temporal invariance is assumed, then the

diagonal sub-matrices of $\mathbf{C}_0^W$ are equivalent, and the off-diagonal submatrices are just the lag-one correlation matrices $\mathbf{C}_1^{\mathbf{Z}} \equiv \mathrm{cov}[\mathbf{Z}(t), \mathbf{Z}(t-1)]$. In this way, one can examine the propagation of EOF spatial patterns in time by noting that the first *n* eigenvalues of a particular eigenvector correspond to the time zero representation of that EOF, and the second *n* eigenvalues correspond to the lag one representation of the same EOF. This approach is closely linked with time-lagged CCA and the minimum /maximum autocorrelation factor (MAF) method in statistics.

-
-
-

TO ACCESS ALL THE **23 PAGES** OF THIS CHAPTER,
Visit: http://www.eolss.net/Eolss-sampleAllChapter.aspx

Bibliography

Bretherton C.S., Smith C. and Wallace J.M. (1992). An intercomparison of methods for finding coupled patterns in climate data. *Journal of Climate* **5,** 541-560. [This paper presents a comprehensive overview of various space-time methods used in climatology.]

Daley, R. (1991). *Atmospheric Data Analysis*, Cambridge: Cambridge University Press. [This book presents essential aspects of meteorological data assimilation and optimal spatial prediction]

Freiberger W., and Grenander, U. (1965). On the formulation of statistical meteorology. *Review of the International Statistical Institute* **33,** 59-86. [A nice early review of statistics in meteorology]

Ghil M., Cohn S.E., Tavantzis J., Bube K., and Isaacson, E.(1981) Application of estimation theory to numerical weather prediction . *Dynamics meteorology: Data Assimilation Methods*, L. Bengtsson, M. Ghil, and E. Källén, eds., New York: Springer–verlag 139-224. [Seminal paper describing the use of Kalman filters in the data assimilation process.]

Haslett J. (1989) Space time modeling in meteorology – A review. *Bulletin International Statistics Institute* **51**, 229-246. [A nice review of spatio-temporal statistical methods in meteorology, focused mainly on spatial predication.]

Hasselmann K. (1988) PIPs and POPs: The reduction of complex dynamical systems using principal interaction and oscillation patterns. *Journal of Geophysical Research* **93**, 11015-11021. [Seminal work describing principal oscillation pattern analysis in a general context.]

Hayashi Y. (1982) Space-time spectral analysis and its applications to atmospheric waves. *Journal Meteorological Society of Japan* **60,** 156-171. [Definitive paper on space-time spectral analysis and the assumptions relative to atmospheric waves]

Preisendorfer R.W. (1988) *Principal Component Analysis in Meteorology and Oceanography*, Amsterdam :Elsevier. [Definitive work on principal component analysis (EOFs) in the geosciences.]

Thiébaux H.J., and Pedder M.A (1987) *Spatial Objective Analysis with Applications in Atmospheric Science,* London: Academic Press. [Classical book describing optimal spatial prediction and data assimilation in the atmospheric sciences]

Von Storch H., Bürger G., Schnur R., and von Storch J.-S. (1995) Principal oscillation patterns: a review.

Journal of Climate **8,** 377-400. [Excellent review paper describing principal oscillation patterns]

von Storch H., and Zwiers .F.W. (1999) *Statistical Analysis in Climate Research*, Cambridge: Cambridge University Press. [Definitive textbook on statistical methods in climatology]

Wikle C.K ., Berliner L.M. and Cressie N. (1998) Hierarchical Bayesian space-time methods, *Environmental and Ecological Statistics* **5**, 117-154.[Seminal paper describing hierarchical Bayesian spatio–temporal models in climate application.]

Wilks D.S. (1995) *Statistical Methods in the Atmospheric Sciences*, San Diego: Academic Press. [Excellent introductory text book describing statistical methods used in atmospheric science]

Biographical Sketch

Christopher K. Wikle is an associate professor of statistics at the University of Missouri - Columbia. His Ph.D. is in both Atmospheric Science and Statistics. He has worked as an air pollution consultant in industry and as a visiting scientist at the National Center for Atmospheric Research in Boulder, Colorado. His primary research interests are in spatio-temporal modeling, hierarchical Bayesian methods, the introduction of physical information into stochastic models, statistical design of environmental monitoring networks, climate dynamics, turbulence, atmospheric waves, and the application of statistics to geophysical and environmental processes.