MATHEMATICAL MODELS OF PUBLIC HEALTH POLICY

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Contents

Summary

Models come to assist us in anticipating the consequences of policy implementation, and thereby help us make optimal choices with minimal damage. It is important to make "the whole" the object of our analysis and take complexity into account. For this purpose various mathematical methodologies have been developed. They mostly deal with more than one variable, since Health Systems are made of a few components, one of which might be the intervener as well.

The possible methods include systems of difference equations, differential equations, cellular automata for understanding and monitoring spatial dynamics, loop analysis as a central method for systems in equilibrium, game theory and Markov processes. A whole spectrum of public health problems can be approached with mathematical models and their analysis, as presented through examples on infectious diseases, regulation of toxic pollutants, and the process of public use of health services.

1. Introduction

Models are constructed to assist us in making decisions. Therefore the first question is: who are the “we” who are making models, why, and for whom, and what sort of decisions? Most policy modeling is done on behalf of governments, their subdivisions, or private organizations such as corporations. The modelers may themselves be employees or consultants of the policy making entity. Often the first question to ask is, is the modeling problem ethically acceptable? That is, is it ethical to accept the objectives of the policy or the side conditions constraining the domain of possible solutions? It is not within the scope of this chapter to propose any specific answer to this question, but only to suggest some criteria. Nor will it address issues of cost; the considerations are purely those related to benefit health of populations.

In its simplest terms, “policy” means the allocation of resources or the imposition of some regulations or constraints to achieve a desired end. It is sometimes so straightforward that the problem itself determines the action to take, and the analysis of “policy” is not needed. Most daily actions are of this sort. If a baby needs changing, we change without any complicated analysis. But with even a little bit of complexity in a problem, decision making is not that obvious. Mathematical models and their analyses come to assist us in clarifying the consequences of policies when many factors are involved; they link various variables involved into a system that lends itself to qualitative and quantitative analysis.

2. Posing the Question and Design of the Answer

A question must be posed wide enough to accommodate an answer that makes the eco-social system as the object of study based on the inseparability of social, biological and physical phenomena in determining health and disease.

In contrast to the reductionist approach, this chapter suggests that it is important to analyze “the whole”. Reduction assumes that to understand the whole it is sufficient to describe as completely as possible the smallest parts and their direct connections. This has its advantages as it has been a highly successful tactic in the small, where the detailed knowledge of the parts really is sufficient, such as in the identification of molecules has having certain specific effects; but failure to see its limitations has led to disasters when the leap is made from physiological facts (e.g. pesticides kill bugs in bottles) to ecological or social claims (therefore application of pesticides will control the bugs in the field or the pesticide-seed-fertilizer-mechanization package will improve the lives of third world farmers and protect national economies).

“In the early 1950’s, the Dayak people of Borneo suffered a malarial outbreak. The World Health Organisation (WHO) had a solution: to spray large amounts of DDT to kill the mosquitoes that carried the malaria. The mosquitoes died; the malaria declined; so far so good. But there were unexpected side effects. Amongst the first was that the roofs of the people’s houses began to fall down on their heads. It seemed that the DDT had also killed a parasitic wasp which had previously controlled thatch-eating caterpillars. Worse, the DDT-poisoned insects were eaten by geckoes, which were eaten by cats. The cats started to die, the rats flourished, and the people were threatened by outbreaks of typhus and plague. To cope with these problems, which it had itself created, the WHO was obliged to parachute 14 000 live cats into Borneo. Operation Cat

Drop, now almost forgotten at the WHO, is a graphic illustration of the interconnectedness of life, and of the fact that the root of problems often stems from their purported solutions. (Quoted in Rachel Wynberg and Christine Jardine, *Biotechnology and Biodiversity: Key Policy Issues for South Africa,* 2000)"

Indeed, failures to take complexity into account (too narrow a time horizon or disciplinary scope) in large scale interventions, may lead to devastating results. Many of the disasters of applied science and technology have been of this sort: pesticides create new pests, antibiotics stimulate the evolution of new pathogens, and hospitals become the foci of infection, straightening rivers increases flood damage, economic growth exacerbates inequality and dependence. ((Levins 1995(a) and Levins 1995(b), (Levins 1998), Awerbuch, Kiszewsky and Levins 2002). Indeed an interdisciplinary approach drawn from the fields of evolution, biogeography, ecology, climatology, behavioral and social sciences is needed (Awerbuch 1994, Awerbuch et al. 1996, McMichael 1997, Levins 1995 (a) and (b), Martens and Rotmans 2000). And if the results are not devastating at times, they may be risky in terms of human health and economically costly, as in the case of attempting to control the spread of Lyme disease in northern Massachusetts (USA) by destroying the deer, the main host of the tick that transmits the disease agent. The program was carried out over a period of a few years during the nineteen-eighties. While the major deer population was destroyed the disease kept growing within the local human population. A mathematical model could have assisted policy makers in predicting that destroying deer and leaving enough of them, above the threshold needed to support the tick population, would not control the disease (Awerbuch and Spielman 1994)

Mathematical models come to assist us in the analysis of complexity by enabling us to link variables from different disciplines and explore their joint dynamics in the short and long run. An example is the system of difference equations used to explore the dynamical relationship between community consciousness and the habitats of mosquitoes that transmit dengue fever.

The breeding sites of urban mosquitoes are formed when people pollute the environment with vases, old tires, cans and other containers which are filled during the rainy season with water, where mosquitoes can lay their eggs. Breeding sites are often removed by public action, which requires not only awareness but also organization. Individual behavior determines the birth rate of breeding sites while collective action predominates in the removal or death rate. This can be expressed as a mathematical model in the form of a system of difference equations:

$$H_{n+1} = a\, H_n e^{-pX_n} + b\, e^{-qX_n} \tag{1a}$$

$$X_{n+1} = c\, X_n + d\, H_n \tag{1b}$$

Where H (for habitats) is the number of breeding sites which change from week to week; and X the level of consciousness which is created when there are many breeding sites observed but also erodes at a rate that is a sociological parameter, a measure of the "half-life" of discipline. a is the fraction of breeding sites that survive from one week to the next and b is the rate of creation of new ones. Both terms are low when awareness is high. The parameters p, measure how sensitive the change in breeding sites is to community awareness (prompting campaigns to clean up the environment) and q to individual consciousness (prompting individuals to not pollute the environment).

Consciousness at time n, X_n, depends on c, the fraction of the consciousness that persists from one week to the next. It is inversely a measure of the erosion rate and in the absence of new breeding sites. The parameter d measures the sensitivity of consciousness to the presence of breeding sites. The pair of difference equations: (1a) and (1b) may be analyzed mathematically for properties such as stability and oscillation. This will enable us to explore the dynamics of prevalence/consciousness systems as affected by the range of parameters and plan educational programs to minimize mosquito breeding sites.

Our purpose here is to demonstrate the possibility of integrating sociological variables into vector epidemiology and explore their joint dynamics.

Simulation studies show that with a low level of community organization (p=0.0001), individual awareness might erode in an oscillatory manner following the number of breeding sites; as shown in Figure 1, the system may show long term oscillations for some parameter values (H_0=500, X_0=100, a=0.5, b=300, c=0.1, d=0.6, q=0.1

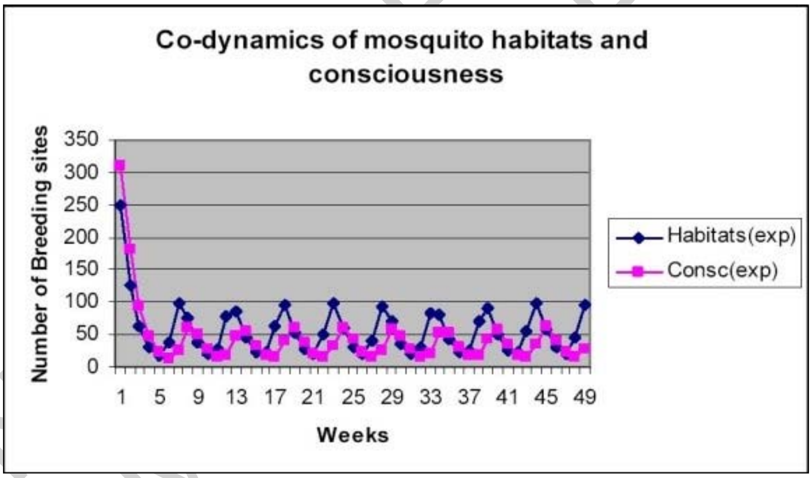

Figure 1: Co-dynamics of mosquito habitats and consciousness (for parameter values: H_0=500, X_0=100, a=0.5, b=300, c=0.1, d=0.6, p=0.0001, q=0.1)

The dynamics of prevalence/community-activity systems might however change dramatically if consciousness is built up over years of education in a political system that encourages community responsibility such as in the case of Cuba.

In such a case the equations can be modified to include a constant g as follows:

$$H_{n+1} = a\, H_n e^{-pX_n} + b\, e^{-qX_n} \qquad (1c)$$

$$X_{n+1} = c\, X_n + d\, H_n + g \qquad (1d)$$

For a value of *g*=20 for example (which is 20%) of the initial value of X_0) the system is stabilized at an equilibrium level of 19 breeding sites as presented in Figure 2:

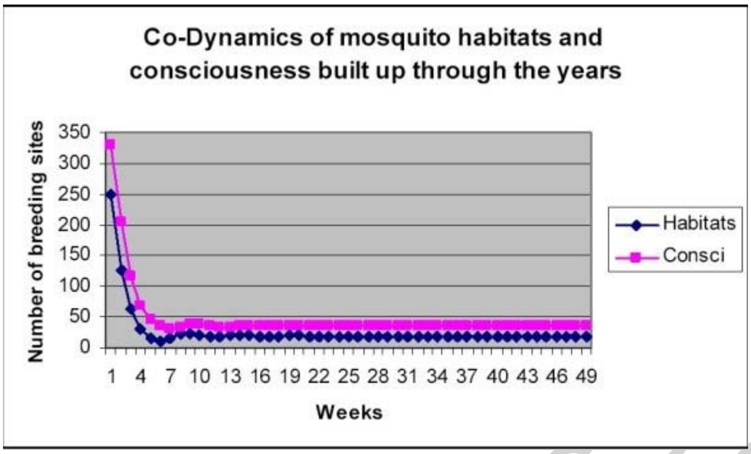

Figure 2: Co-dynamics of mosquito habitats and consciousness built up through the years (for parameter values: H_0=500, X_0=100, *a*=0.5, *b*=300, *c*=0.1, *d*=0.6, *p*=0.0001, *q*=0.1, *g*=20)

If community consciousness is sensitive as well, at *p*=0.1 and *q*=0.1, based on Eqs. (1a) and (1b), an equilibrium level of both consciousness and breeding sites is possible to achieve.

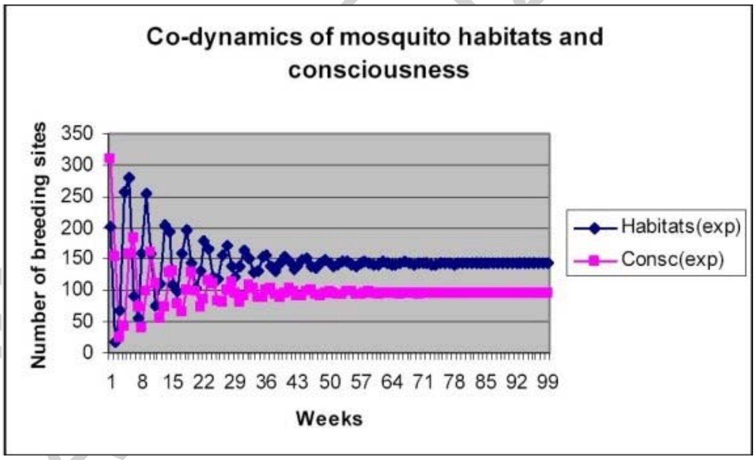

Figure 3: Co-dynamics of mosquito habitats and consciousness (for parameter values: H_0=500, X_0=100, *a*=0.5, *b*=300, *c*=0.1, *d*=0.6, *p*=0.0001, *p*=0.1, *q*=0.1)

The addition of constant education such as when for example g=20, using Eqs. (1c) and (1d) the equilibrium for the number of breeding sites is lowered from about 150 to 125. The results of this analysis identify the persistence parameters as targets of intervention.

When implementing an intervention it is important to assess the long term consequences that have an impact on various aspects of the human environment, as any change percolates through a complex network of physical, biological and social interactions that affect health; the change may feed-back and feed-forwards in a counterintuitive manner, and its effect can show up at points far removed from the original entry into the system;

in addition the results of the intervention we expect to see may be reversed. Sometimes the immediate effect of a change is different from the long term effect; sometimes the local changes may be different from the region-wide alterations. The same may have quite different effects in different places or times. It is important therefore, to develop new models or modify old ones to guide policies for particular situations in place and time.

The study of the consequences of policy implementation is a study of the short- and long term dynamics of complex systems, a domain where our common sense intuitions are often unreliable and new intuitions have to be developed in order to make sense of often paradoxical results. Thus mathematics of complexity has to be studied as an object of interest in its own right (Levins 1973, Pattee 1973, Puccia and Levins 1985) and developed as a tool for policy makers.

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Biographical Sketches

Dr. Tamara Awerbuch, Lecturer, Harvard School of Public Health (HSPH). Tamara Awerbuch holds a Ph.D. from MIT, and for the last two decades she has been working at the HSPH as a health scientist and biomathematician whose main interests focus on bio-social interactions that cause disease. She has been conducting research on the conditions that lead to the emergence, maintenance, and spread of epidemics. She presented her work in many international conferences and at the Isaac Newton Institute of Mathematical Sciences in Cambridge, England where she was invited to participate in the Program on Models of Epidemics. She is a founding member of the New and Resurgent Disease Group at Harvard, and established international collaborations such as with Israeli, Cuban and Brazilian scientists. She was a co-investigator in the project: "Why New and Resurgent Diseases Caught Public Health by Surprise and a Strategy to Prevent This," supported by the Robert Wood Johnson Foundation. Some of her research papers were the result of collaboration with students through the course Mathematical Models in Biology. She is interested in Public Health education and developed educational software for adolescents based on models for determining the probability that an individual will become infected with HIV given certain sexual behaviors.

Dr. Richard Levins, John Rock Professor of Population Sciences, Harvard School of Public Health (HSPH).Richard Levins studied plant breeding and mathematics at Cornell University, farmed in Puerto Rico and obtained his doctorate in zoology from Columbia University. He has taught at the University of Puerto Rico and the University of Chicago before coming to his present position as John Rock Professor of Population Sciences at the Harvard School of Public Health. Levins is currently on the Advisory Board of the International Society for Ecosystem Health and is a member of the American Academy of Arts and Sciences. He has received awards as a pioneer of the ecology movement of Puerto Rico, for his long term contributions to the development of ecological agriculture in Cuba, and the Edinburgh Science Medal (Scotland) for contributions to science and the broader society. He has received awards as a pioneer of the ecology movement in Puerto Rico, for long term contributions to the development of agricultural ecology in Cuba, the Edinburgh Science Medal(Scotland) for contributions to science and the broader society, the Lukacs 21st Century Award for contributions to statistical and mathematical ecology, and an honorary doctorate in environmental science from the University of Havana. His theoretical interests have been applied to problems of community development as part of the Board of Directors of OXFAM-America and chair of their subcommittee on Latin America and the Caribbean from 1989 to 1995. Working from a critique of the industrial-commercial pathway of development, he promoted alternative development pathways that emphasize economic viability with equity, ecological and social sustainability and empowerment of the dispossessed. As part of the New World Agriculture and Ecology Group, he has helped to develop modern agroecology, concentrating on the whole-system approaches to gentle pest management. The "Dialectical Biologist," co-authored with Richard Lewontin, presented the authors' approach to the study of the philosophy, sociology and history of science.