DECISION TREES AND INFLUENCE DIAGRAMS

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Summary

This chapter describes decision trees and influence diagrams. We start with a small decision problem called Medical Diagnosis. Next we describe the decision tree representation and solution technique and illustrate it using the Medical Diagnosis problem. Then we state some strengths and weaknesses of the decision tree representation and solution technique. Next we describe the influence diagram representation and solution technique and illustrate it using the Medical Diagnosis problem. Then we describe some strengths and weaknesses of the influence diagram representation technique. Finally we conclude with a summary.

1. Introduction

The main goal of this chapter is to describe decision trees and influence diagrams, both of which are formal mathematical techniques for representing and solving one-person decision problems under uncertainty. Decision trees have their genesis in the pioneering work of von Neumann and Morgenstern (1944) on extensive form games. Decision trees graphically depict all possible scenarios. The decision tree representation allows computation of an optimal strategy by the backward recursion method of dynamic programming. Howard Raiffa (1968) calls the dynamic programming method for solving decision trees “averaging out and folding back.”

Influence diagram is another method for representing and solving decision problems. Influence diagrams were initially proposed by Ron Howard and Jim Matheson (1981) as a method only for representing decision problems. The motivation behind the formulation of influence diagrams was to find a method for representing decision problems without any preprocessing. Subsequently, Scott Olmsted (1983) and Ross Shachter (1986) devised methods for solving influence diagrams directly, without first having to convert influence diagrams to decision trees. In the last decade, influence diagrams have become popular for representing and solving decision problems.

2. A Medical Diagnosis Problem

In this section, we will state a simple symmetric decision problem that involves Bayesian revision of probabilities (a decision problem is said to be *asymmetric* if there exists a decision tree representation such that the number of scenarios in the representation is less than the product of the cardinalities of the state spaces of the decision and chance variables in the representation, and a decision problem is said to be *symmetric* if it is not asymmetric). This will enable us to show the strengths and weaknesses of the various methods for such problems. A physician is trying to decide on a policy for treating patients suspected of suffering from a disease *D*. *D* causes a pathological state *P* that in turn causes symptom *S* to be exhibited. The physician first observes whether or not a patient is exhibiting symptom *S*. Based on this observation, he/she either treats the patient (for *D* and *P*) or not. The physician's utility function depends on his/her decision to treat or not, the presence or absence of disease *D*, and the presence or absence of pathological state *P*. The prior probability of disease *D* is 10%. For patients known to suffer from *D*, 80% suffer from pathological state *P*. On the other hand, for patients known not to suffer from *D*, 15% suffer from *P*. For patients known to suffer from *P*, 70% exhibit symptom *S*. And for patients known not to suffer from *P*, 20% exhibit symptom *S*. We assume *D* and *S* are probabilistically conditionally independent given *P*. Table 1 shows the physician's utility function.

Physician's Utilities (υ)		**States**			
		Has pathological state (p)		**No pathological state ($\sim p$)**	
		Has disease (d)	**No disease ($\sim d$)**	**Has disease (d)**	**No disease ($\sim d$)**
	Treat (t)	10	6	8	4
Acts					
	Not treat ($\sim t$)	0	2	1	10

Table 1: The physician's utility function for all act–state pairs

3. Decision Trees

In this section, we describe a decision tree representation and solution of the Medical Diagnosis problem. Also, we describe the strengths and weaknesses of the decision tree representation and solution techniques.

3.1. Decision Tree Representation

Figure 1 shows the preprocessing of probabilities that has to be done before we can complete a decision tree representation of the Medical Diagnosis problem. In the probability tree on the left, we compute the joint probability distribution by multiplying the conditionals. For example,

$$\Pr(d, p, s) = \Pr(d)\ \Pr(p|d)\ \Pr(s|p) = (0.10)(0.80)(0.70) = 0.0560. \qquad (1)$$

In the probability tree on the right, we compute the desired conditionals by additions and divisions. For example,

$$\begin{aligned}\Pr(s) &= \Pr(s, p, d) + \Pr(s, p, \sim d) + \Pr(s, \sim p, d) + \Pr(s, \sim p, \sim d)\\ &= 0.0560 + 0.0945 + 0.0040 + 0.1530 = 0.3075, \qquad (2)\\ \Pr(p|s) &= \frac{\Pr(s, p)}{\Pr(s)} = \frac{\Pr(s, p, d) + \Pr(s, p, \sim d)}{\Pr(s)} = \frac{0.0560 + 0.0945}{0.3075} = 0.4894,\end{aligned}$$

and

$$\Pr(d|s, p) = \frac{\Pr(s, p, d)}{\Pr(s, p)} = \frac{\Pr(s, p, d)}{\Pr(s, p, d) + \Pr(s, p, \sim d)} = \frac{0.0560}{0.0560 + 0.0945} = 0.3721. \qquad (3)$$

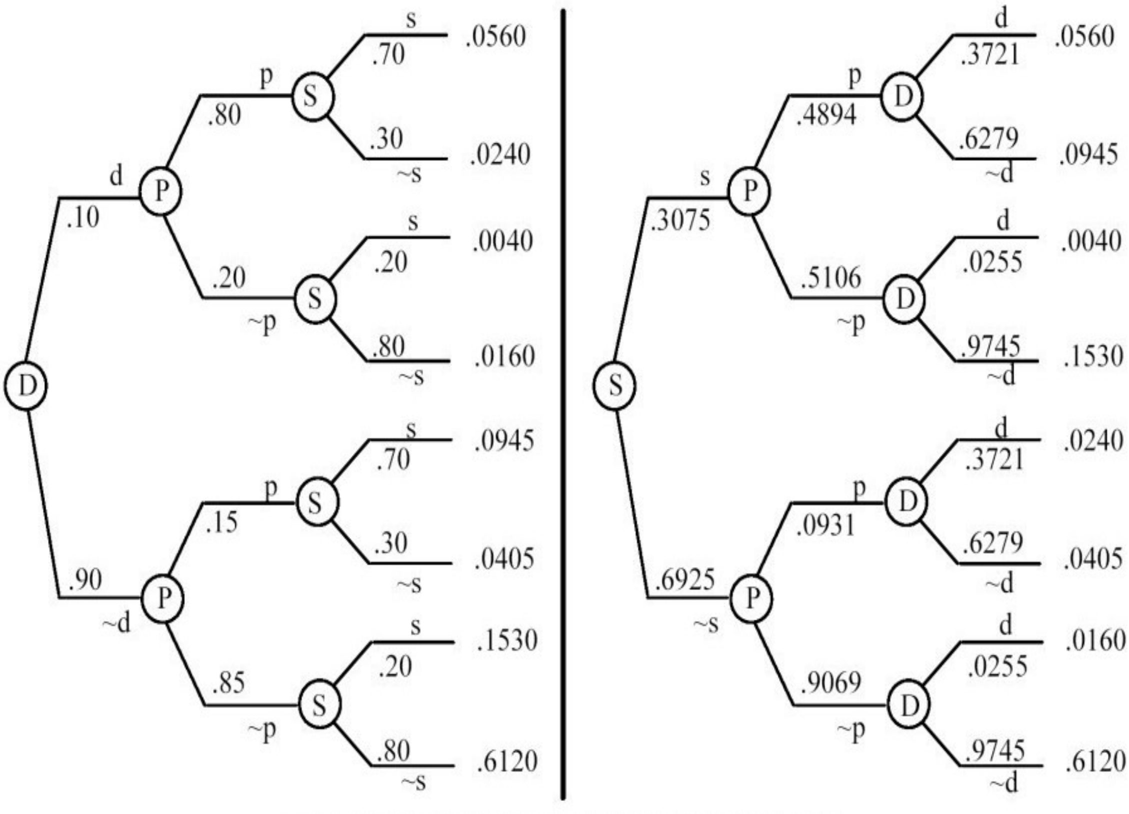

Figure 1: The preprocessing of probabilities in the Medical Diagnosis problem

Figure 2 shows a complete decision tree representation of the Medical Diagnosis problem. Each path from the root node to a leaf node represents a *scenario*. This tree has 16 scenarios. A decision problem is said to be *asymmetric* if there exists a decision tree representation such that the number of scenarios in the decision tree representation is less than the product of the cardinalities of the states spaces of the chance and decision variables in the problem. The *Medical Diagnosis* problem is symmetric since the number of scenarios is $16 = |\Theta_S||\Theta_T||\Theta_P||\Theta_D| = 2 \cdot 2 \cdot 2 \cdot 2$ (Θ_S denotes the set of all possible values of S and $|\Theta_S|$ denotes its cardinality).

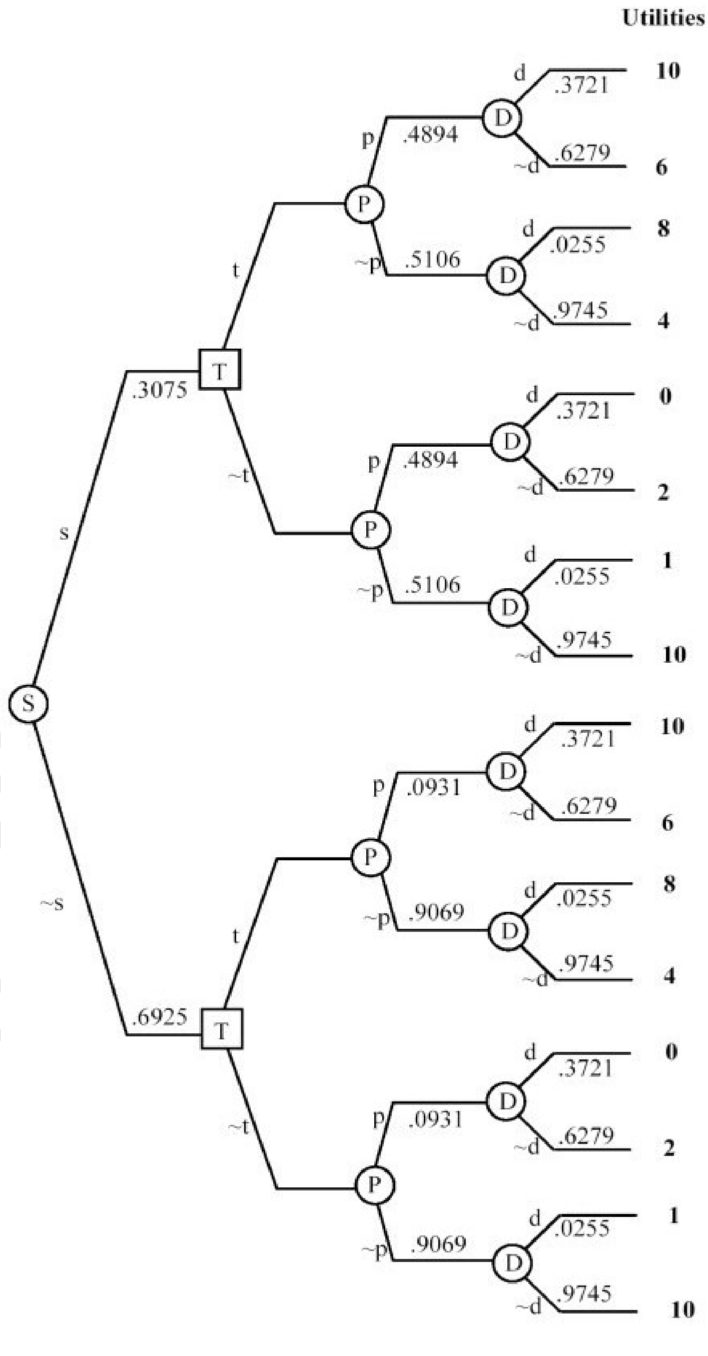

Figure 2: A decision tree representation of the Medical Diagnosis problem

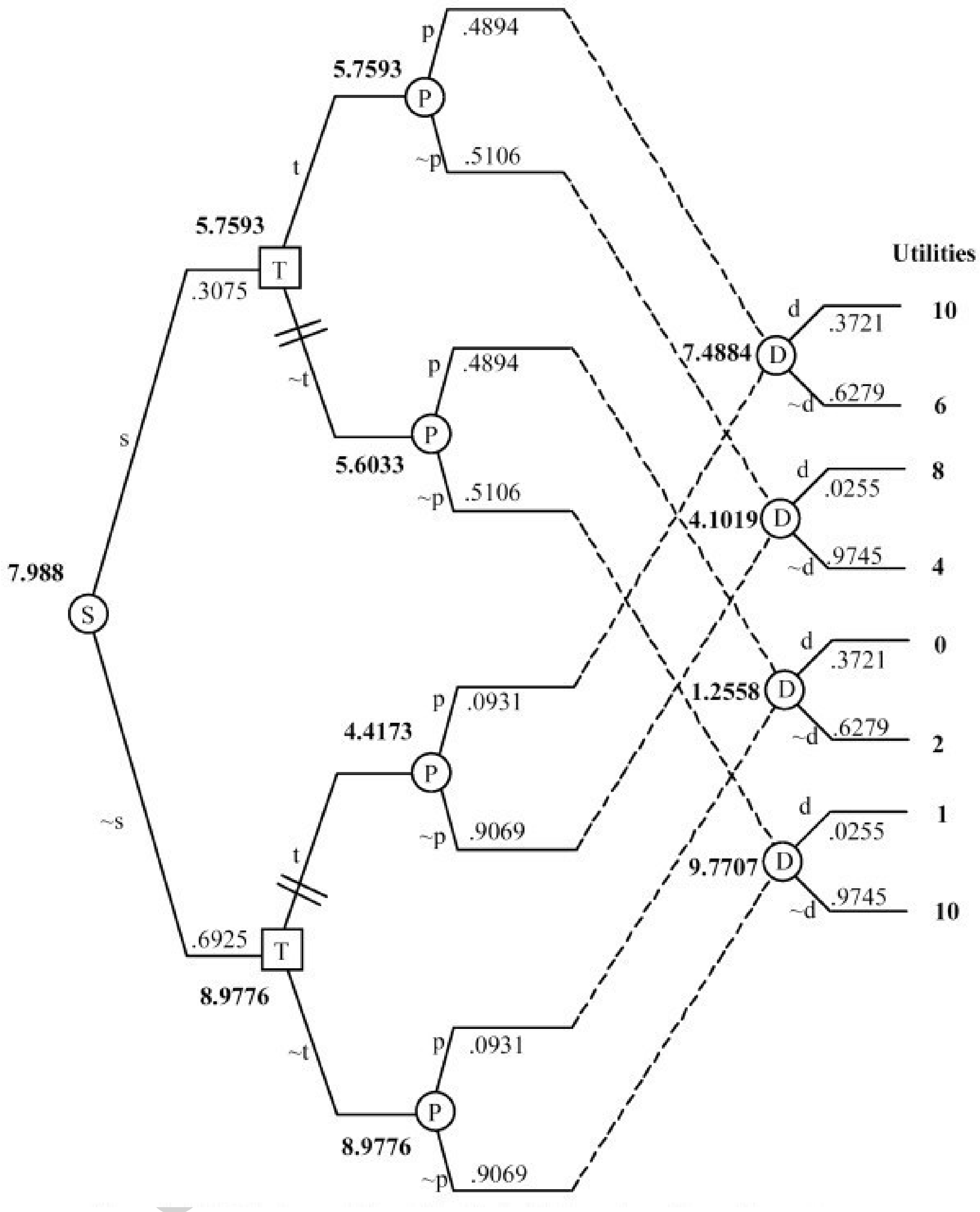

Figure 3: A decision tree solution of the Medical Diagnosis problem using coalescence

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Biographical Sketch

Prakash P. Shenoy is the Ronald G. Harper Distinguished Professor of Artificial Intelligence in Business, University of Kansas at Lawrence. He received a B.Tech.. in Mechanical Engineering from the Indian Institute of Technology, Bombay, India, in 1973, and an M.S. and a Ph.D. in Operations Research from Cornell University in 1975 and 1977 respectively. His research interests are in the areas of artificial intelligence and decision sciences. He has published many articles on management of uncertainty in expert systems, decision analysis, and the mathematical theory of games. His articles have appeared in journals such as *Operations Research, Management Science*, *Artificial Intelligence*, and *International Journal of Approximate Reasoning*. He has received several research grants from the Database and Expert Systems (DES), and Decision, Risk and Management Science (DRMS) programs of the National Science Foundation, the Research Opportunities in Auditing program of the Peat Marwick Main Foundation, the Higher Education Academic Development Donations program of Apple Computer, Inc., and the Information Sciences Department of Hughes Research Laboratories. He served as the North-American editor of *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, and as an associate editor of *Operations Research*, *Management Science*, and *International Journal of Approximate Reasoning*, and as an ad-hoc referee for over 30 journals and conferences in Artificial Intelligence and Management Science/Operations Research. He served as Program Co-Chair of the *Thirteenth Conference on Uncertainty in Artificial Intelligence* held at Brown University, Providence, 1997, and as Conference Chair of the *Fourteenth Conference on Uncertainty in Artificial Intelligence* held at University of Wisconsin-Madison in 1998.