HISTORY AND PEDAGOGY OF MATHEMATICS IN FRANCE

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Contents

Summary

Although interest in the history of mathematics has continued and even expanded during the last forty years in France, it has had various motives and taken various forms. In particular, the benefits anticipated in the 1970s differ to some extent from those expected in 2010. This will be illustrated by the training and the material initiated in France by the Institutes for Research into the Teaching of Mathematics (IREMs) during the period 1975-2010.

The National Commission inter-IREM (CII-IREM) "Epistemology and history of mathematics" provides a framework for the epistemological and historical activities of these Institutes. The first training courses run by the IREMs and the resulting publications soon had practical repercussions. In fact, from the 1980s, textbooks in junior secondary schools (pupils aged 11-15) and senior secondary schools (pupils aged 15-18) began to include historical elements while secondary school syllabuses referred to the value of a historical approach.

The benefits of history to teaching are linked not only to the content of the teaching but also to the concerns of teachers. From this point of view, the history, which interests teachers, has three attributes: reorienting, epistemological and cultural (Barbin 1997).

We will present these various benefits, which have been developed in the different actions implemented in France since the 1980s. We will examine the contributions of history of mathematics in teachers training.

Next, we will explain what we call an "introduction of a historical perspective" in mathematics teaching and we will report some experiments of mathematics teachers, who propose to read ancient texts of mathematics in classroom.

1. End of the 1970: A Therapy against Dogmatism

1.1. The Reform of "Modern Mathematics"

The 1970s was the age of the "Modern Mathematics" Reform. The instigators of this reform denounced the historical style of previous teaching, claiming that it did not provide a unified concept of mathematics (Barbin, 1987). According to the terminology of that time, the Reform gave the latest "show" of mathematics. However, its implementation was soon disputed, particularly in the IREMs, both because it presented mathematics as a language and because mathematics had become a "discipline of selection".

For the teachers, historical research was thus "a therapy against dogmatism, a resource to enable them to acquire and master their knowledge better". We stated that "for the pupils, it paved the way for mathematics to stop playing the role of the cold-hearted monster which standardizes, judges and condemns, and to regain its status as a cultural activity inseparable from other human practices" (IREM, 1982). Mathematics should no longer be seen as a finished product but as a historical process, nor should it be understood as a language but as an intellectual activity (Barbin, 1989).

The Institutes for Research into the Teaching of Mathematics (IREMs) were created at the request of the instigators of the Modern Mathematics Reform, particularly the Association of Mathematics Teachers in State Education (APMEP), at the end of the 1960s. They were set up following the introduction of the Reform at the beginning of 1970. They are University Research Institutes, dependent on a University and linked to the Mathematics Department of that University. There is usually one Institute per Academy, which is the local education authority of the State Education system in France. From the beginning, the IREMs had four main objectives: pedagogical research into mathematics teaching, in-service teacher training, participation in the initial training of teachers, and the production of teaching material. It should be noted that mathematics is the only discipline that has such Institutes. Teachers of other subjects, such as philosophy and geography, have called for similar organizations, but without success. One of the first tasks of the IREMs was to "recycle" mathematics teachers by training them in the "modern mathematics" that they were required to teach. Hundreds of teachers were involved, so the training was part of their service. The trainers were university or secondary school teachers who were thus seconded from some of their teaching duties.

1.2. The IREMs: Centers of Research and Training

Before long, the members of the IREMs criticized the way in which the Reform had been implemented. They suggested other types of training, which corresponded more to pedagogical research, such as teaching through project work, and interdisciplinary

studies, like mathematics and language or mathematics and physical sciences. This led the directors of the IREMs to set up National Inter-IREM Commissions, bringing together members of different Institutes who were working on the same topics. The national structure was strengthened by the organization of national conferences, which enabled research projects to be coordinated and supported.

In 1990 and 1991, the Ministry of Education created the University Institutes for Teacher Training (IUFM), to be responsible for training secondary school teachers of all subjects, as well as educational advisers. Their legal status was that of a public administrative body. In 2005, they became a department within a University.

The CII-IREM Commission "Epistemology and history of mathematics" was set up in May 1975 on the initiative of Jean-Louis Ovaert and Christian Houzel. Half the participants are senior secondary school teachers while the other half is of junior secondary school teachers and academics. Although the majority are mathematics teachers, there are also philosophy and physical science teachers, as well as researchers into the history of science. They work in the various IREMs or IUFMs, where they organize education and training courses. Most of them belong to a research group in an IREM.

This Commission meets two or three times a year in Paris and organizes a conference every two years in a provincial city. The topics of these conferences correspond to current interests or work in progress. The 20th National CII-IREM conference will be held in May 2013 on "Mediterranean mathematics". The proceedings and other material from each conference are published. The Commission also publishes books and anthologies. In July 1993, it organized the first European Summer University (ESU) in Montpellier, which attracted 244 participants from 29 countries.

As a reaction against the selective role of mathematics, the IREMs began to discuss the relationship between mathematics and society, organizing three conferences on this topic at the end of the 1970s. We summarized the benefits of the history of mathematics in 1982, writing that "from the historian's point of view […], mathematics is by no means a dead subject but one full of life; engaged in intra- and extra-mathematical research; inseparable from problems in astronomy, physics, optics, technology and artistic creation; caught up in philosophical and theological controversies; confronted by authorities and institutions" (IREM, 1982).

The first national Inter-IREM conference, organized by the IREM of Caen in 1977, concerned the introduction of a historical perspective into the teaching of mathematics. However, this introduction was envisaged only after teachers had been trained. For this purpose and on the occasion of the International Congress on Mathematical Education (ICME) at Berkeley in 1980, we gathered together articles from different IREMs in a work entitled *Mélanges*.

2. Since 1980: other Types of Panacea for the History of Mathematics

After the Reform was abandoned, several successive "new syllabuses" were introduced for the teaching of mathematics but, unlike the Modern Mathematics Reform, these

were not guided by an overall plan. On the contrary, they were the result of various eliminations and additions, which led to knowledge and processes being rather dispersed. Hence, even though the syllabuses were made lighter, they still seemed too heavy for the time allocated, which moreover had been much reduced. After the Modern Mathematics Reform, which was based on a strong axiomatic concept, it was suggested in the 1980s that teaching should be based on "deductive blocks". Since then, the idea of deduction has been extremely diluted. In junior secondary school, reasoning is often reduced to linking one or two steps. In addition, assertions have a confused status: are they definitions? properties? propositions? Lastly, the "new syllabuses" are often interpreted as reducing mathematics to a "service discipline".

The history of mathematics thus became a sort of "therapy against heterogeneity", used to construct and connect different types of mathematical knowledge from a range of mathematical and non-mathematical problems, to analyze the construction of knowledge based on or contrary to other bodies of knowledge, to identify long-lasting knowledge, and to understand the links between mathematics and other scientific activities. In 1985, the subject of the Inter-IREM conference in Montpellier was "The role of problems in history and in mathematics". It was followed in 1993 by the publication of the book *History of problems, history of mathematics* in which different chapters deal with the problem of irrationality, with solving equations as well as with representation in perspective. "Historical exercises" are also provided for readers (IREM, 1993).

In the proceedings of the 2002 CII-IREM conference in Rennes about "Mathematics in the long term", we wrote that "to consider history on the basis of major issues is a way of understanding both the continuity of certain concepts and the differences between successive approaches" (IREM, 2002). History shows that mathematics was not invented to provide support for educational activities but that it was first and foremost an instrument for understanding and controlling the world. This point is essential because it legitimizes the idea of a mathematical education for everyone.

It may seem paradoxical that, after having suggested during the time of the "Modern Mathematics Reform" that mathematics is an activity, some would later denounce a "pedagogical activism" which neglects the constructions of a mathematical knowledge (Bkouche, 1992). To explain this apparent paradox, it has to be situated at the beginning of the 1990s, when the University Institutes for Teacher Training were set up (IUFM). At the same time, a number of university posts were created to carry out the training, but in 1992 there were only three new posts in the history of science. On the other hand, these Institutes were powerful channels to spread "new educational methods", such as teaching by activities or by groups of pupils, or differentiated education. Thus, in a professional dissertation on teaching by activities, a newly-qualified teacher was pleased that he had always been able to find activities for his teaching but was sorry that this was impossible for trigonometry because "it serves no purpose" (Barbin, 2010). This statement reveals an ignorance of history because trigonometry, such as measuring angles using ratios of measurements of segments, is very ancient.

This notion can be found in the calculations of the slope of the pyramids in Egyptian mathematics around 1650 BC, in Euclidean geometry and in the astronomy of Ptolemy.

Trigonometry can be used to locate one's position in space or to measure inaccessible distances (Guichard, 2010). Conversely, a knowledge of angles enables distances to be measured. Thus, in 1765, the mathematician Alexis Clairaut taught angles on the basis of the triangulation used to construct geographical maps. However, the assertion of the young teacher seems to suggest that he imagined that teaching knowledge through activities was only an educational obligation, or that only academic subjects need be taught.

Here, the history of mathematics could be used as a "therapy against a pedagogical view" of the teaching of mathematics, because it shows the real scope of the subjects taught. In other words, it is well worth enriching teaching through activities, or better still through problems, by a knowledge of history. There is no longer a paradox (Barbin, 2010).

3. Contribution of an Epistemological History to Teaching

The benefits of history to teaching are linked not only to the content of this teaching but also to the concerns of teachers. From this point of view, the history, which interests teachers, has three attributes: reorienting, epistemological and cultural (Barbin 1997). These various benefits have been developed in the different actions implemented in France since the 1980s.

3.1. The Virtue of Reorienting the Teachers

History possesses the virtue of reorienting us, "to surprise us by the obvious" as the historian Paul Veyne wrote. First and foremost, because mathematics never was and never will be the same as it is taught today. It is the work of men, women and communities. Secondly, mathematics reflects the concerns and the mathematical concepts of the age in which it was constructed. Lastly, it was produced in a cultural and geographical area and then circulated. The reorientation is mathematical as well as cultural. This can enable us to understand the difficulties of our pupils who, unlike their teachers, are in unknown territory. It can also help us to listen to their questions and interpret their mistakes better. The term "dépaysement" was introduced in 1997 (Barbin, 1997) and translated into English as "reorientation" in the ICME Study Report (Fauvel and Van Maanen, 2000).

Using the history of mathematics in the IREMs is associated with an epistemological study, which is not linked to the question of the foundations of mathematics but how mathematics works. It concerns a "historical epistemology" which follows the trail blazed by Gaston Bachelard in *The formation of the scientific mind*. We will give three examples: reciprocal transformations of problems and concepts, the status of proofs and methods, and modifications of the notion of number.

In teaching, a problem leads to the application of a concept or a piece of knowledge, usually the one which was dealt with in the previous lesson. Then, another problem follows which enables another concept or piece of knowledge to be used, and so on. However, history shows that a problem can undergo transformations, and that solutions require transformations of concepts. Thus, proofs on tangents are geometrical in Greek

texts while, in the 1630s, the problem of finding the tangent to a curve became a kinematic one for Roberval and an optical one for Descartes. As a result, the notions of tangent and curve are changed. Roberval saw a curve as the trajectory of a point in movement and the tangent as the direction of movement of a point. Descartes associated an equation to a curve and searched for a circle tangential to the curve at a point (Barbin, 2010).

A second example of historical epistemology concerns the status of proofs and methods. Until the 2000s, junior secondary school pupils were asked to "prove" by reasoning and by applying logical deduction. However, they were also asked to obtain geometrical results using algebraic calculus, and later vector calculus. What is the status of these types of calculus in relation to a demonstrative discourse? History enables each of them to be understood as a "method", a notion which has largely disappeared from teaching. These questions were tackled at the IREM Conference in Besançon in 1989 on "The mathematical proof in history" and in Reims in 1993 on "Calculus and the analytical approach".

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Biographical Sketch

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