THE ECONOMICS OF BARGAINING

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Summary

This article presents the main principles of bargaining theory, along with some examples to illustrate the potential applicability of this theory to a variety of real-life bargaining situations. It will be shown that such models can be adapted, extended, and modified in order to explore other issues concerning bargaining situations.

1. Introduction

Bargaining is ubiquitous. Married couples negotiate over a variety of matters such as who will do which domestic chores. Government policy is typically the outcome of negotiations amongst cabinet ministers. Whether or not a particular piece of legislation meets with the legislature's approval may depend on the outcome of negotiations amongst the dominant political parties. National governments are often engaged in a variety of international negotiations on matters ranging from economic issues (such as the removal of trade restrictions) to global security (such as the reduction in the stockpiles of conventional armaments, and nuclear nonproliferation and test bans) and environmental and related issues (such as carbon emissions trading, biodiversity conservation, and intellectual property rights). Much economic interaction involves negotiations on a variety of issues. Wages and prices of other commodities (such as oil, gas, and computer chips) are often the outcome of negotiations amongst the concerned parties. Mergers and acquisitions require negotiations over, amongst other issues, the price at which such transactions are to take place.

What variables (or factors) determine the outcome of negotiations such as those mentioned above? What are the sources of bargaining power? What strategies can help improve one's bargaining power? What variables determine whether parties to a territorial dispute will reach a negotiated settlement, or engage in military war? How can one enhance the likelihood that parties in such negotiations will strike an agreement quickly to minimize the loss of life through war? What strategies should one adopt to maximize the negotiated sale price of one's house? How can one negotiate a better deal (such as a wage increase) from one's employers?

Bargaining theory seeks to address the above and many similar real-life questions concerning bargaining situations.

2. Bargaining Situations and Bargaining

Consider the following situation. An individual, called Aruna, owns a house that she is willing to sell at a minimum price of £50 000; that is, she "values" her house at £50 000. Another individual, called Mohan, is willing to pay up to £70 000 for Aruna's house; that is, he values her house at £70 000. If trade occurs—that is, if Aruna sells the house to Mohan—at a price that lies between £50 000 and £70 000, then both Aruna (the "seller") and Mohan (the "buyer") would become better off. This means that, in this situation, these two individuals have a common interest to trade. At the same time, however, they have conflicting (or divergent) interests over the price at which to trade: Aruna, the seller, would like to trade at a high price, while Mohan, the buyer, would like to trade at a low price.

Any exchange situation, such as the one just described, in which a pair of individuals (or organizations) can engage in mutually beneficial trade but have conflicting interests over terms of trade is a bargaining situation. Stated in general terms, a bargaining situation is a situation in which two or more players have a common interest to cooperate, but have conflicting interests over exactly how to cooperate. (A "player" can be either an individual or an organization (such as a firm, a political party, or a country).)

There are two main reasons for being interested in bargaining situations. The first, practical reason is that many important and interesting human (economic, social, and political) interactions are bargaining situations. As mentioned above, exchange situations (which characterize much of human economic interaction) are bargaining situations. In the arena of social interaction, a married couple, for example, is involved in many bargaining situations throughout the relationship. In the political arena, a bargaining situation exists, for example, when no single political party on its own can form a government (such as when there is a hung parliament): the party that has obtained the most votes will typically find itself in a bargaining situation with one or more of the other parties. The second, theoretical reason for being interested in bargaining situations is that understanding such situations is fundamental to understanding the workings of markets and the appropriateness, or otherwise, of prevailing monetary and fiscal policies.

The main issue that confronts the players in a bargaining situation is the need to reach agreement over exactly how to cooperate. Each player would like to reach some agreement, rather than disagree and not reach any agreement, but each player would also like to reach an agreement that is as favorable to that individual as possible. It is thus feasible that the players will strike an agreement only after some costly delay, or indeed fail to reach an agreement—as is witnessed by the history of disagreements and costly delayed agreements in many real-life situations (as exemplified by the occurrences of trade wars, military wars, strikes, and divorce).

Bargaining is any process through which the players try to reach an agreement. This process is typically time consuming, and involves the players making offers and counteroffers to each other. Any theory of bargaining focuses on the efficiency and distribution properties of the outcome of bargaining. The former property relates to the possibility that the players fail to reach an agreement, or that they reach an agreement after some costly delay. Examples of costly delayed agreements include when a wage agreement is reached after lost production due to a long strike, and when a peace settlement is negotiated after the loss of life through war. The distribution property relates to the issue of exactly how the gains from cooperation are divided between the players.

The principles of bargaining theory set out in this article determine the roles of various key factors (or variables) on the bargaining outcome (and its efficiency and distribution properties). As such, they determine the sources of a player's bargaining power.

2.1 An Outline of this Article

If the bargaining process is "frictionless"—by which we mean that neither player incurs any cost from haggling—then each player may continuously demand that agreement be struck on terms that are most favorable to that player. (For example, in the exchange situation described above, Aruna may continuously demand that trade take place at the price of £69 000, while Mohan may continuously demand that it take place at £51 000.) In such circumstances, the negotiations are likely to end up in an impasse (or deadlock), since the negotiators would have no incentive to compromise and reach an agreement. Indeed, if it did not matter when the negotiators agree, then it would not matter whether they agreed at all. In most real-life situations, the bargaining process is not frictionless. A basic source of a player's cost from haggling comes from the twin facts that bargaining is time consuming and that time is valuable to the player In Section 4 below, we shall discuss the role of the players' degrees of impatience on the outcome of bargaining. A key principle that will be discussed is that a player's bargaining power is higher the less impatient that player is relative to the other negotiator. For example, in the exchange situation described above, the price at which Aruna sells her house will be higher the less impatient she is relative to Mohan. Indeed, patience confers bargaining power.

A person who has been unemployed for a long time is typically quite desperate to find a job, and may be willing to accept work at almost any wage. The high degree of impatience of the long-term unemployed can be exploited by potential employers, who may thus obtain most of the gains from employment. As such, an important role of minimum wage legislation would seem to be to strengthen the bargaining power of the long-term unemployed. In general, since a player who is poor is typically more eager to strike a deal in any negotiations, poverty (by inducing a larger degree of impatience) adversely affects bargaining power. No wonder, then, that the richer nations of the world often obtain relatively better deals than the poorer nations in international trade negotiations.

Another potential source of friction in the bargaining process comes from the possibility that the negotiations might breakdown into disagreement because of some exogenous and uncontrollable factors. Even if the possibility of such an occurrence is small, it nevertheless may provide appropriate incentives to the players to compromise and reach an agreement. The role of such a risk of breakdown on the bargaining outcome is discussed in Section 5.

In many bargaining situations, the players may have access to "outside" options and/or "inside" options. For example, in the exchange situation described above, Aruna may have a nonnegotiable (fixed) price offer on her house from another buyer; and, she may derive some "utility" (or benefit) while she lives in it. The former is her outside option, while the latter is her inside option. When, and if, Aruna exercises her outside option, the negotiations between her and Mohan terminate forever in disagreement. In contrast, her inside option is the utility per day that she derives by living in her house while she temporarily disagrees with Mohan over the price at which to trade. As another example, consider a married couple who are bargaining over a variety of issues. Their outside options are their payoffs from divorce, while their inside options are their payoffs from remaining married but without much cooperation within their marriage. The role of outside options on the bargaining outcome is discussed in Section 6, while the role of inside options is discussed in Section 7.

Important questions addressed in Sections 4–7 are why, when, and how to apply Nash's bargaining solution—the latter is described and studied in Section 3. It is shown that under some circumstances, when appropriately applied, Nash's bargaining solution describes the outcome of a variety of bargaining situations. These results are especially important and useful in applications, since it is often convenient for applied economic and political theorists to describe the outcome of a bargaining situation—which may be one of many ingredients of their economic models—in a simple (and tractable) manner.

An important determinant of the outcome of bargaining is the extent to which information about various variables (or factors) are known to all the parties in the bargaining situation. For example, the outcome of union–firm wage negotiations will typically be influenced by whether or not the current level of the firm's revenue is known to the union. The role of such asymmetric information on the bargaining outcome is studied in Section 8.

In the preceding chapters, the focus is on "one-shot" bargaining situations. In Section 9, we study repeated bargaining situations in which the players have the opportunity to be involved in a sequence of bargaining situations. We conclude in Section 10.

(It may be noted that a bargaining situation is a game situation in the sense that the outcome of bargaining depends on both players' bargaining strategies: whether or not an agreement is struck, and the terms of the agreement (if one is struck), depend on both players' actions during the bargaining process. It is therefore natural to study bargaining situations using the methodology of game theory (see *Game Theory*).)

3. The Nash Bargaining Solution

A bargaining solution may be interpreted as a formula that determines a unique outcome for each bargaining situation in some class of bargaining situations. In this section, we introduce the bargaining solution created by John Nash. (Nash's bargaining solution and the concept of a Nash equilibrium are unrelated concepts, other than the fact that both concepts are the creations of the same individual.) The Nash bargaining solution is defined by a simple formula, and it is applicable to a large class of bargaining situations—these features contribute to its attractiveness in applications. However, the most important reason for studying and applying the Nash bargaining solution is that it possesses sound strategic foundations: several plausible (game-theoretic) models of bargaining vindicate its use. These strategic bargaining models will be studied in later sections, where we shall address the issues of why, when, and how to use the Nash bargaining solution. A prime objective of the current section, on the other hand, is to develop a thorough understanding of the definition of the Nash bargaining solution, which should facilitate its characterization and use in any application.

Two players, A and B, bargain over the partition of a cake (or surplus) of size π, where $\pi > 0$. The set of possible agreements is $X = (x_A, x_b): 0 \le x_A \le \pi \textit{ and } x_B = \pi - x_A$, where x_i is the share of the cake to player i ($i = A, B$). For each $x_i \in [0, \pi]$, $U_i(x_i)$ is player i's utility from obtaining a share x_i of the cake, where player i's utility function $U_i : [0, \pi] \to \Re$ is differentiable, strictly increasing, and concave. If the players fail to reach an agreement, then player i obtains a utility of d_i, where $d_i \ge U_i(0)$. There exists an agreement $x \in X$ such that $U_A(x) > d_A$ and $U_B(x) > d_B$, which ensures that there exists a mutually beneficial agreement.

The utility pair $d = (d_A, d_B)$ is called the *disagreement point*. In order to define the Nash bargaining solution of this bargaining situation, it is useful to first define the set Ω of possible utility pairs obtainable through agreement. For the bargaining situation described above, $\Omega = \{(u_A, u_B)$: there exists $x \in X$ such that $U_A(x_A) = u_A$ and $u_B(x_B) = u_B\}$.

Fix an arbitrary utility u_A to player A, where $u_A \in [U_A(0), U_A(\pi)]$. From the strict monotonicity of U_i, there exists a unique share $x_A \in [0, \pi]$ such that $U_A(x_A) = u_A$; that is, $x_A = U_A^{-1}(u_A)$, where U_A^{-1} denotes the inverse of U_A. Hence:

$$g(u_A) \equiv U_B\left(\pi - U_A^{-1}(u_A)\right) \tag{1}$$

is the utility player B obtains when player A obtains the utility u_A. It immediately follows that:

$$\Omega = \left\{(u_A, u_B) : U_A(0) \le u_A \le U_A(\pi) \text{ and } u_B = g(u_A)\right\}. \tag{2}$$

The Nash bargaining solution (NBS) of the bargaining situation described above is the unique pair of utilities, denoted by (u_A^N, u_B^N), that solves the following maximization problem:

$$\max_{(u_A, u_B) \in \Theta} (u_A - d_A)\ (u_B - d_B) \tag{3}$$

where:

$$\Theta \equiv \left\{(u_A, u_B) \in \Omega : u_A \ge d_A \text{ and } u_B \ge d_B\right\} \equiv \left\{(u_A, u_B) : U_A(0) \le u_A \le U_A(\pi)\right.$$

$$\left. u_B = g(u_A),\ u_A \ge d_A \text{ and } u_B \ge d_B\right\}. \tag{4}$$

The maximization problem stated above has a unique solution, because the maximand $(u_A - d_A)(u_B - d_B)$—which is referred to as the Nash product—is continuous and strictly quasiconcave, g is strictly decreasing and concave, and the set Θ is nonempty.

Hence, the Nash bargaining solution is the unique solution to the following pair of equations:

$$-g'(u_A) = \frac{u_B - d_B}{u_A - d_A} \text{ and } u_B = g(u_A) \tag{5}$$

where g' denotes the derivative of g.

Example 1 (Split the Difference Rule)

Suppose $U_A(x_A) = x_A$ for all $x_A \in [0, \pi]$ and $U_B(x_B) = x_B$ for all $x_B \in [0, \pi]$. This means that for each $u_A \in [0, \pi]$, $g(u_A) = \pi - u_A$, and $d_i \ge 0$ ($i = A, B$). It follows from Eq. (5) that:

$$u_A^N = d_A + \frac{1}{2}\left(\pi - d_A - d_B\right) \ \ and \ \ u_B^N = d_B + \frac{1}{2}\left(\pi - d_A - d_B\right) \qquad (6)$$

which may be given the following interpretation. The players agree first to give player i ($i = A, B$) a share d_i of the cake (which gives player i a utility equal to the utility that i obtains from not reaching agreement), and then they split equally the remaining cake $\pi - d_A - d_B$. Notice that players i's share is strictly increasing in d_i and strictly decreasing in d_j ($j \neq i$).

Example 2 (Risk Aversion)

Suppose $U_A(x_A) = x_A^{\gamma}$ for all $x_A \in [0, \pi]$, where $0 < \gamma < 1$, $U_B(x_B) = x_B$ for all $x_B \in [0, \pi]$, and $d_A = d_B = 0$. This means that for each $u_A \in [0, \pi]$, $g(u_A) = \pi - u_A^{1/\gamma}$. Using Eq. (5), it is easy to show that the shares x_A^N and x_B^N of the cake in the NBS allotted to players A and B, respectively, are as follows:

$$x_A^N = \frac{\gamma\pi}{1+\gamma} \text{ and } x_B^N = \frac{\pi}{1+\gamma} \qquad (7)$$

As γ decreases, x_A^N and x_B^N increase. In the limit, as $\gamma \to 0$, $x_A^N \to 0$ and $x_B^N \to 1$. Player B may be considered risk neutral (since B's utility function is linear), while player A is risk averse (since A's utility function is strictly concave), where the degree of A's risk aversion is decreasing in γ. Given this interpretation of the utility functions, it has been shown that player A's share of the cake decreases as A becomes more risk averse.

3.1 An Application to Bribery and the Control of Crime

An individual C decides whether or not to steal a fixed amount of money π, where $\pi > 0$. If C steals the money, then the probability is ξ that C is caught by a policeman P. The policeman is corruptible, and bargains with the criminal over the amount of bribe b that C gives P in return for not reporting the crime to the authorities. The set of possible agreements is the set of possible divisions of the stolen money, which (assuming money is perfectly divisible) is $\{(\pi - b, b)$: $0 \leq b \leq \pi\}$. The policeman reports the criminal to the authorities if and only if they fail to reach agreement. In that eventuality, the criminal pays a monetary fine. The disagreement point $(d_C, d_P) = (\pi(1 - \nu), 0)$, where $\nu \in [0,1]$, is the penalty rate. The utility to each player from obtaining x units of money is x.

It immediately follows that the NBS is $u_C^N = \pi[1 - (\upsilon/2)]$ and $u_P^N = \pi\upsilon/2$. The bribe associated with the NBS is $b^N = \pi \ \nu/2$. Notice that, although the penalty is never paid to the authorities, the penalty rate influences the amount of bribe that the criminal pays the corruptible policeman.

Given this outcome of the bargaining situation, we now address the issue of whether or not the criminal commits the crime. The expected utility to the criminal from stealing the money is $\xi\pi\ [1 - (\nu/2)] + (1 - \xi)\pi$, because with probability ξ, C is caught by the policeman (in which case the utility is u_C^N) and with probability (1 - ξ), C is not caught by the policeman (in which case C keeps all of the stolen money). Since the utility from not stealing the money is zero, the crime is not committed if and only if π $[1 - (\xi\nu/2)] \leq 0$. That is, since $\pi > 0$, the crime is not committed if and only if $\xi\nu \geq 0$. Since $\xi < 1$ and $0 < \nu \leq 1$ implies that $\xi\nu < 1$, for any penalty rate $\nu \in [0,1]$ and any probability $\xi < 1$ of being caught, the crime is committed. This analysis thus vindicates the conventional wisdom that if penalties are evaded through bribery, then they have no role in preventing crime.

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Biographical Sketch

Abhinay Muthoo is professor of economics at the University of Essex. He has previously taught at the London School of Economics (LSE) and the University of Bristol. He was educated at the LSE and the University of Cambridge. Professor Muthoo has published papers on bargaining theory, amongst other topics, in the world's top academic journals. His recent book on bargaining theory published in 1999 by Cambridge University Press has received noteworthy acclaim from leading economists and academicians.