PHASE TRANSITIONS AND SPONTANEOUSLY BROKEN SYMMETRIES

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Keywords: Spontaneously broken symmetries, global and local gauge symmetries, Nambu-Goldstone and Higgs bosons, phase transitions, order and control parameters, ferromagnetism, superconductivity, electroweak theory.

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Summary

The concepts of spontaneously broken symmetries and phase transitions are widespread in physics. In addition to the examples chosen here, there are many other applications of spontaneously broken symmetries, *e.g.* superconductivity, superfluidity and Bose-Einstein condensation in condensed matter physics, and mean-field descriptions of correlated many-body systems as used extensively in nuclear physics, just to mention a few.

Spontaneously broken symmetries occur whenever the fundamental equations of a physical system possess a symmetry that is not displayed by the ground state. The phenomenon of spontaneous symmetry breaking (SSB) is closely related to the occurrence of phase transitions between two phases with different symmetry properties. SSB shows a remnant of a deeper symmetry of the physical laws that are hidden from sight, but which in principle can be restored by increasing the energy (or temperature) of the system.

1. Introduction

The concept of symmetry is a powerful tool that has inspired many generations of scientists in their search for beauty, harmony, order and regularity in Nature and in the understanding of the fundamental laws of physics. Nevertheless, at times it has also led physicists astray, trying to find higher, or more fundamental, symmetries before the time is ripe, *i.e.* without the availability of sufficient empirical information on the phenomena one seeks to understand.

At present, symmetries form one of the cornerstones of physics: its basic laws are based upon symmetry. The deep connection between three fundamental concepts in physics, symmetries, conservation laws and variational principles, was established in the beginning of the 20th century by the German scientist Emmy Noether (1882-1935). Nowadays it is known as the Noether theorem which states that for every continuous symmetry of the laws in physics, there exists a corresponding conservation law and vice versa, for every conservation law *there exists a continuous symmetry and vice versa for every conservation law there exists a continuous symmetry*. As a consequence, all conservation laws reflect fundamental symmetries of Nature. In classical physics one has the familiar examples of the conservation of angular momentum, momentum and energy which are related to the space-time symmetries of rotations and translations in space and time, respectively.

However, not only symmetries themselves, but also their breaking has provided deep insights into the nature of physical laws. The importance of symmetry breaking was realized as early as 1894 by Pierre Curie (1859-1906): *the asymmetry of effects must be found in their causes* and *asymmetry is what creates a phenomenon*. He also discussed some important aspects of what later became known as spontaneous symmetry breaking: *the phenomena generally do not exhibit the symmetries of the laws that govern them*. Symmetry breaking can occur in different ways. It can be broken explicitly as for example parity violation by the weak interaction, or spontaneously as in ferromagnetism, electroweak theory, superconductivity, superfluidity and Bose-Einstein condensation. In the case of spontaneously broken symmetries, the fundamental equations of a system possess a symmetry that is not displayed by the ground state.

The first time that the concept of spontaneous symmetry breaking (SSB) came to be perceived as a general principle was when Yoichiro Nambu (1921) introduced this mechanism into particle physics using an analogy with superconductivity (the name was coined a few years later in a paper by Baker and Glashow). SSB is a general mechanism which plays an important role in many different branches of physics.

The phenomenon of SSB is most easily understood by examining a few examples. First, consider a ball which is located on top of a hill. The ball is in a completely symmetric state. However, this configuration is not stable: any small disturbance may cause the ball to roll down the hill in one direction or another. At the moment the ball starts to roll down, the initial symmetry is broken, because the direction in which the ball rolls down has been chosen arbitrarily from all possible directions.

Another example is provided by Abdus Salam’s analogy of a dinner party at which the guests are seated around a circular table, and a napkin is placed between each pair of neighbors. The table setting is symmetrical until someone takes a serviette from his left or his right side. After that, the symmetry is broken, and the other guests can no longer choose between left or right napkins. The left-right symmetry is spontaneously broken.

This chapter briefly reviews the concept of spontaneously broken symmetries and phase transitions by examining the cases of ferromagnetism, global and local gauge symmetries and the electroweak unification.

2. Ferromagnetism

One of the best-known examples in physics of spontaneously broken symmetries is that of a ferromagnet. It was discovered by Pierre Curie that beyond a certain temperature ferromagnetic materials lose their magnetic properties, *i.e.* the ability to have a net magnetization in the absence of an external magnetic field. This so-called critical temperature T_c (or the Curie point) is different for each material.

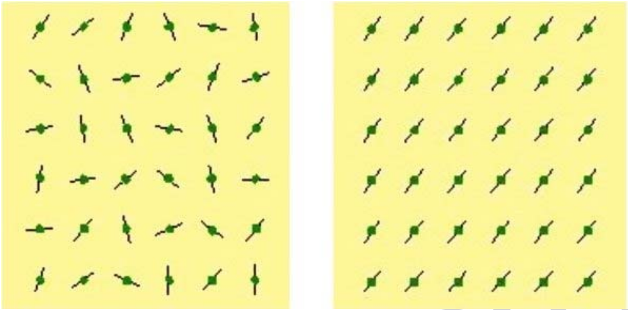

Figure 1. Ferromagnet at high temperature with $T > T_c$ (left) and at low temperature with $T < T_c$ (right).

The atoms in a ferromagnet interact through a spin-spin interaction

$$H = -\sum_{ij} \alpha_{ij} \vec{S}_i \cdot \vec{S}_j,$$

which is invariant under rotations. For temperatures below the critical value $T < T_c$, the magnetic moments of the atoms are partially aligned within magnetic domains in ferromagnetic materials. As the temperature is increased, this alignment is destroyed by thermal fluctuations, until the net magnetization vanishes at $T = T_c$. For temperatures above the critical point $T > T_c$ the ground state of the system is symmetric, *i.e.* the atoms are randomly oriented. There is no preferred direction in space. However, for temperatures below the critical value $T < T_c$, the rotational symmetry is broken since now the ground state consists spins which are aligned within a certain domain. The orientation of the magnetization is random. Each of the possible directions is equally likely to occur, but nevertheless only one of them is chosen at random. The rotational symmetry of the ferromagnet is manifest for $T > T_c$, but is broken or hidden by the arbitrary selection of a particular (nonsymmetrical) ground state for $T < T_c$ (see Figure 1). According to a famous image by Coleman *a little man living inside such a ferromagnet would have a hard time detecting the rotational invariance of the laws of nature*. The ferromagnet provides an example of spontaneous symmetry breaking: the system itself has rotational symmetry, but the ground state is not invariant under that symmetry.

The vanishing of magnetization at the critical temperature $T = T_c$ is a second order phase transition in which the magnetization (order parameter) changes discontinuously as a function of the temperature (control parameter). At the critical temperature, the magnetic susceptibility becomes infinite. In general, the spontaneous breaking of any continuous symmetry is marked by a nonzero order parameter.

Any situation in physics in which the ground state (*i.e.* the state of minimum energy) of a system has less symmetry than the system itself, exhibits the phenomenon of spontaneous symmetry breaking. For example, the state of minimum energy for an iron magnet is that in which the atomic spins are all aligned in the same direction, giving rise to a net macroscopic magnetic field. By selecting a particular direction in space, the magnetic field has broken the rotational symmetry of the system. However, if the energy of the system is raised, the symmetry may be restored, *e.g.* the application of heat to an iron magnet destroys the magnetic field and restores rotational symmetry.

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Biographical Sketch

Roelof Bijker, date of birth: March 21, 1956. Academic position: Full professor, Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Mexico. Distinctions: Investigador Nacional Nivel 3, PRIDE Nivel D; Cátedra Patrimonial de Excelencia Nivel 2 (1994-1996); Chair of the Nuclear Physics Division of the Sociedad Mexicana de Física (1999-2001); Member of the Academia Mexicana de Ciencias (2000); Visiting Professor: University of Genova, Italy (2001/2002); Medalla Marcos Moshinsky (2002); Chair of the Escuela Latinoamericana de Física (2004). Education: University of Groningen, the Netherlands, Bachelor's degree (1977), Master's degree (1980) and Ph.D. in physics (1984). Employment: Full professor, ICN-UNAM (2000-present); Associate professor, ICN-UNAM (1994-2000); Research Associate, University of Utrecht, the Netherlands (1989-1994), Bartol Research Institute, University of Delaware, USA (1987-1989) and University of Pennsylvania, USA (1984-1987). Scientific output: 97 articles published in refereed journals, 41 in conference proceedings, 5 review articles and 5 sets of lecture notes with more than 1850 citations; More than 60 invited and plenary talks at international conferences and more than 50 seminars presented at universities and research institutes; Editor of 2 books and 7 supplements of the Revista Mexicana de Física; Referee of 18 scientific journals and research proposals in Mexico, USA and Colombia; Organizing Committee of 14 scientific events. Teaching: Professor of 15 courses in undergraduate programs and graduate schools; 4 invited courses at Summer Schools in Physics in Mexico and Turkey; Students: 1 tesina, 1 Master's thesis and 1 Ph.D. thesis. Member of jury of professional exams in Mexico, USA, the Netherlands, Spain and Italy. Research interests: Nuclear and hadronic physics; spectroscopy; (super)symmetries; electromagnetic, weak and strong couplings; form factors.