LINEAR ANALYSIS OF STRUCTURAL SYSTEMS

Aslam Kassimali
Department of Civil and Environmental Engineering, Southern Illinois University-Carbondale, Carbondale, Illinois 62901-6603, USA

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Summary

The basic principles of linear analysis of statically determinate and indeterminate framed structures, subjected to static loading, are presented with the main emphasis on the intuitive classical methods. An introduction to modern matrix methods of structural analysis is also included.

1. Introduction

Structural analysis involves evaluating the response of a structure due to specified loads or other external effects, such as temperature changes and support movements. For structural design, the response characteristics generally of interest are: support reactions, stresses or stress resultants (i.e., axial forces, shears and bending moments) and deflections. This chapter covers some of the commonly used methods for linear analysis of structures in static equilibrium.

Linear structural analysis is based on two fundamental assumptions, namely, (a) material linearity – i.e., the structures are composed of linear elastic material, and (b) geometric linearity implying that the structural deformations are so small that the equations of equilibrium can be expressed in the undeformed geometry of the structure.

The advantage of making these assumptions is that the equations relating applied loads to the resulting structural deformations become linear which can be solved conveniently. Another important advantage of linear load-deformation relations is that the *principle of superposition* can be used to simplify the analysis. This principle states that the combined response due to several loads acting simultaneously on a structure equals the algebraic sum of the responses due to each load acting individually on the structure. Linear analysis generally yields accurate results for most common structures under service loading conditions. However, because of its inherent limitations, linear analysis cannot predict structural response in the large deformation and/or failure range, nor it can detect instability conditions (buckling) of structures.

Since Galileo's pioneering work in the 17th century, numerous methods for analysis of structures have been developed. Only a few of these methods, deemed to be most relevant to the current structural engineering practice, are covered in this chapter. A more comprehensive treatment of the subject can be found in various textbooks listed in the bibliography.

This chapter is organized as follows. The types of loads commonly considered in structural design are described in Section 2. In Section 3, the concept of static determinacy and indeterminacy of structures is introduced. The analysis of cables subjected to distributed loading is covered next in Section 4, and the virtual work method for calculating deflections of beams and plane frames is presented in Section 5. The remaining two Sections, 6 and 7, cover the analysis of statically indeterminate beams and plane frames using the classical and modern (computer-based) methods, respectively.

2. Loads on Structures

A structure is designed so that it can support all the loads it is subjected to while serving its purpose throughout its lifespan. Loads on structures are usually grouped into three categories, namely, (a) *dead loads* due to the weight of the structure and any permanent attachments, (b) *live loads* due to the use or occupancy of the structure, and (c) *environmental loads* caused by the effects of the environment in which the structure is situated.

The minimum loads and load combinations for the design of common civil engineering structures can be found in national/international standards, such as, *ASCE Standard Minimum Design Loads for Buildings and Other Structures* (ASCE/SEI 7-05) and the *International Building Code* (IBC-2006). Local/regional building codes generally incorporate these national/international standards and may specify additional provisions warranted by local/regional conditions. Local/regional building codes are usually legally enforceable documents enacted to safeguard the public, and the engineer must follow the load estimation requirements of the building code for the area in which the structure is to be built.

In the following, the types of loads commonly considered in designing civil engineering structures are described, and the basic concepts of estimating their magnitudes are introduced. Because of space limitations, the details of load estimation are not covered. The information provided herein is mainly based on the *ASCE Standard Minimum Design Loads for Buildings and Other Structures* (ASCE/SEI 7-05), and the reader is referred to this document for complete details.

2.1. Dead Loads

Dead loads consist of the weights of the structure and all other material and equipment permanently attached to it. They include the weights of beams, columns, floors, roofs, ceilings, walls, stairways, and permanent service equipment. The weight of the structure can be computed by using the dimensions of its components and the unit weights of materials. As the dimensions of the structural components are not known in advance of design, the weight of the structure is initially assumed, and is revised after the structure has been analyzed and the dimensions of its components determined. The unit weights of some commonly used construction materials are given in Table 1. More comprehensive tables of material unit weights, and of the weights of various types of roof, ceiling and floor finishes, are available in civil engineering handbooks. The weights of fixed service equipment, such as heating, ventilating, and air conditioning systems, plumbing, electrical systems, and fire sprinkler systems, etc., can be obtained from the manufacturers' specifications.

Construction Material	Unit Weight (kN/m^3)
Aluminum	25.9
Brick	18.8
Concrete, plain	22.6
Concrete, reinforced	23.6
Iron, cast	70.7
Structural steel	77.0
Wood	6.3

Table 1. Unit weights of some common construction materials

2.2. Live Loads

Live loads are caused by the use or occupancy of the structure. They are primarily characterized by their movability, although the magnitudes of live loads may also vary. The magnitudes of design live loads for buildings are usually specified in codes as uniformly distributed surface loads. For example, the ASCE/SEI 7-05 specifies that the floors of apartments and residential dwellings (except balconies and stairs) be designed for a minimum distributed live load of 1.92 kPa. Some codes also specify concentrated loads to be used in lieu of distributed loads if they cause larger stresses in structural members. Additional details, including provisions for roof live loads and reduction in live loads, can be found in building codes.

Since the position of a live load may change, each structural member is designed for the load position that causes the maximum stress in that member. The concept of *influence lines* can be used for determining such critical positions of live loads.

Some live loads, such as those due to elevators and other machinery, may act rapidly on the structure causing stresses larger than those produced if the same loads would have been applied gradually. This dynamic effect of loads is called *impact*, and its effect is taken into account by increasing the magnitudes of such live loads by impact percentages or factors. For example, both ASCE/SEI 7-05 and IBC-2006 specify that the building elevator loads be increased by 100 percent to account for impact.

2.3. Wind Loads

Wind loads are caused by the effects of wind on the structure. The flow of wind induces pressures on the vertical surfaces on the windward side of structures, whereas, the inclined windward surfaces may be subjected to either pressures or suctions (negative pressures) depending on their slopes. On the structures' horizontal surfaces and those on the leeward side, the wind flow causes suctions or uplifts (Figure 1).

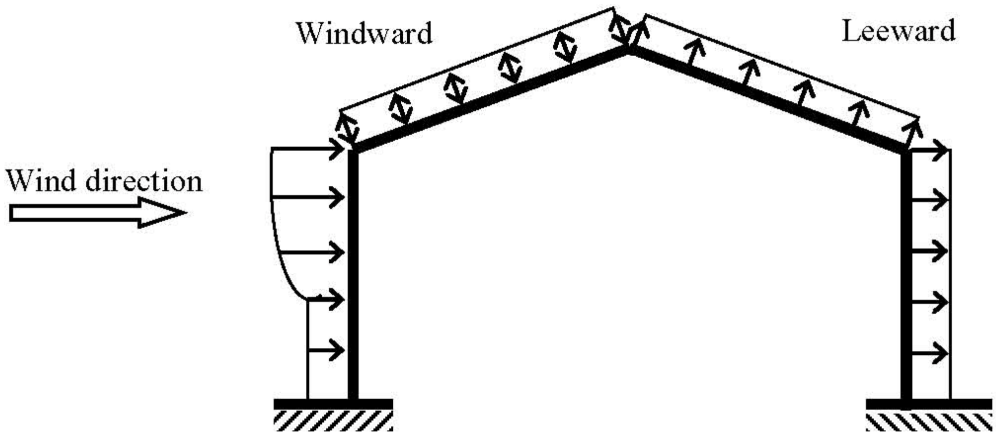

Figure 1. Typical distribution of wind loading

The magnitudes and distributions of wind loads used in designing structures depend on: (a) the wind speed based on the meteorological data for the geographic location of the structure, (b) obstructions in the structure's surrounding terrain, e.g., nearby buildings, etc., (c) the structure's geometric and vibration characteristics, and (d) the importance of the structure. While the procedures specified in various codes for estimating design wind loads usually vary in detail, most of them are based on the foregoing considerations. The ASCE/SEI 7-05 provides detailed tables, charts and equations for estimating wind loads for various types of structures.

2.4. Snow Loads

In many parts of the world, snow loads need to be considered for designing structures. The magnitudes of the design snow loads depend on: (a) the amount of snowfall based on the meteorological data for the geographic location of the structure, (b) the structure's exposure to wind, (c) the slope and shape of the structure's roof, (d) whether or not the structure is heated, and (d) the importance of the structure. Since for some structures, the snow load acting only on a part of the roof may cause larger stresses than when the entire roof is loaded, the codes generally recommend that the effect of unbalanced snow loads also be considered in design. The detailed procedures for estimating snow loads for structures can be found in building codes or standards such as ASCE/SEI 7-05.

2.5. Earthquake Loads

In seismically active regions, structures must be designed to resist earthquake effects. Earthquakes usually cause the structures to vibrate in lateral directions producing horizontal shears that must be considered in their designs. To accurately predict the stresses that may develop in a structure in the case of an earthquake, a dynamic analysis, considering the structure's mass and stiffness properties, should be performed. However, for regular building configurations of low- to medium-heights, the codes usually permit the use of equivalent static loads to design for earthquake resistance. In this simplified approach, a system of static lateral loads is applied to the structure to approximate the dynamic earthquake effect, and a static analysis is performed to determine the structure's stresses. The magnitude and distribution of the equivalent lateral load system that a building is designed to resist depend on: (a) the ground motion expected at the location of the building, (b) the structure's mass and stiffness properties, (c) the soil characteristics at the building site, and (d) the importance of the structure. The detailed procedures for calculating equivalent lateral loads for earthquake design can be found in building codes or standards such as ASCE/SEI 7-05.

2.6. Other Effects and Load Combinations

Some other effects/loads considered in designing civil engineering structures include rain and ice loads, soil and hydrostatic pressures, and secondary effects due to temperature changes, differential support settlements, etc.

Since the different types of design loads might act simultaneously on the structure, the structure is designed to withstand the most critical combination of loads that is likely to occur in its lifespan. The various load combinations to be considered in structural design are usually based on past experience and probability analysis, and can be found in building codes.

3. Statically Determinate and Indeterminate Structures

3.1. Beams

A beam is considered to be statically determinate if all of its support reactions can be determined by applying the equations of equilibrium and condition. The equations of condition ensure that any restrictions on internal forces, due to connections used to connect various parts (or members) of the structure, are satisfied. In the analysis of civil engineering structures, the equations of condition are usually used to satisfy the conditions of zero bending moments at the locations of internal hinges.

Consider, for example, a beam composed of two members AB and BC connected by an internal hinge at B, and supported by four external support reactions, as shown in Figure 2. As the beam is in equilibrium under the action of applied loads and support reactions, it must satisfy the three equations of equilibrium ($\sum F_X = 0$, $\sum F_Y = 0$, and $\sum M = 0$). Furthermore, since the internal hinge at B cannot transmit moment, it provides one independent equation of condition that the bending moment at B is zero ($\sum M_{\mathrm{B}}^{\mathrm{AB}} = 0$ or $\sum M_{\mathrm{B}}^{\mathrm{BC}} = 0$). Since all four unknown reactions of the beam of Figure 2 can be determined by solving the three equilibrium equations plus one equation of condition, the beam is considered to be statically determinate. Thus, in general, if a beam is supported by r (number of) support reactions and has e_{c} (number of) equation of conditions, and if $r = 3 + e_{\mathrm{c}}$, then the beam is statically determinate because all of its unknown reactions can be determined by solving the three equations of equilibrium plus e_{c} equations of condition. However, if $r > 3 + e_{\mathrm{c}}$, then all of the unknown reactions (r) cannot be determined from the available equations ($3 + e_{\mathrm{c}}$) and the beam is considered to be statically indeterminate. For such beams, the degree of indeterminacy is given by $i = r - (3 + e_{\mathrm{c}})$. Finally, if $r < 3 + e_{\mathrm{c}}$, then the beam is not supported by a sufficient number of reactions to prevent all possible movements in its plane, and is therefore referred to as statically unstable.

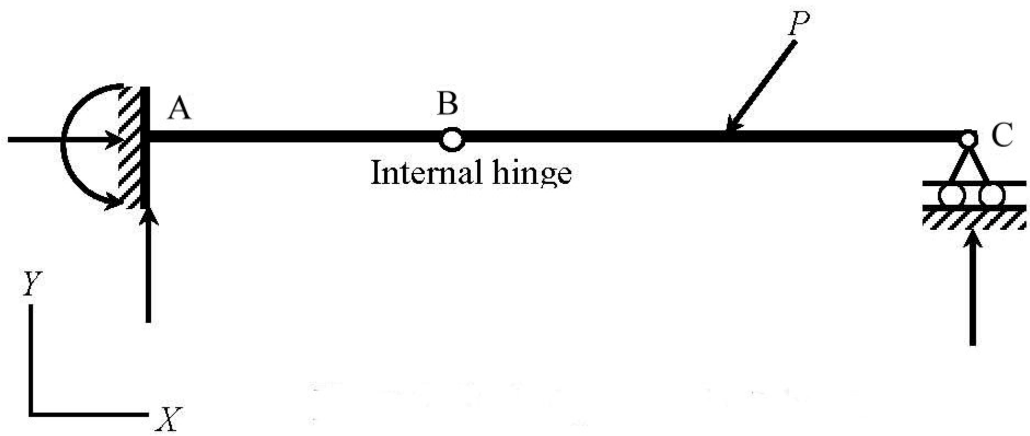

Figure 2. Statically determinate beam

The conditions of static determinacy and indeterminacy stated in the preceding paragraph, although necessary, are not sufficient in the sense that a beam may be supported by enough reactions ($r \geq 3 + e_{\mathrm{c}}$), but may still be unstable due to improper arrangement of supports or internal hinges. Such a beam is referred to as geometrically unstable. In order for a beam to be geometrically stable, it must be supported by at least three support reactions, all of which must neither be parallel nor concurrent. Also, any part of the beam containing three hinges in a row should be supported at a minimum of one intermediate point to avoid geometric instability.

3.2. Plane Frames

A frame is considered to be statically determinate, if all of its support reactions and internal member end forces can be determined by applying the equations of equilibrium and condition. Consider an arbitrary plane frame composed of m members and j joints. The frame is supported by r support reactions, and has e_c equations of condition. The analysis of the frame involves evaluation of r unknown external reactions and six internal end forces for each member (i.e., axial force, shear and bending moment, at each member end). Thus, a total of $r+6m$ unknown quantities needs to be determined. As three equilibrium equations can be written for each member and each joint, the total number of equations (equilibrium plus condition) available is $3(m+j)+e_c$. By comparing the number of unknowns to be determined with the number of available equations, the conditions for static determinacy, indeterminacy, and instability of plane frames can be expressed as:

$$
\begin{aligned}
3m+r &= 3j+e_c && \text{statically determinate frame} \\
3m+r &> 3j+e_c && \text{statically indeterminate frame} \qquad (1) \\
3m+r &< 3j+e_c && \text{statically unstable frame}
\end{aligned}
$$

For statically indeterminate frames, the degree of indeterminacy is given by $i=(3m+r)-(3j+e_c)$.

As in the case of beams, each internal hinge located between the ends of a plane frame member provides one equation of condition. If a frame contains a hinged joint connecting several members, then the number of equations of condition at the joint equals the number of members meeting at the joint minus one.

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Biographical Sketch

Aslam Kassimali was born in Karachi, Pakistan. He received his Bachelor of Engineering (B.E.) degree in civil engineering from the University of Karachi (N.E.D. College) in Pakistan in 1969. In 1971, he earned a Master of Engineering (M.E.) degree in civil engineering from the Iowa State University in Ames, Iowa, USA. After completing further studies and research at the University of Missouri at Columbia in the USA, he received Master of Science (M.S.) and Ph.D. degrees in civil engineering in 1974 and 1976, respectively. His practical experience includes working as a Structural Design Engineer for Lutz, Daily and Brain, Consulting Engineers (USA), from January to July 1973, and as a Structural Engineering Specialist and Analyst for Sargent & Lundy Engineers in Chicago (USA) from 1978 to 1980. He joined Southern Illinois University – Carbondale (USA) as an Assistant Professor in 1980, was promoted the rank of Professor in 1993, and was awarded the title of Distinguished Teacher in 2004. He is currently a Professor and Distinguished Teacher in the Department of Civil & Environmental Engineering at Southern Illinois University in Carbondale, Illinois (USA). He has authored and co-authored four textbooks on structural analysis and mechanics, and a number of papers in the area of nonlinear structural analysis. Dr. Kassimali is a member of the American Society of Civil Engineers (ASCE).