DISTRICT HEATING NETWORKS CALCULATION AND OPTIMIZATION

Michele Calì and **Romano Borchiellini**
Polytechnic of Turin, Italy

Keywords: network models, fluid networks, graph theory, energy transport, thermoeconomic models, district heating

Contents

Summary

After introducing a general definition of the concept of network, an analysis is provided of the networks constituted by pipes that transport a fluid in space and in time, interacting at the same time with the external environment and exchanging mass and energy with it.

The first step in studying the behavior of a pipe network is to identify the objectives that must be attained and the physical and logical problems that must be solved; this aim can be achieved by building a suitable *model* of the network. The next step must be the operation of the model (which, for example, in the case of numerical models consists of using an appropriate algorithmic procedure) to obtain the required answers to the set problem. Then the most common engineering objectives, the simulation and the design calculation or optimization, are identified and discussed.

Because the most relevant action is the representation of the real network with a model, the principal phases of this activity are analyzed in depth. These phases are the building of the topological model, a representation of spatial shapes and of the interconnections of the elements that constitute the network, and the building of the physical model, which includes the transport of each physical quantity that can be related to the fluid

flow, as mass, energy, exergy, entropy, and also cost (value) flow rates.

The models of these different problems are shown in a unified approach that is based on the so-called "nodal method," which makes extensive use of matrix algebra.

Finally a simplified example with numerical results is reported to demonstrate how it is possible to utilize the methodology. It contains the simulation analysis of a simple pipe network in which the fluid changes of pressure, temperature, and value are due respectively to friction losses, to heat transfer, and to thermodynamic irreversibilities.

1. Introduction

The concept of a network has a very general meaning and does not pertain only to the world of exact sciences. It is a powerful tool for representing and studying a multiplicity of real situations that range from scientific phenomena to technical plants and social relations. Models of transportation, communications, and relations among persons, as well as symbolic relations, correspond to each kind of network. Hence, generally speaking, a network is a model of a real situation, also called a system, where a selected plurality of entities, which can be either material or immaterial, interact together and with other entities not belonging to the system studied. As in every model that is a simplified image of reality, in a network model the unnecessary details can be omitted and only the relations necessary for the analysis are represented.

A network is usually represented by an abstract topological object that is a system diagram, called a graph, essentially constituted by two sets of elements, nodes and edges, also called branches. A node can be the image of a place, a connection, a machine, a processing unit, and many other things of this nature while an edge can be the image of a one-to-one link, sometimes oriented, between two nodes. The main advantage of using this kind of representation is that topology has developed a coherent set of powerful theorems to describe and manipulate graphs.

Depending on the real situation represented, nodes and edges can be associated with quantities such as state, potential, and charge to nodes, as well as length, capacity, cost, and loss to edges. For each kind of network the relations between the node and branch quantities are established applying the general and phenomenological laws characteristic of the particular point of view under study.

Here we restrict our discussion only to problems related to pipe networks, but the considerations developed can also be applied to more complex fluid plants.

2. Pipe Networks

A pipe network is a particular plant built to transport a fluid in space and in time, while interacting with other systems (including the environment), exchanging mass and energy with them. During the transportation the fluid undergoes physical and thermodynamic processes.

According to the thermodynamic theory, it is well known that for any substance the mass is a property defined as *extensive* because it is proportional to the quantity of matter. As with the mass, many other extensive properties can be considered for the matter, for example, volume, kinetic, potential and internal energy, entropy, exergy, and—extending the definitions from the thermodynamic to other theories—also value or cost in any arbitrary scale. Consequently, a flow rate of each related extensive property corresponds to the fluid movement.

Fluid networks consist of a large number of interconnected and interacting components that can be classified into a few general categories, each one identified by the main function or action that it exerts. The most common are:

- pipes to transport fluids in space and reservoirs to transport in time (or to store),
- pumps and fans to raise the pressure expending mechanical power,
- turbines and motors, apparatuses to obtain mechanical power lowering the fluid pressure, and
- valves to control the pressure drops and the flow rate.

The fluid moves through every component, modifying its thermodynamic and physical state, interacting with the environment and eventually with other external systems, and gaining or losing energy and value.

Examples of this kind of network are waterworks, gas and oil pipelines, metropolitan district heating and cooling plants, air and water heating and air-conditioning systems in buildings, fire safety plants, drainage plants, ventilation systems of motorways, railway tunnels, and mines, fuel feeding in airplanes and submarines, and many more.

3. The Engineering Problem

In the analysis of a fluid network (as in all engineering calculations) the first step is to clarify the aim of the whole procedure. For example, to:

- identify the objectives that must be attained by using questions,
- identify the physical and logical problem to solve,
- select the tools to reach this aim,
- build a suitable model of the plant, which is almost always numerical, but sometimes can be also analogical or even to scale, and
- operate or solve the model to obtain the expected answers to the proposed questions.

One rough distinction can be made between two extreme situations identified by the statement that the network exists or does not exist as a real object.

In the first case, called "simulation," one supposes that the majority of dimensional and design characteristics of the components (for example materials used, cost, length, diameter and material of pipes, efficiency curves and power requests of the pumps, pressure, temperature and flow rate boundary conditions) are well known. The calculation is then oriented to build a model of the network that adequately represents a known condition of real behavior; then one can use the model to examine what happens in different situations that it not possible or convenient to reproduce in the real plant. A characteristic of this calculation procedure is that the number of equations in the model is exactly equal to the number of unknowns.

In the second case, called “design calculation,” one supposes that only some dimensional and design characteristics of the components are known or fixed before building the network. In this case the number of equations in the model is less than the unknowns, so it is necessary to utilize the so-called “optimization procedure,” which allows the calculation of all the unknowns by finding the maximum or the minimum of a constrained objective function. Examples of the most common objective functions are equations that state the total economic capital, operating and maintenance cost, the total weight, and the space occupied. A scheme of the two procedures is shown in Figure 1.

Many other situations can be identified that represent a mix of simulation and design calculations. Nevertheless in both cases it is necessary to have a model available, and that means a tool that represents the network behavior starting from a quantified description of the network (knowledge of connections, dimensions, and characteristics of all components, and of boundary conditions).

4. The Model

As can be inferred from Figure 1, the building of the model has a central role in the analysis of a pipe network. In this section it is shown how it is possible to build a model that integrates in a homogeneous mathematical structure the problem of studying the transport in a fluid network of mass, energy, and value, the latter as defined in the context of thermoeconomic theory.

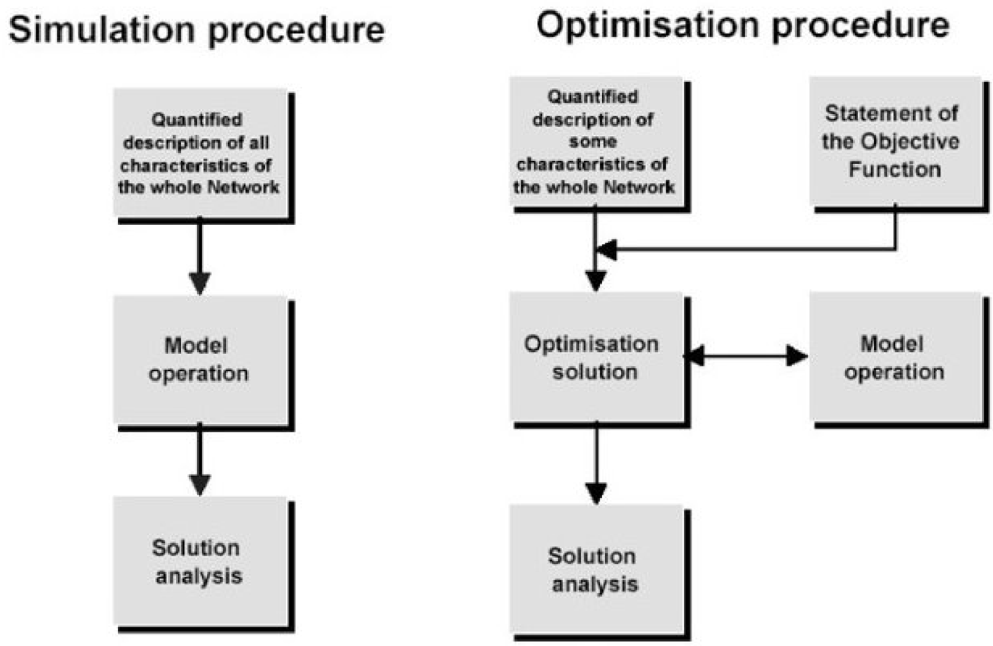

Figure 1. Schemes of the solution procedures

In Figure 2 a flowchart shows that the building of a complete model must proceed through some significant steps that include, in order, first the topological model and then the formulation of general and phenomenological laws as required by the analysis. In what follows more details of this procedure will be explained.

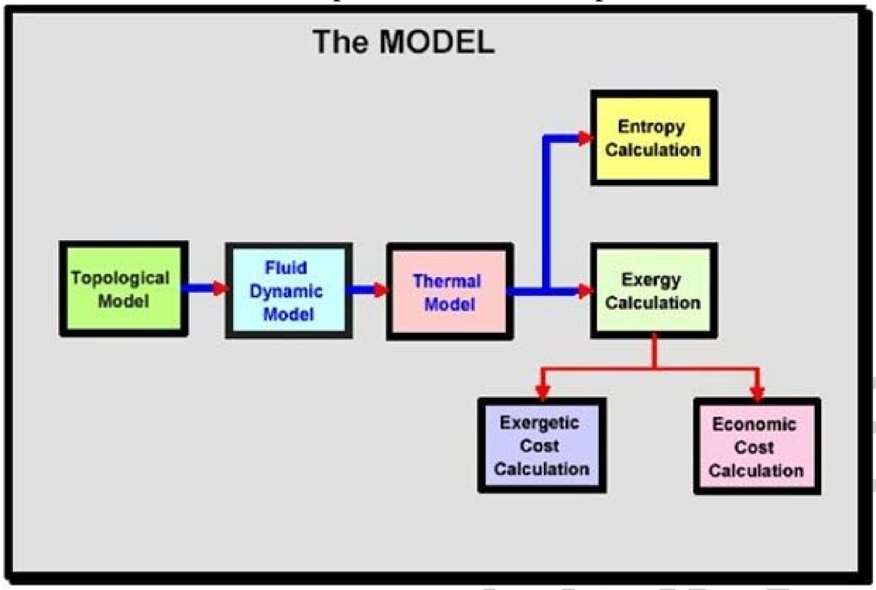

Figure 2. Elements of a pipe network model

The model described here is based on a graph representation of the network. From this assumption it follows that for each extensive quantity that must be taken into account (a) it is possible to identify a state variable that can be put in relation with the flow rates of extensive quantity, and (b) the flow in pipes is substantially one-dimensional.

4.1. The Topological Model

The first step in building the model of a pipe network is to represent its complexity with a graph. A graph is a topological object made only of two elements, oriented edges and nodes; lines interconnected in the nodes represent the edges. The main problem is to establish which components of the physical network nodes can model and which ones can be modeled by edges. Although this choice is not univocal, the most common representation of most diffused fluid networks components are summarized in Table 1.

Edges	**Nodes**
Pipes, ducts	Pipe junctions
Valves, fittings	Fluid reservoirs
Pumps, Fans	

Table 1. Components modeled by edges and nodes

In any case the responsibility for the optimal selection is entrusted to the analyst, who must take into account the final purpose of his work.

To explain how the instruments of the graph theory can help to manipulate network models, a little terminology is included in the glossary.

In Figure 3 a very simple pipe network is represented together with the corresponding topological model.

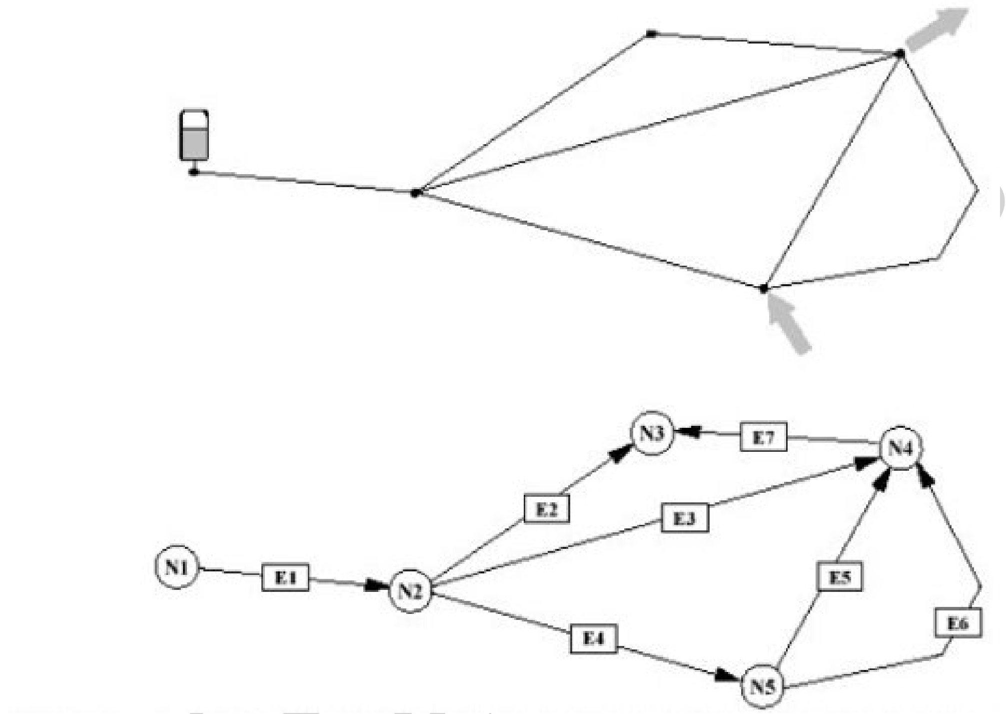

Figure 3. A simple pipe network with one reservoir at constant pressure, one inflow and one outflow (in gray)

In fluid networks the edges must be oriented; this means that a preferential direction is selected from the starting to the arrival node. If in the solution negative flow rates are found, the real direction of the edge flow in the model is opposite to the edge orientation. In a fundamental paper on this subject, written by Yevdokimov, it has been demonstrated that the incidence matrix A contains the complete information about the topological structure of the network; all the other information can be obtained with quite simple matrix algebra starting from knowledge of the incidence matrix A alone.

To *solve* a fluid network means to calculate the values of the state variables in each point and the mass flow rates as well as the rates of each quantity related to the mass in each pipe. In the literature the methods used to develop this calculation can be summarized in three groups: the nodal method, the loop method, and the so-called "linear method." They differ first in the choice of unknowns to calculate and also in the numerical procedure.

In the nodal method the unknowns are the values of the state variables in nodes; the flow rates are calculated afterwards. In the loop method the unknowns are the flow rates in each loop; the values in the nodes are calculated afterwards. Finally in the linear method the unknowns are the flow rates in each pipe and, as in the loop method, the values in the nodes are calculated afterwards.

In this article we describe only the nodal method but it is possible to extend the same considerations to the other methods.

Referring to the definition in the glossary, the incidence matrix of the network in Figure 3 is:

$$\mathbf{A} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \end{vmatrix}$$

-
-
-

Bibliography

ASHRAE, (1992). *Handbook of Fundamentals*. Atlanta, Georgia: ASHRAE (American Society of Heating Refrigeration, Air-conditioning Engineers). [A well-known handbook where the complete procedure to calculate the pressure losses in pipes is well analyzed.]

Blasius H. (1908). Grenzschichten in Flussikeiten mit kleiner Reibung. *Zeitung der Mathematische und Physische*, **56**,(1). [A fundamental paper studying the theory of the viscous losses in pipes.]

Boyne G.G. (1970). *The Design & Analysis of Gas Distribution Networks*. PhD thesis, Dept. of Civil Engineering Heriot-Watt University, Edinburgh. [A deep analysis of optimization procedures that can be used in pipe network optimization.]

Borchiellini R., Ferro V., and Giaretto V. (1994). Transient thermal analysis of main road tunnels. *Aerodynamics and Ventilation of Vehicle Tunnels*, (ed. I.J. Cockram) pp. 17–31. London: Mechanical Engineering Publication Ltd. [An application of extended network calculation to tunnels ventilation.]

Calì M., Borchiellini R. (1987). Una metodologia unificata per il calcolo fluidodinamico di reti complesse per il trasporto di liquidi e gas. *Condizionamento dell'Aria Riscaldamento Refrigerazione*, **31**(1), 51–61. [This contains a general exposition of a method that can be applied to water and gas networks.]

Chinneck J. W., Chandrashekar M. (1984). Models of large-scale industrial energy system: I. Simulation. *Energy*, **9**(1), 21–34. [One of the first papers where mass and energy flow rates calculation in extended networks are treated together.]

Chinneck J. W., Chandrashekar M. (1984). Models of large-scale industrial energy system: II. Optimization and Synthesis. *Energy* **9**(8), 679–692. [The continuation of the previous paper.]

Cross H., (1936). *Analysis of Flow in Network of Conduits or Conductors*. Bulletin 286, November 1936. University of Illinois Engineering Experimental Station, Urbana, Ill. [A fundamental paper where an iterative procedure to calculate flows in extended networks is proposed.]

Ferro V., Borchiellini R., Giaretto V. (1991). Description and Application of a Tunnel Simulation Model, Aerodynamics and Ventilation of Vehicle Tunnels. *Aerodynamics and Ventilation of Vehicle Tunnels.* (ed. A. Haerter) pp. 487–512. London: Elsevier Applied Science. [An application of extended network calculation to tunnels ventilation.]

Kralik J., Stiegler P., Vostry Z. and Zavorka J. (1984). A universal dynamic simulation model of gas pipeline networks. *IEEE Transactions on Systems, Man, and Cybernetics,* SMC-**14**(4), July–August, 597–606. [Here an application to gas networks is exposed.]

Lozano M.A., Valero A. (1993). Theory of the exergetic cost. *Energy,* **18**(9), 939–960. [A fundamental paper where the theory of thermoeconomic analysis is exposed.]

Nikuradse J., (1932). Gesetzmassigkeiten der turbulenten Strömung in glatten rohren, *Ver. Dtsch. Ing. Forschungsheft,* **356**. 1–22. [Another fundamental paper where the theory of the viscous losses in pipes is studied].

Sabet M.H., and Helweg O.J. (1985). Cost-effective operation of urban water supply system using dynamic programming. *Water Resources Bulletin,* **21**(1), 75–81. [An application of optimal network calculation.]

Shamir U., Howard C.D.D. (1968). Water distribution systems analysis. *Journal of Hydraulic Division, American Society of Civil Engineering, Hydraulics,* **1**, 219–234. [This contains the procedure to solve the fluid dynamic problem with the nodal method.]

Verda V. (2001). *Thermoeconomic Diagnosis of an Urban District Heating System, Based on Cogenerative Steam and Gas Turbine*, Doctoral Thesis, Dottorato di Energetica, Politecnico di Torino. [An extended fluid dynamic and thermoeconomic analysis of a district heating plant.]

Wood D.J., Charles C.O.A. (1972). Hydraulic network analysis using linear theory, *Journal of Hydraulic Division, American Society of Civil Engineering, Hydraulics,* **7**, 1157–1170. [This contains the procedure to solve the fluid dynamic problem with the linear method.]

Yevdokimov A.G. (1969). A Theory of the Solution of steady-state Network Problem with special reference to Mine Ventilation Networks, *International Journal of Numerical Methods in Engineering* **1** 279–299. [This is a fundamental paper where the author developed the topological theory of pipe networks and the loop method to solve in a very general way the fluid dynamic problem.]

Biographical Sketches

Michele Calì, born in 1946, is Full Professor at the Department of Energy at the Engineering Faculty of the Polytechnic of Turin, in Italy.

After having completed studies in the classic lyceum, he obtained the degree of Mechanical Engineer in Politecnico di Torino in 1970. After military service (1971–1972), he joined the former Institute of Technical Physics, in the Department of Energy, in the Engineering Faculty of the Politecnico di Torino as assistant professor and subsequently associate professor in 1982 and full professor in 1986. He lectures on thermodynamic, heat transfer, HVAC, and energy transforming plants.

His research activities are principally related to the modeling and analysis of engineering systems that transport and convert energy. He has studied and developed theoretical methods of applied analysis, published in more than 100 papers. These include papers on extended fluid networks matrix analysis, on conduction and convection heat transfer problems solved using finite element method, and on the analysis of experimental data with inverse methods. There are works on thermal behavior of buildings, on the analysis of many kinds of fluid networks as building air and water heating systems, district heating plants, and urban gas distribution. Since the late 1990s he has extended his interest to the thermoeconomic and environomic analysis of energy systems.

Among his publications there is a book on engineering thermodynamic and various teaching books.

Romano Borchiellini took his degree in Mechanical Engineering in 1983 at the Polytechnic of Turin, Italy. After his military service (1983–1984), he worked as consultant engineer in a HVAC system design company (1984–1990).

He joined the Department of Energy as researcher in 1990 and in 1997 he was appointed Associate Professor and was charged with teaching undergraduate, graduate, and post-graduate courses in thermodynamics, heat and mass transfer, and HVAC systems. Since February 2001, Romano Borchiellini has been a Full Professor at the Department of Energy of the Polytechnic of Turin, Italy.

His research activity is related mainly to the development and application of mathematical models in heat and mass transfer applications and in energy system analysis. Among the models investigated, the graph network approach for calculating fluid transport system, energy system, HVAC system, and tunnel ventilation system is one of the most deeply analyzed and used.

Among his publications are more than 30 papers in journals and international conferences.