DYNAMICAL SYSTEMS, INDIVIDUAL-BASED MODELING, AND SELF-ORGANIZATION

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Contents

Summary

To understand the workings of systems from diverse areas of research, e.g. ecology, economy, the social sciences, and biology, it is important to take into account the interactions between the individual elements of the system under study. This is especially true when the behavior of the system is a form of *self-organization*, the development of order that is not caused by external organizing forces, but a result of interactions between the elements of the system. This realization has lead to the use of individual-based models. Computer simulations of such models allow the investigator to test different hypotheses about the system under study.

The mathematical theory for such systems and systems that change over time in general, is that of dynamical systems. Valuable concepts have been developed to increase understanding of such systems. This article explains the basis concepts of dynamical systems theory, and shows their role in examples from physics, chemistry, biology, ecology, traffic, and the brain.

1. Introduction

Some systems in this world are best understood by considering the parts they consist of and the possible interactions between them, and combining these to infer the expected behavior of the system. Examples of such systems are the cantilever, the pulley, and the mechanical clock.

As soon as the number of elements in a system becomes more than a few however, this approach becomes infeasible. The classical approach then is to measure global properties of the system. A good example of this is thermal energy. Whereas it would be virtually impossible to measure the energy of each individual particle of a substance, it is straightforward to measure the temperature of the substance. If the substance is in a state of thermodynamic equilibrium, its thermal energy can be computed from the temperature.

The principle of using a global measure (temperature, in the example) as a characteristic for the behavior of the complete system is used in almost all sciences. Some other examples are indices of stock exchanges in economy, mortality and growth rates in ecology, and welfare statistics in sociology.

The validity of such global measures stands or falls with the presence of equilibrium. Measures of systems that are not near equilibrium are unlikely to be representative of the complete system. Moreover, the development over time of non-equilibric systems can differ widely from that of the same system near equilibrium, as will be seen in this article.

Growing awareness of these and related issues, in combination with the widespread availability of computers, has lead researchers in many different fields to use different types of models. Next to the equilibrium-based models that have been common for long, other models begin to emerge that explicitly represent the individual elements in the system of interest. Such models allow researchers to study the effects of the local interactions between these elements, and to understand how global behavior can be highly unpredictable as a result of these interactions.

The mathematical theory appropriate for systems whose behavior is determined by interacting elements is that of dynamical systems. This branch of mathematics studies systems that change over time, and has developed concepts that may improve understanding of the real world systems that are modeled. This article will introduce some of the basic concepts of this theory, and then give examples of real world systems that exhibit self-organization and chaos.

2. Individual-based modeling

One of the clearest examples where modeling the individual elements of a system has been recognized and used as an alternative to classic, equilibrium based models, is ecology. Here, measuring and modeling the individual elements of a system instead of global properties of the system is known as *individual-based modeling*.

To convey the point that modeling interactions between individuals is required for understanding global behavior of the system, an example concerning ethological research into the foraging behavior of ants will be briefly described. Ants transporting food tend to use the same routes. The choice of these routes is not given in by a central coordinating force, nor by any individual ant. Rather, this choice is the result of interactions between the ants through the environment. While walking, ants deposit a pheromone. After food has been found, the amount of pheromone that is deposited during walking increases. Since ants are attracted by this substance, they are more likely to walk over pheromone trails than in other places. While differences in the amounts of pheromone on different paths are initially small, these differences are enlarged by the fact that the paths that already have more pheromones attract more ants, and thus gather even more pheromones. This positive feedback principle increases initially small differences such that eventually all ants follow the same path. Due to the increase in deposits caused by finding food, and due to the shorter travel times and hence higher frequencies of shorter paths, self organization causes ants to find short paths to food sources.

Individual-based modeling has been used to investigate the relative influences of space perception, memory, pheromone trails, sensitivity of the sensory system, stimulation of nestmates, and food distribution on foraging. By measuring the values of parameters in the subject of study, such as the speed of ants with and without food, variance in heading distribution, and pheromone deposition interval, realistic simulations are produced that allow researchers to test hypotheses about the behavior of the animals.

The behavior of ants is only one example where individual interactions are important in understanding the global behavior of a system. Topics that have been investigated with individual models include economy, anthropology, sociology, linguistics, and ecology research on mammals, fish, birds, insects, bacteria, mixed ecosystems, and forests. Another example of an individual based model described later in this article concerns traffic flow.

Individual-based models are interesting in that they allow investigating the effect of interactions among the elements of a system. However, the dynamical systems perspective can also provide insight when used to model the dynamics of higher level properties of a system, such as population size, in cases where classic, equilibrium based models are not applicable. Apart from stable equilibria, observed fluctuations of population dynamics have been explained using periodic cycles, quasi-periodic cycles, and chaos. An example of this is given by the population dynamics of the flour beetle. These have been shown to be subject to a chaotic regime. Using a nonlinear demographic model, the dynamics under laboratory conditions were first predicted, to determine when chaotic should be expected. Testing this prediction experimentally confirmed the transition to chaos.

These results suggest that human intervention in ecological systems demands a firm understanding of the population dynamics, since the effects of actions can widely differ from expectations when chaotic population dynamics are mistakenly assumed to be approximately linear.

3. Basic notions of dynamical systems theory

In order to understand emergence and self-organization, it is necessary to study some basic notions of dynamical systems theory.

3.1 Dynamical system

Dynamical systems are systems that change over time. Only in systems that change over time can emergence and self-organization take place. All processes that can occur in nature can be described as dynamical systems, albeit usually very complex dynamical systems.

3.2 State variable

The state of a dynamical system can be described by a number of *state variables*.

3.3 Dimension

The minimum number of variables that completely captures the state of the system is referred to as the *dimension* of the system. An example of a simple dynamical system is that of a pendulum that swings in a plane. In the case of the pendulum, the state variables are the pendulum's position and its velocity. The pendulum is therefore a two-dimensional system.

Apart from the state variables, there can be other factors that can influence the behavior of dynamical systems. However, these do not change over time and are therefore not part of the system's state.

3.4 Control parameter

These other factors are called the *control parameters* of the system. In the example of the pendulum, the control parameters are the length of the pendulum, the friction and the strength of gravity.

3.5 Control law

Finally, a dynamical system is governed by *control laws*. Control laws determine the next state of the system for any given state. In the pendulum example, the control law is determined by Newton's second law of motion. The state variables of a dynamical system by definition give a complete description of the state of the system.

3.6 Determinism

Furthermore, in a *deterministic system*, the control laws always give the same successor state for any chosen state. Thus, there are no random influences, and the state of the system at any point in time completely determines the future behavior of the system. In real world systems however, it is generally impossible to determine the values of the state variables exactly, since this demands infinite precision of the measurements. In addition, the laws governing the system may be such that it is impossible to compute the future states of the system exactly, even if they are determined. This is the case in the *n-body problem* for instance, for $n \geq 3$.

3.7 Non-determinism

Systems in which there are random influences also exist. Such systems are called *non-deterministic* or *stochastic*, and it is in general not possible to predict their future behavior exactly. For these systems, it is not possible to find a description with more state variables such that the system becomes deterministic. In other words, the non-determinism is not caused by hidden state variables. However, very complicated deterministic dynamical systems will often be modeled as non-deterministic systems with fewer state variables, in which case the non-determinism is caused by hidden state variables.

In order to make a mathematical description of a dynamical system it is necessary to decide whether to model time as continuous, or as divided into discrete slices. Physical systems will generally be modeled using continuous time, but for simulation purposes it is useful to create computer models that work with discrete time. In the case of continuous time, the equation of any dynamical system can be written as follows:

$$\dot{\mathbf{x}} = f(\mathbf{x}, t), \tag{1}$$

where **x** is the vector of state variables, *f* is the function that describes the behavior of the system and *t* is time.

3.8 Differential equation

This is a so-called *differential equation*. The notation with bold face for vectors and overdots for derivatives is standard in dynamical systems literature.

3.9 Difference equation

In the case of discrete time, the equation is as follows:

$$\mathbf{x}_{t+1} = f(\mathbf{x}_t, t) \tag{2}$$

which is a *difference* equation.

3.10 Autonomous dynamical systems

The above equations are of the most general form. Many dynamical systems that are encountered in practice have control laws that are independent of time, so that the value of *f* only depends on the state variables $\mathbf{x}_t$. Dynamical systems of this kind are called *autonomous* dynamical systems.

Example 3.10.1: Pendulum

A pendulum can best be described in continuous time. Its state variables are its position *u* and its velocity *v*. The functions approximating its behavior for small angles are:

$$\begin{aligned} \dot{u} &= v \\ \dot{v} &= -g\frac{u}{l} - bv \end{aligned} \tag{3}$$

where *g* is the acceleration due to gravity, *l* is the length of the pendulum and *b* is a factor representing friction. It can be observed that these equations do not depend on time (the pendulum is therefore an autonomous dynamical system) and that the state variables occur only in linear combinations.

3.11 Linear dynamical systems

Equation 3 is a *linear* dynamical system. The advantage of linear dynamical systems is that they can be solved analytically. This means that expressions for its variables can be found that depend only on time, and hence that the system's state for any point in the future can be calculated directly. This is not generally true for non-linear dynamical systems, and it will be shown below that only non-linear dynamical systems can show emergent and self-organizing behavior.

However, for now it is convenient to stick to the example of the pendulum in order to illustrate an important way of representing dynamical systems graphically.

3.12 Phase plot

This is by means of a *phase plot* of the behavior of the system. A phase plot is a plot of the state variables of a system against each other.

3.13 State space (Phase space)

The space of all possible system states is called the *state space* or *phase space* of the system, hence the name phase plot. The phase plot shows at a glance what paths connect the different states.

3.14 Trajectories

Such paths are called *trajectories*, and depict what states the system will pass through when starting in a given state. Unfortunately, many of the more interesting dynamical systems have more than two degrees of freedom, and their phase spaces are impossible to plot exactly. In these cases, projections of the phase space may provide insight into the system's behavior.

Example 3.14.1 Phase plots of the pendulum

Examples of phase plots of the pendulum system are given in figure 1. In the leftmost frame, the phase plot of a strongly damped pendulum with a length of one meter is shown. Two trajectories are shown, beginning at different positions and velocities. The arrows indicate the direction in which the system evolves. Different trajectories can not cross in a deterministic system.

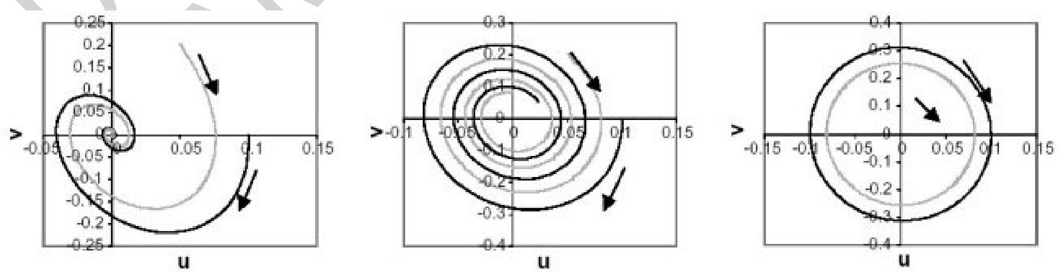

Figure 1: Phase plots of the pendulum

In the second frame, trajectories of a pendulum with equal length, but with less damping are shown. This shows the role a control parameter can play. Although the behavior of the system is different, it is not qualitatively different. All trajectories in the system still converge towards a single point, where the pendulum is at rest in its lowest position. Such a point is called a *point attractor* of the system. The role of attractors in a dynamical system will be discussed in more detail below.

3.15 Dissipative systems

The damped pendulum is also an example of a *dissipative* system. Imagine a cluster of initial conditions, having a certain volume in phase space, and following this cluster over time. If the volume of the cluster decreases over time, the system is called dissipative. The criterion of dissipation is important because attractors are only present in dissipative systems.

The physical interpretation of a dissipative system is that a certain quantity in the system (usually its energy) is dissipated from the system. In the example of the pendulum, dissipation is caused by friction; friction decreases the speed of the pendulum, and thereby shrinks the range of possible speeds and the range of possible locations of the pendulum. When friction is removed (which is physically impossible), the system's behavior changes qualitatively. This situation is depicted in the rightmost frame of figure 1. Such a system is no longer dissipative and its trajectories become cycles.

3.16 Bifurcation

A change of the attractors due to a change of the control parameters of a system is called a *bifurcation*. The particular bifurcation of the pendulum system is not a very interesting one. As will be shown below, more interesting bifurcations take place in non-linear dynamical systems.

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Biographical Sketches

Edwin de Jong (Rhenen, The Netherlands, 1972) received the MSc from Delft University of Technology (The Netherlands) in January 1996. He then joined the Artificial Intelligence Laboratory at the Vrije Universiteit Brussel headed by Prof. Luc Steels to do PhD research. Edwin is interested in intelligent behavior; how animals and humans produce it and how machines might. His research concerns artificial intelligence, cognitive science, and dynamical systems and has addressed coordination in multi-agent systems, reinforcement learning, concept formation, and the evolution of communication. Edwin de Jong is editor for Belgium of the Newsletter of the Belgian Dutch Association for Artificial Intelligence (BNVKI) and is member in the BNVKI, the International Society for Adaptive Behavior (ISAB), and the European Network of Excellence in Evolutionary Computing (EvoNet).

Bart G. de Boer was born in 1970 in the Netherlands and works at the AI-lab of the Vrije Universiteit Brussel. He studied computer science at Leiden University from 1988–1994 and got his master's of science degree with a thesis on learning classifier systems. He did a Ph. D. at the AI-lab of the Vrije Universiteit Brussel supervised by Prof. Luc Steels on the subject of self-organization in vowel systems. He successfully defended his thesis "Self-organisation in vowel systems" in 1999. His research interests are into self-organization in systems of speech sounds for explaining universal tendencies of speech sounds in human languages.