# BOUND-BASED IDENTIFICATION

**Eric Walter**
*CNRS-Supélec-Université Paris-Sud, France.*

**Keywords:** bounded error, calibration, estimation, guaranteed estimation, identification, interval analysis, linearity in the parameters, maximum likelihood, nonlinear estimation, parameter estimation, parameter bounding, set estimation, set inversion, set-membership estimation.

## Contents



## Summary

A standard procedure to estimate the parameters of a mathematical model from experimental data is to minimize a cost function quantifying the distance between the output of this mathematical model and the observed behavior of the system to be modeled. This cost function may be deduced from hypotheses about the statistical distribution of the noise corrupting the data.

The present topic-level paper describes an alternative approach based on the hypothesis that a set of acceptable errors has been defined. The task is then to characterize the set of all values of the parameter vector that are such that the error remains acceptable. The methods for achieving this task depend on whether the error is affine or not in the parameters, just as in the case of more conventional estimation by minimization of a quadratic cost function.



## 1. Introduction

Mathematical models turn out to be extremely helpful when decisions have to be taken in complex situations, for instance for fault diagnosis or to compute a control input in order to obtain appropriate behavior of a system. Very often, such models consist of a set of algebraic or differential or difference equations that depend on a vector $\theta$ of unknown parameters.

This vector has then to be *estimated* (or *identified*, or *calibrated*), based on the available prior information (or hypotheses) and experimental data. Assume that these experimental data have been collected in a vector $\mathbf{y}$. To make $\theta$ uniquely identifiable from $\mathbf{y}$, the number of data points should be at least equal to the number of unknown parameters, *i.e.*,

$$\dim \mathbf{y} \geq \dim \theta\,, \tag{1}$$

but usually many more data points are collected, in the hope of averaging out the effect of measurement noise and other perturbations.

Therefore, it becomes in general impossible to find a value of θ such that the corresponding model behaves exactly as the system described. This is why a standard procedure is to take as the *estimate* $\hat{\theta}$ of the parameter vector the value of θ that minimizes some cost function $J(\theta)$

$$\hat{\theta} = \arg\min J(\theta) \tag{2}$$

The cost function $J(\theta)$ may, for instance, be a weighted sum of the squares of the differences $e_i(\theta)$ between the entries $y_i$ of **y** and the corresponding model outputs $y_{\mathrm{m}}(i, \theta)$, which may be computed analytically or by simulation software. One then speaks of a quadratic cost function

$$J(\theta) = \sum_{i=1}^{\dim \mathbf{y}} w_i [e_i(\theta)]^2\,, \tag{3}$$

where

$$e_i(\theta) = y_i - y_{\mathrm{m}}(i, \theta)\,, \tag{4}$$

and where the positive weights $w_i$ should be chosen in such a way as to express the relative degree of confidence attached to the various data points $y_i$. The index $i$ only serves to designate a given element in the database.

It may correspond to the $i$th time instant at which a measurement was performed on a single-input single-output system, but this is not necessarily so; and static systems together with multi-input and multi-output systems can be considered in the same framework. A maximum-likelihood approach can be used to deduce $J(\theta)$ from information (or hypotheses) on how the measurements are perturbed. A typical type of hypothesis is

**H1:** the data satisfy

$$y_i = y(i, \theta^*) + \varepsilon_i, \quad i = 1, \ldots, \dim \mathbf{y}\,, \tag{5}$$

where $\theta^*$ is some “true” value of θ, forever unknown, and the $\varepsilon_i$s are independently identically distributed according to a Gaussian law with zero mean and variance $\sigma^2$.

Under H1, a maximum-likelihood estimate of θ is obtained by minimizing the cost defined by Eq. (3) with $w_i$ = 1 whatever the value of $\sigma^2$. This is one of the many justifications of the almost universal use of quadratic cost functions in parameter estimation.

However, a maximum-likelihood approach may also lead to a nonquadratic criterion, even in the case of Gaussian noise, when the variance of the noise is unknown and parametrized. When a prior distribution is available for $\theta$, various options based on the Bayes formula are available to incorporate this information into the cost function, thus allowing a reduction in the number of experimental data points to be collected.
The purpose of the present group of three papers is to describe an alternative approach to parameter estimation, known as *bounded-error estimation* or *set-membership estimation* or *parameter bounding*.

-
-
-



**Bibliography**

Chernousko F.L. (1994). *State Estimation for Dynamic Systems*, 304 pp. Boca Raton: CRC Press. [State estimation contains parameter estimation and the tracking of time-varying parameters as special cases; see also the book by Kurzhanski and Valyi]

Combettes P.L. (1993). The Foundations Of Set Theoretic Estimation. *Proceedings of the IEEE*, **81**(2), 182-208. [An interesting survey, with a number of references]

Deller J., Nayeri M., and Odeh S. (1993). Least-Square Identification with Error Bounds for Real-Time Signal Processing and Control. *Proceedings of the IEEE*, **81**(6), 815-849. [A signal-processing view of bound-based estimation]

Fogel E. and Huang Y.F. (1982). On The Value Of Information In System Identification - Bounded Noise Case. *Automatica*, **18**(2), 229-238. [The paper that revived interest in ellipsoidal outer bounding]

Jaulin L., Kieffer M., Didrit O., and Walter E. (2001). *Applied Interval Analysis*, 379 pp. London: Springer-Verlag. [Interval analysis, and its application to estimation, control and robotics, with information about interval constraint propagation; contains details about the theoretical properties, applications and implementation of Sivia]

Kurzhanski A. and Vályi I. (1997). *Ellipsoidal Calculus for Estimation and Control*, 321 pp. Boston: Birkhäuser. [A useful complement to the book by Chernousko]

Milanese M., Norton J., Piet-Lahanier H., and Walter E., eds. (1996). *Bounding Approaches to System Identification*. 565 pp. New York: Plenum Press. [30 chapters and 565 pages about bound-based identification, from theory to applications, by specialists from the East and West]

Norton J.P., ed. (1994). Special Issue on Bounded-Error Estimation: Issue 1. *International Journal of Adaptive Control and Signal Processing*, **8**(1), 1-118. [A special issue entirely dedicated to bound-based identification.]

Norton J.P , ed. (1995). Special Issue on Bounded-Error Estimation: Issue 2. *International Journal of Adaptive Control and Signal Processing*, **9**(1), 1- 134. [A special issue entirely dedicated to bound-based identification]

Schweppe F.C. (1973). *Uncertain Dynamic Systems*, 563 pp. Englewood Cliffs, NJ: Prentice-Hall. [An excellent book by one of the fathers of  the topic]

Walter E., ed. (1990). Special Issue on Parameter Identification with Error Bounds. *Mathematics and*

*Computers in Simulation*, **32**(5&6), 447-607. [A special issue entirely dedicated to bound-based identification]

Walter E. and Pronzato L. (1997). *Identification of Parametric Models from Experimental Data*, 413 pp. London: Springer-Verlag. [Bound-based identification is just one of the topics considered, in part of Chapter 5, devoted to the quantification of parameter uncertainty]

**Biographical Sketch**

**Eric Walter** was born in Saint Mandé, near Paris, France, in 1950. He was awarded a Doctorat d'Etat in automatic control theory in 1980. He is Directeur de Recherche at the CNRS (the French National Center for Scientific Research). His research interests revolve around parameter estimation and its applications to robotics, chemistry, chemical engineering, image processing, pharmacokinetics, medicine and robust control. Along with Luc Pronzato, he has written *Identification of Parametric Models from Experimental Data*, Springer, London, 1997, and with Luc Jaulin, Michel Kieffer and Olivier Didrit he has written *Applied Interval Analysis*, Springer, London, 2001. He is now head of the Laboratoire des Signaux et Systèmes.