ADAPTIVE CONTROL

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Contents

Summary

Control theory is concerned with modifying the behavior of dynamical systems so as to achieve desired goals. These goals include maintaining relevant outputs of a system around constant values, ensuring that they follow specified time functions, or more generally assuring that the overall system optimizes a performance criterion. If a suitable mathematical model of the system is available, the above goals can be achieved by computing a control input based on the observed outputs of the system. Powerful analytical techniques exist for computing the latter, when the characteristics of the system as well as those of the disturbances affecting it are completely known. This implies that the equations describing the system and the parameters contained in them are specified. However, in most practical systems, many of the parameters are either unknown or vary with time. It is to cope with such uncertainties that the field of adaptive control theory was developed.

In the 1950s and 1960s, most of the research was focused on gradient based methods for adjusting the control parameters, when the plant parameters are assumed to be constant. Following a landmark paper that demonstrated that such methods could result in instability, interest shifted to the search for stable methods. Starting around the early 1970s the generally accepted philosophy has been to design a controller which would assure stability of the overall system and then adjust the parameters of the system within this framework to optimize performance.

The field hit its high notes in the 1970s and 1980s and gradually became part of mainstream control theory. Systematic methods for designing stable adaptive observers and controllers for linear time-invariant systems in the 1970s, and detailed investigations of their robustness properties in the 1980s contributed to this. This period also witnessed the study of multivariable adaptive control and the adaptive control of systems with stochastic inputs. During the following years, interest shifted to nonlinear adaptive control and adaptive control in distributed systems where research is continuing at the present time. The chapter traces these many historical developments in the field and examines the major contributions made to it.

1. Introduction

The biological implications of the term “adaptive” lend the topic of adaptive systems an aura that was responsible for the great interest evinced in it over four decades ago, as well as the fascination it continues to have for researchers at the present time. It is an important area in modern control, dealing as it does, with the control of systems in the presence of uncertainties, structural perturbations, and environmental variations. Problems of adaptation occur in many diverse fields such as evolution, ecology, psychology, biology, economics and control. While they appear in various guises, the fundamental questions remain the same i.e. achieving satisfactory response in the presence of great uncertainty, complexity, nonlinearity, and time-variations. During the past four decades, many significant theoretical problems in the field have been solved. The spectacular advances in computer technology have resulted in adaptive control strategies being successfully applied in a wide variety of practical problems. Today, at the beginning of the 21st century, it is a discipline of considerable theoretical elegance, challenging practical problems, and holds great promise for the future. While we are still very far from designing autonomous systems which can take care of themselves, nevertheless a considerable amount of insight has been gained regarding the underlying concepts, and the theoretical questions associated with them. The main objectives of this chapter are to provide a brief outline of the history of the field, trace some of the major developments that have taken place in it since the 1960s, and examine new directions in which it is currently evolving.

2. Basic Concepts and Definitions

Biological systems are known to cope easily and efficiently with changes in their environments. As interest in control theory shifted over the years to the control of systems with greater uncertainty, efforts were naturally made to incorporate in them characteristics similar to those in living systems. This resulted in the introduction of numerous words such as adaptation, learning, pattern recognition, and self organization into the control literature, among which "adaptation" was the first.

The term "adaptive system" was formally introduced into the control literature by Drenick and Shahbender in 1957. Soon after that, sessions in numerous conferences and workshops were organized to define an adaptive system precisely. This resulted in a profusion of definitions, each containing some property that its proponent considered peculiar to adaptive systems. Yet close to five decades after the first paper appeared, a simple, universally accepted definition is still elusive. It is now generally recognized that the definition of adaptation is multi-faceted and not easily compressed into a simple statement without loss of vital content.

Today, our view of adaptive systems is that they are nonlinear systems, derived from linear or nonlinear systems in which the parameters are adjusted using input-output data. This process makes the parameters themselves state variables, thereby enlarging the dimension of the state space of the original system. To the designer, such a system is adaptive. To an external observer, the modified system is merely a nonlinear feedback system. This led Truxal in 1963 to define an adaptive system as one designed from an adaptive viewpoint.

The following are basic concepts which are essential for any discussion of adaptive control:

Regulation and Tracking: The objective of control is to maintain the output variables of a given plant (or process) at desired values or within prescribed limits of such values. If the desired values are constant, the problem is one of regulation. If they are functions of time, the problem is one of tracking.

Direct and Indirect Control: These are two philosophically distinct approaches to controlling a system under uncertainty. In indirect control, the unknown plant parameters are first estimated and the appropriate control input is generated. In direct control, the control parameters are directly adjusted (based on additional information about the system) to optimize a performance index based on the output error. Indirect control can consequently be considered as parameter adaptive, while direct control is performance adaptive.

MRAC and STR: Model Reference Adaptive Control (MRAC) and the Self Tuning Regulator (STR) are two distinct approaches to adaptive control. In the former, which arose during investigations on deterministic (continuous-time) servomechanisms, the specifications are given in terms of a reference model. The MRAC problem is then to design a controller so that the output of the plant follows the output of the reference model. In contrast to MRAC, STR arose in the study of the stochastic regulation problem. STR can be either direct or indirect, and can be applied to either discrete-time or continuous-time systems. However, much of the STR literature is concerned with discrete-time plants using an indirect approach. In spite of the seeming differences between MRAC and STR, a direct correspondence exists between problems in the two areas.

The Adaptive Control Problem: Given a process whose parameters are known imprecisely, the adaptive control problem can be stated qualitatively as one of designing a controller which will result in the output following a desired output rapidly and with sufficient accuracy. As stated earlier, the desired output can be either a constant or a time-varying function. In the ideal case, when no external disturbances or noise are present, the theoretical objective is to make the output error tend to zero. In practice, however, the designer attempts to keep the output error within prescribed limits.

Strictly speaking, adaptive control is needed when the plant parameters are unknown and vary with time. However, for the sake of mathematical tractability, attention was confined during the first two decades to time-invariant systems with unknown parameters. The accepted philosophy was that if an adaptive system was fast and accurate when the plant parameters are constant, it would also prove satisfactory when the parameters varied with time, provided the latter occurred on a relatively slower time-scale.

3. Historical Background

The early days of adaptive control in the late 1950s and early 1960s coincided with one of the most active periods in the history of automatic control. The state vector representation of dynamical systems was introduced in the United States and system theoretic properties such as controllability and observability were defined. The linear quadratic regulator problem was resolved, and researchers became better acquainted with the outstanding work of Pontryagin and his co-workers on optimal control. The works of Lurie, Aizerman, Popov, Kalman, Lefschetz and LaSalle stimulated a great deal of interest in stability theory. Advances in stochastic estimation were accelerated by the introduction of the Bucy-Kalman filter, and Bellman's dynamic programming method was increasingly being used for sequential planning and optimal resource allocation.

While the above advances naturally had a profound impact on researchers on adaptive control, the research in the field nevertheless remained distinct from mainstream control theory in the United States in the 1960s. Motivated by a desire to make systems

adaptive, numerous ingenious schemes were suggested which were qualitative and experimental in nature. As a result, many of the theoretical concepts of control theory were either ignored or were absorbed very slowly by the newly developing field. A notable exception was the book, “Adaptive Control Processes- A Guided Tour” by Bellman, which was published in 1961. It provided numerous insights into problems that arise in the control of unknown systems, and for the first time attempted to enumerate the fascinating possibilities implied by the innocuous term “unknown”. According to Bellman, adaptive control processes represent the last of a series of three stages (after deterministic and stochastic) in the evolution of control processes. This, along with Truxal’s definition in 1963 (given earlier) that adaptive control is only in the eye of the designer, is quoted often in control circles and both have stood the test of time.
The close connection between identification and control in the context of partially known systems was stated succinctly around this time (1965) by Feldbaum, who introduced the term “dual control”. It defines the dual role played by the control input which must aid in the estimation of system characteristics even while attempting simultaneously to improve performance.

3.1. Gradient Based Adaptive Methods:

Research in the 1960s in adaptive control was confined mainly to two areas: (a) extremum adaptation (b) sensitivity models

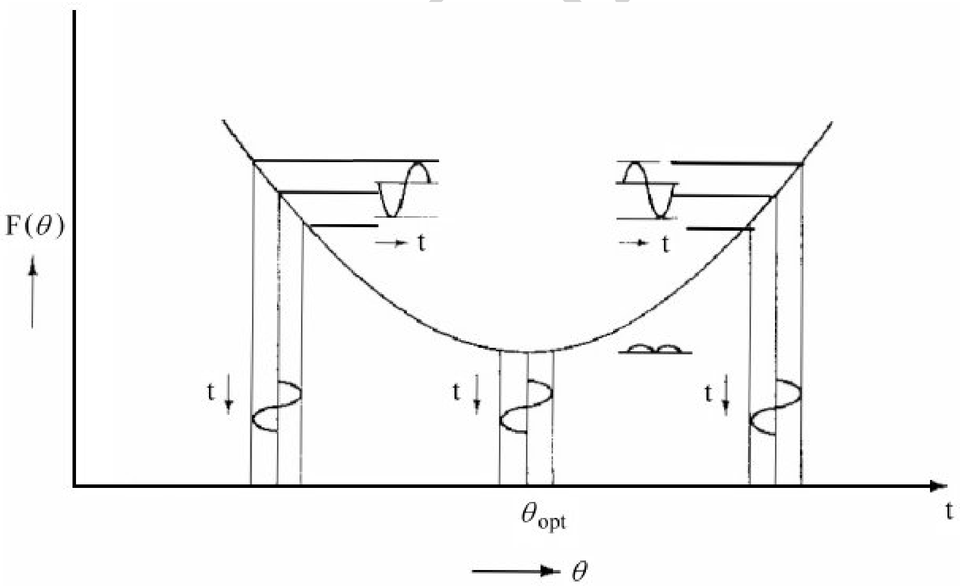

Figure1: Extremum Adaptation

a)Extremum Adaptation:

This was perhaps the most popular among the various methods investigated in the early 1960s. It had considerable appeal to researchers due to its simplicity, applicability to nonlinear plants, robustness and the fact that it does not require explicit identification of plant parameters. Draper and Li suggested the scheme in

> 1951 for optimizing the performance of an engine and for several years following that it was extensively investigated and its principal features were studied exhaustively.

The extremum adaptation method is also referred to as the parameter perturbation method. It is a direct method which involves perturbation and correlation (to determine the gradient of the performance parameter at the operating point) and adjustment of the parameter (along the negative gradient) to minimize the cost function. For example, the parameter perturbation is illustrated in Figure 1. If the optimal value of the parameter (corresponding to the minimum) is θ_{opt}, increasing the parameter if $\theta > \theta_{\text{opt}}$ $\theta > \theta_{\text{opt}}$ increases the performance index and decreases it if $\theta < \theta_{\text{opt}}$. The objective is therefore to determine the gradient of the function with respect to θ. If the plant contains a vector of adjustable parameters, the partial derivatives with respect to the latter are obtained by correlation. In practical problems, assumptions had to be made about the frequency of perturbation, period of correlation, and bandwidth of the system. The speed of adaptation was generally slow even for a single adjustable parameter and only local stability could be established. With increasing number of parameters the problem became substantially more complex and the method gradually passed into history.

b)Sensitivity Methods:

> Another gradient based method which enjoyed a wide following at this time was the sensitivity method which used sensitivity models. In contrast to parameter perturbation methods which are direct methods, sensitivity methods are indirect methods. They assume significantly more about the plant to be controlled and as a result yield faster and more accurate adaptive control systems.

An example of an adaptive system using the above approach is shown in Figure 2.

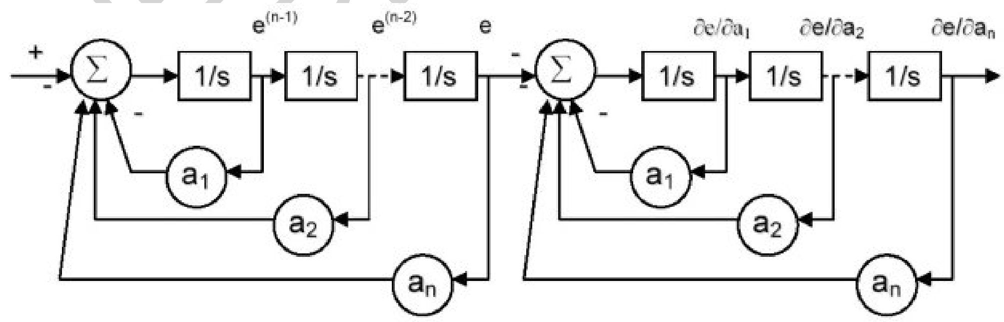

Figure 2: Adaptive system implementing sensitivity methods

$a_1, a_2, ...a_n$ are n parameters of a linear time-invariant system described by the differential equation

$$\frac{d^n e}{dt^n} + a_1 \frac{d^{n-1} e}{dt^{n-1}} + ... + a_n e = r(t),$$

where $e(t)$ is the output error of a dynamical system which is shown on the left side of Figure 2. The parameter vector θ is defined as $\theta^{\mathrm{T}} = \left[a_1, a_2, ...a_n\right]$. The objective is to determine the gradient $\frac{\partial e}{\partial \theta}$ of the error $e(t)$ with respect to θ at every instant of time and use it in turn to adjust θ. A sensitivity model is shown on the right side of Figure 2, and the input to this model is $e(t)$. The structure of the sensitivity model is seen to be identical to the error model (1). The state variables of the sensitivity model are seen to be the partial derivatives of the error with respect to the elements of θ.

While sensitivity methods are very elegant, they have serious drawbacks to adaptive control. In particular, they treat the adaptive system as linear with constant (or at best slowly time-varying) coefficients. This in turn, made stability analysis of such systems very difficult. For this reason, they gradually faded away from the scene and gave way to more rigorous methods which are described in the following sections.

Even though sensitivity methods are no longer in vogue, they have had a major impact on adaptive control, and many of the concepts and structures used in the latter have had their origins in these methods.

3.2. The MIT Rule and Park's Proof of Instability:

In 1958, Whitaker, Yamron and Kezer proposed a model reference adaptive scheme which used a reference model whose output represented the desired output of the plant. The parameter θ of the controller was adjusted based on the error e between the reference output and the output of the plant. The adaptive law attempted to adjust the parameter θ along the negative gradient of e^2 with respect to θ as follows:

$$\dot{\theta} = -\gamma e \frac{\partial e}{\partial \theta} \qquad \gamma > 0 \qquad (1)$$

However, since the plant parameters are unknown, $\frac{\partial e}{\partial \theta}$ could not be directly determined and hence was replaced by a differential signal according to a rule called the MIT (Massachusetts Institute of Technology) rule.

In 1966, in a paper of great historical significance, Patrick Parks conclusively demonstrated using a specific example that the use of the MIT rule could result in instability. At the same time, he also demonstrated that the system could be made stable using a design procedure based on Lyapunov's method. This tolled the death knell of gradient based adaptive systems and witnessed a gradual shift on the part of researchers to the design of adaptive systems based on stability methods.

4. Stable Adaptive Systems

In the gradient methods described thus far for the study of adaptive systems, the emphasis was on their performance. Once the design of the controller was completed, the stability of the adaptive system was analyzed and conditions for local stability were established. Since adaptive systems are nonlinear and time-varying, determining even sufficient conditions for stability was not always possible.

In view of the importance of stability in control system design, it was suggested in 1963 by Grayson that a reversal of the procedure adopted earlier would be more efficient. He argued that adaptive systems should be designed to be globally stable for all values of a parameter vector $\gamma \in \mathrm{S}$ and that optimization of the system could then be carried out by choosing $\gamma_{\mathrm{opt}} \in \mathrm{S}$ to optimize a performance criterion.

For example, if a dynamic system can be described by the differential equation

$$\begin{aligned} \dot{x} &= f\left[x, p, \theta, t\right] \\ y &= h\left[x, p, \theta, t\right] \end{aligned} \qquad (2)$$

where $x(t) \in R^n$ and $y(t) \in R$ are respectively the state vector and output of the system, p is an unknown parameter vector (representing the uncertainty in the plant) and $\theta(t)$ is a control parameter vector, let an adaptive law of the form

$$\dot{\theta} = g\left[y, \gamma, t\right] \qquad (3)$$

exist such that the nonlinear system described by Eqs. (2) and (3) is globally asymptotically stable for all values of $\gamma \in S$ where S is a compact set. This implies that stability and performance can be decoupled and that performance can be improved by choosing γ appropriately in S.

The above procedure is very similar to that used in the design of optimal regulators, where the optimal controller parameters are chosen in such a fashion that the poles of the overall system lie in the open left half of the complex plane.The suggestion that adaptive systems should be designed from a stability viewpoint was enthusiastically received by the adaptive control community. Numerous papers appeared in the control literature and the names of Shackcloth and Butchart, Parks, Monopoli, Philipson, Winsor and Roy are associated with them.

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Biographical Sketch

Kumpati S. Narendra received a B.E. degree with honors from Madras University in 1954, and the M.S. and Ph.D degrees from Harvard University in Applied Physics in 1955 and 1959 respectively. He was a member of the Harvard faculty from 1961 to 1965, when he joined the Yale faculty, and was made Professor in 1968.Currently he is the Harold W. Cheel Professor of Electrical Engineering and the Director of the Center for Systems Science at Yale University. He received an honorary Doctor of Science degree from his alma mater in 1995, and an honorary Doctor of Science degree from the National University of Ireland in June, 2007.

Professor Narendra is the author of over 200 technical papers and three books, and the editor of four others in the areas of stability theory, adaptive control, learning automata, and artificial neural

networks. He received the Franklin V. Taylor Award of the IEEE SMC Society in 1972, the George S. Axelby Best Paper Award of the Control Systems Society in 1987, and the best paper award of the Neural Network Council in 1991.

His recent research is focused on the control of complex systems using multiple models.

Professor Narendra has received numerous honors for his work. These include the Ragazzini Education Award of the American Automatic Control Council (AACC), The Neural Networks Leadership Award, and The Bode Prize of the IEEE Control Systems Society. In 2003, he was awarded the Richard E. Bellman Control Heritage Award of the AACC, which is the highest professional achievement for U.S. control systems engineers and scientists.

Professor Narendra has been a consultant to several industrial laboratories for four decades. He has lectured widely around the world, and has taken part in numerous national and international conferences.