STOCHASTIC ADAPTIVE CONTROL

T. E. Duncan
Department of Mathematics, University of Kansas, Lawrence, KS 66045, USA

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Contents

Summary

This chapter describes some results in stochastic adaptive control. Stochastic adaptive control is the control of an unknown stochastic system. To identify an unknown linear stochastic system in discrete or continuous time, a weighted least squares algorithm is used that converges to a random variable and a modification of this family of estimates converges to the true parameter value.

An adaptive pole placement problem is solved to stabilize an unknown ARMAX system. A linear quadratic Gaussian control problem is solved for an unknown linear stochastic system by using the weighted least squares estimates for a certainty equivalence control.

This control law achieves the optimal cost for the known system. An adaptive control problem for a finite Markov chain is solved. Some generalizations of adaptive control problems are described.

1. Introduction

Stochastic adaptive control is the control of a partially known or completely unknown stochastic system. A physical system is often subject to perturbations and a model for a physical system is only an approximation so there are unmodeled dynamics.

These perturbations or unmodeled dynamics are often described by noise entering the model. For physical systems, it is often important or even necessary to control the system to have some desirable behavior. Typically in the mathematical modeling of a physical system there are unknown or unspecified parameters. The control of an incompletely known stochastic system is a problem of stochastic adaptive control. The fact that a mathematical model is stochastic allows for the representation of these perturbations or unmodeled dynamics.

The control of an incompletely known system, that is adaptive control, is a fundamental problem in control theory; often perturbations or unmodeled dynamics of a system require a stochastic model. These problems of stochastic adaptive control occur in a broad cross-section of the control of physical systems.

Some well known applications are guidance and control systems, fault diagnosis, telecommunications, medicine, thermal processes, electric power, finance, process and production control and traffic systems (see Automation and Control Applications).

In this chapter, some distinct stochastic adaptive control problems are described. Initially the adaptive control of an ARMAX model is given. An ARMAX (autoregressive moving average with exogenous inputs) model is a discrete time linear stochastic system.

It is assumed that the parameters of this model are unknown. A weighted least squares algorithm is used to estimate the unknown parameters. This family of estimates converges and with an additional attenuating excitation added to the control the family of estimates is strongly consistent.

An adaptive pole placement procedure is used to stabilize the unknown system. For a linear quadratic Gaussian control problem for this unknown ARMAX system, a certainty equivalence control achieves the optimal cost for the known system.

An adaptive control problem is solved for a continuous time linear stochastic system described by a stochastic differential equation where the parameter matrices are unknown. A weighted least squares algorithm in continuous time converges and with an additional attenuating excitation added to the control the family of estimates is strongly consistent, that is, converges to the true parameter value.

For a linear quadratic Gaussian control problem for this unknown linear stochastic system, a lagged certainty equivalence control achieves the optimal cost for the known system. Some generalizations of adaptive control to nonlinear systems and stochastic partial differential equations are briefly described.

2. Adaptive control of Markov Chains

An elementary and important family of stochastic control problems is the control of finite state Markov chains. An early topic in stochastic adaptive control was the adaptive control of these finite state Markov chains.

A Markov chain is described by its transition probabilities and for the adaptive control problem it is assumed that the transition probabilities are controlled and that they depend on an unknown parameter.

Let $\mathcal{S}$ be a finite state space for the Markov chain and U be the finite set of control actions. For each parameter $\alpha \in \mathcal{A}, p(i, j, u, \alpha)$ is the probability of making a transition from state i to state j using the control u. The state of the system at time t, x_t, is observed for all t and the control $u_t \in U$ is chosen based on x_t, that is, $u_t = \phi(\alpha, x_t)$. Since α is unknown, it is necessary to estimate it at each t, which is denoted $\hat{\alpha}_t$, and the adaptive control is $u_t = \phi(\hat{\alpha}_t, x_t)$.

The unknown parameter is estimated by the maximum likelihood method so that $\hat{\alpha}_t$ satisfies

$$\begin{aligned} P(x_0,...,x_t \mid x_0,u_0,...,u_{t-1},\hat{\alpha}_t) &= \prod_{i=0}^{t-1} p(x_i, x_{i+1}, u_i, \hat{\alpha}_t) \\ &\geq \prod_{i=0}^{t-1} p(x_i, x_{i+1}, u_i, \alpha) \\ &= P(x_0,...,x_t \mid x_0,u_0,...,u_{t-1},\alpha) \qquad (1) \end{aligned}$$

for all $\alpha \in \mathcal{A}$. If the likelihood function is maximized at more than one value of α, then a unique value is chosen according to some fixed priority ordering.

The following assumptions are used

A1. $\mathcal{A}$ is a compact set

A2. There is an $\varepsilon > 0$ such that for each pair (i, j) either $p(i, j, u, \alpha) > \varepsilon$ for all (u, α) or $p(i, j, u, \alpha) = 0$ for all (u, α).

A3. For each pair (i, j), there is a sequence $i_0, i_1, ..., i_r$ such that for all $(u, \alpha), p(i_{s-1}, i_s, u, \alpha) > 0 \;\; s = 1, ..., r+1$ where $i_0 = i$ and $i_{r+1} = j$.

The assumption A2 guarantees that the probability measures $P(x_0,...,x_n \mid x_0,u_0,...,u_{n-1},\alpha)$ for $\alpha \in \mathcal{A}$ are mutually absolutely continuous. The assumption A3 guarantees that the Markov chain generated by the transition probabilities $p(i, j, \phi(\alpha, i), \alpha)$ has a single ergodic class, that is, all states communicate with all states.

The following theorem verifies the convergence of the family of maximum likelihood estimates.

Theorem 1 *If* A1 *and* A3 *are satisfied and* $(\hat{\alpha}_t, t \in \mathbb{N})$ *is the family of maximum likelihood estimates* (1), *then there is a random variable* α^* *and a finite random time* T *such that*

1. $\hat{\alpha}_t = \alpha^*$ *a.s.*(*almost surely*) *for* $t \geq T$
2. $p(i, j, \phi(\alpha^*, i), \alpha^*) = p(i, j, \phi(\alpha^*, i), \alpha_0)$ *a.s. for all* $i, j \in \mathcal{S}$ *where* α_0 *is the true parameter value.*

To obtain strong consistency of the family of maximum likelihood estimates another assumption is introduced

A4. If $\alpha, \alpha' \in \mathcal{A}$ and $\alpha \neq \alpha'$, then there is an $i \in \mathcal{S}$ such that

$$[p(i,1,u,\alpha),..., p(i,r,u,\alpha)] \neq [p(i,1,u,\alpha'),..., p(i,r,u,\alpha')]$$

for each $u \in U$.

This assumption means that the transition vectors are distinct and is sometimes called an identifiability condition. The contrast function for the maximum likelihood estimates is

$$f(i, j, u, \alpha) = \begin{cases} -\text{In}\, p(i, j, u, \alpha) & \text{if } p(i, j, u, \alpha) \neq 0 \\ 1 & \text{if } p(i, j, u, \alpha) = 0. \end{cases}$$

The following theorem provides strong consistency of the family of maximum likelihood estimates and optimality of an adaptive control.

Theorem 2 *If* A1-A4 *are satisfied,* $p(i, j, u, \alpha), c(i, j, u), \phi(\alpha, i)$ *and* $f(i, u, \alpha)$ *are continuous functions,* $p(i, j, u, \alpha) > 0$ *for all* i, j, α, u *then*

(i) *For any control*

$$\lim_{t \to \infty} \hat{\alpha}_t = \alpha_0 \quad a.s.$$

(ii) *If* $\phi(\hat{\alpha}_t, x_i) = \prod_t (x_0, u_0, ..., x_t)$ *and* $\phi(\alpha_0, x_t)$ *is an optimal control, then*

$$\lim_{t \to \infty} \frac{1}{t} \sum_{i=0}^{t-1} c(x_i, x_{i+1}, u_i) = J(\alpha_0) \quad a.s.$$

where $(\hat{\alpha}_t, t \geq 0)$ *is the family of maximum likelihood estimates and* $u_t = \phi(\hat{\alpha}_t, x_t)$.

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Biographical Sketch

T. E. Duncan is Professor of Mathematics at the University of Kansas. He received the B.E.E. degree from Rensselaer Polytechnic Institute in 1963 and the M.S. and Ph.D. degrees from Stanford University in 1964 and 1967, respectively. He has held regular positions at the University of Michigan (1967-1971), the state University of New York, Stony Brook (1971-1974), and the University of Kansas (1974-present). He has held visiting positions for an academic year at the University of California, Berkeley (1969-1970), the University or Bonn, Germany (1978-1979), and Harvard University (1979-1980) and shorter visiting positions at many other institutions.

Dr. Duncan is the author or co-author of more than one hundred fifty published papers. He is currently a corresponding Editor of SIAM Journal on Control and Optimization and is a member of AMS, IEEE, MAA and SIAM. He is an IEEE Fellow.