

ORBITAL RESONANCES IN PLANETARY SYSTEMS

Renu Malhotra

Lunar & Planetary Laboratory, The University of Arizona, Tucson, AZ, USA

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Summary

There are two main types of resonance phenomena in planetary systems involving orbital motions: (i) *mean motion resonance*: This is intuitively the most obvious type of resonance; it occurs when the orbital periods of two planets are close to a ratio of small integers; (ii) *secular resonance*: this is a commensurability of the frequencies of precession of the orientation of orbits, as described by the direction of pericenter and the direction of the orbit normal. It is often possible to identify an unperturbed subsystem and separately a resonant perturbation, which facilitates the use of perturbation theory and other analytical and numerical tools. Resonances can be the source of both stability and instability, and play an important role in shaping the overall orbital distribution and the ‘architecture’ of planetary systems. This chapter provides an overview of these resonance phenomena, with simple models that elucidate our understanding.

1. Introduction

Consider the simplest planetary system consisting of only one planet, of mass m_1 , orbiting a star of mass m_0 . Let r_0 and r_1 denote the inertial coordinates of these two bodies. This system has six degrees of freedom, corresponding to the three spatial degrees of freedom for each of the two bodies. Three of these degrees of freedom are made ignorable by identifying them with the free motion of the center-of-mass. The remaining three degrees of freedom can be identified with the coordinates of the planet relative to the star and the problem is reduced to the familiar planetary problem described by the Keplerian Hamiltonian,

$$\mathcal{H}_{\text{Kepler}} = \frac{|\mathbf{p}|^2}{2m} - \frac{GMm}{|\mathbf{r}|} \quad (1)$$

where \mathcal{G} is the universal constant of gravitation, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ is the position vector of the planet relative to the star, $\mathbf{p} = m d\mathbf{r} / dt$ is the linear momentum of the reduced mass,

$$m = \frac{m_0 m_1}{m_0 + m_1}, \quad (2)$$

and $M = m_0 + m_1$ is the total mass. In this Hamiltonian description, \mathbf{r} and \mathbf{p} are canonically conjugate variables. The general solution of this classic two-body problem is well known in terms of conic sections; the bound solution is called the *Keplerian* ellipse. In this chapter, we will be concerned with only the bound orbits.

The three degrees of freedom for the Kepler system can also be described by three angular variables, one of which measures the motion of the planet in its elliptical orbit and the other two describe the orientation of the orbit in space. The size, shape and orientation of the orbit are fixed in space, and there is only one non-vanishing frequency, namely, the frequency of revolution around the orbit. The orbital elements illustrated in Figure 1 are related to the set of action-angle variables for the two-body problem derived by Charles Delaunay (1816–1872) [see Chapter 1],

$$\left. \begin{aligned} L &= \sqrt{\mathcal{G}Ma}, & \ell &= \text{mean anomaly} \\ G &= \sqrt{\mathcal{G}Ma(1-e^2)}, & \omega &= \text{argument of pericenter,} \\ H &= \sqrt{\mathcal{G}Ma(1-e^2)} \cos i, & \Omega &= \text{longitude of ascending node,} \end{aligned} \right\} \quad (3)$$

where a , e and i are the semimajor axis, eccentricity and inclination, respectively, of the bound Keplerian orbit. The mean anomaly, ℓ , is related to the orbital frequency (mean motion), n , which in turn is related to the semimajor axis by Kepler's third law of planetary motion:

$$\dot{\ell} = n = \left(\mathcal{G}M / a^3 \right)^{\frac{1}{2}}. \quad (4)$$

In Eqs. (3), L, G, H are the action variables and ℓ, ω, Ω are the canonically conjugate angles, known as the *mean anomaly*, *argument of pericenter* and *longitude of ascending node*, respectively. As defined in Eq. (3), the action variables have dimensions of specific angular momentum.

The Kepler Hamiltonian can be expressed in terms of the orbital elements and the Delaunay variables:

$$\mathcal{H}_{\text{Kepler}} = -\frac{\mathcal{G}Mm}{2a} = -\frac{(\mathcal{G}M)^2 m}{2L^2}. \quad (5)$$

For the case of nearly co-planar and nearly circular orbits, we will also make use of a set

of modified Delaunay variables defined by the following canonical set:

$$\left. \begin{aligned} \Lambda &= L, & \lambda &= \ell + \omega + \Omega, \\ \Gamma &= L - G, & \gamma &= -\omega - \Omega \equiv -\varpi, \\ \Upsilon &= L - G - H, & \nu &= -\Omega. \end{aligned} \right\} \quad (6)$$

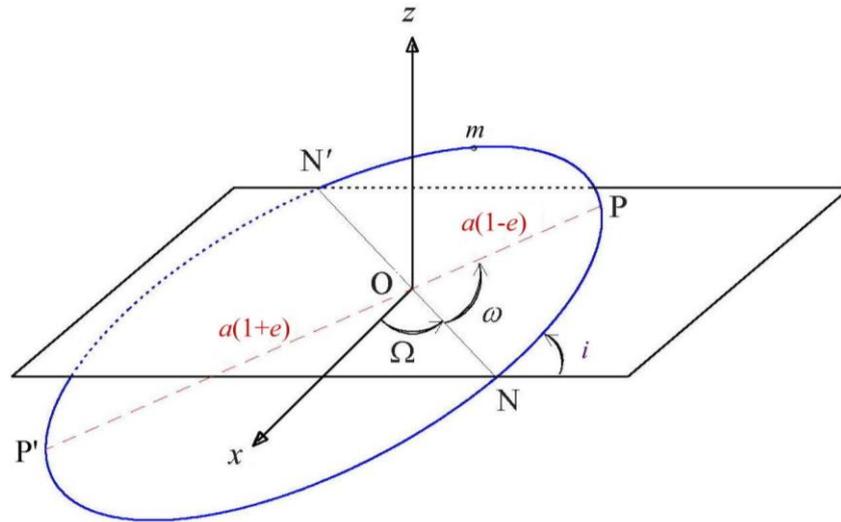


Figure 1. The Keplerian orbit: a planet, m , traces out an ellipse of *semimajor axis* a and *eccentricity* e , with the Sun at one focus of the ellipse (which is the origin of the heliocentric coordinate system indicated here). The plane of the orbit has *inclination* i with respect to the fixed reference plane, and intersects the latter along the line of nodes, NN' , where ON defines the *ascending node*; the longitude of ascending node, Ω , is the angle from the reference direction x to ON ; it is measured in the reference plane.

The pericenter is at P , so the distance OP is $a(1-e)$; the apocenter is at P' and the distance OP' is $a(1+e)$; the *argument of perihelion* ω is the angle from ON to OP ; it is measured in the orbital plane. The *true anomaly* is the instantaneous angular position of the planet measured from OP .

For multiple planets around the star, it is desirable to describe the system as a sum of two-body Keplerian Hamiltonians plus the smaller interaction part (the potential energy of the planet-planet interactions). However a similar approach with coordinates relative to the central mass (called ‘heliocentric coordinates’ in the context of the solar system, more generally ‘astrocentric coordinates’) does not yield a Hamiltonian that is a sum of two-body Keplerian parts plus an interaction part, as we might naively expect. This is because the kinetic energy is not a diagonal sum of the squares of the momenta in relative coordinates. This problem is overcome by using a special coordinate system invented by Carl Jacobi (1804–1851), in which we use the coordinates of the center-of-mass, and then, successively, the coordinates of the first planet relative to the star, the coordinates of the second planet relative to the center-of-mass of the star and the first planet, and so on. For a system of N planets orbiting a star, let \mathbf{r}_i ($i=0,1,\dots,N$) denote the coordinates of the star and the N planets in an inertial reference frame; then the

Jacobi coordinates are given by

$$\tilde{\mathbf{r}}_0 = \frac{\sum_{j=0}^N m_j \mathbf{r}_j}{\sum_{j=0}^N m_j}; \quad \tilde{\mathbf{r}}_i = \mathbf{r}_i - \mathbf{R}_{i-1}, \quad \text{with } \mathbf{R}_i = \frac{\sum_{j=0}^i m_j \mathbf{r}_j}{\sum_{j=0}^i m_j}, \quad (7)$$

and the conjugate momenta,

$$\tilde{\mathbf{p}}_i = \tilde{m}_i \dot{\tilde{\mathbf{r}}}_i, \quad \text{with } \tilde{m}_i = \frac{m_i \sum_{j=0}^{i-1} m_j}{\sum_{j=0}^i m_j}. \quad (8)$$

Then the Hamiltonian for the N -planet system is given by

$$\mathcal{H} = \frac{\tilde{p}_0^2}{2 \sum_{j=0}^N m_j} + \sum_{i=1}^N \frac{\tilde{p}_i^2}{2 \tilde{m}_i} - \sum_{i=1}^N \frac{G m_0 m_i}{r_{i0}} - \sum_{0 < i < j} \frac{G m_i m_j}{r_{ij}}, \quad (9)$$

and r_{i0} is the distance between the star and the i^{th} planet, and r_{ij} is the distance between planet i and planet j . Because r_{i0} and r_{ij} do not depend upon the center-of-mass position, this Hamiltonian is independent of $\tilde{\mathbf{r}}_0$, and it follows that $\tilde{\mathbf{p}}_0$ is a constant. Thus, the first term in Eq. (9), which is the center-of-mass kinetic energy, is a constant. By construction, the remaining kinetic energy terms are a diagonal sum of the squares of the new momenta. We can now obtain a Hamiltonian that is a sum of N unperturbed Keplerian Hamiltonians and a small perturbation:

$$\mathcal{H} = \sum_{i=1}^N \left[\frac{\tilde{p}_i^2}{2 \tilde{m}_i} - \frac{G m_0 m_i}{\tilde{r}_i} \right] - \sum_{0 < i < j} \frac{G m_i m_j}{r_{ij}} + \sum_{i=1}^N \left[\frac{G m_0 m_i}{\tilde{r}_i} - \frac{G m_0 m_i}{r_{i0}} \right] \quad (10)$$

In deriving Eq. (10) from Eq. (9), we omitted the constant center-of-mass kinetic energy term and we added and subtracted $\sum_{i=1}^N \frac{G m_0 m_i}{\tilde{r}_i}$. In Eq. (10), we can recognize the first

series as a sum of N independent Keplerian Hamiltonians. The second series describes the direct planet-planet interactions. The last series consists of terms that are differences of two large quantities; these difference terms are each of order $\sim m_i m_j$, i.e., of the same order as the terms in the direct planet-planet interactions, and is referred to as the ‘indirect’ perturbation. Thus, the Hamiltonian of Eq. (10) is of the form

$$\mathcal{H} = \sum_{i=1}^N \mathcal{H}_{\text{Kepler}}^{(i)} + \mathcal{H}_{\text{interaction}} \quad (11)$$

which is suitable for the tools of perturbation theory.

An important special case of the perturbed system is when one of the bodies is of infinitesimal mass, a ‘test particle’. The test particle does not affect the massive bodies but is perturbed by them. Let the unperturbed orbit of the test particle be a Keplerian ellipse about m_0 . Then the specific energy of the test particle can be written as a sum of its unperturbed Keplerian Hamiltonian, $\mathcal{H}_{\text{tp}} = -\frac{\mathcal{G}m_0}{2a}$, where the subscript tp abbreviates ‘test particle’, and an interaction part owing to the perturbations from N planets,

$$\mathcal{H}_{\text{tp,interaction}} = -\sum_{j=1}^N \mathcal{G}m_j \left[\frac{1}{|\mathbf{r}_{\text{tp}} - \mathbf{r}_j|} - \frac{(\mathbf{r}_{\text{tp}} - \mathbf{r}_0) \cdot (\mathbf{r}_j - \mathbf{r}_0)}{|\mathbf{r}_j - \mathbf{r}_0|^3} \right] \quad (12)$$

The perturbations, $\mathcal{H}_{\text{interaction}}$, cause changes in the Keplerian orbital parameters. The Delaunay variables are of course no longer action-angle variables, but they provide a useful canonical set; we will make use of it in the following sections. Qualitatively, the perturbed Keplerian orbit gains two slow frequencies, the precession of the direction of pericenter and the precession of the line of nodes (equivalently, the pole) of the orbit plane; these are slow relative to the mean motion, n .

Resonance

A *secular resonance* involves a commensurability amongst the slow frequencies of orbital precession, whereas a *mean motion resonance* is a commensurability of the frequencies of orbital revolution. The timescales for secular perturbations are usually significantly longer than for [low order] mean motion resonant perturbations, but there is also a coupling between the two which leads to resonance splittings and chaotic dynamics. The boundaries (or separatrices) of mean motion resonances are often the sites for such interactions amongst secular and mean motion resonances.

A mean motion resonance between two planets occurs when the ratio of their mean motions or orbital frequencies n_1, n_2 is close to a ratio of small integers, $(p+q)/p$ where $p \neq 0$ and $q \geq 0$ are integers. The case $q=0$ is sometimes called a corotation or co-orbital resonance; a prominent example in the solar system is the Trojan asteroids that share the mean motion of Jupiter but librate approximately $\pm 60^\circ$ from Jupiter’s mean longitude. When $q > 0$, it is called the order of the resonance; this is because the strength of the resonant potential is proportional to e^q or i^q when the eccentricities e and inclinations i of the planets are small. Inclination resonances occur only for even values of q . In a resonant configuration, the longitude of the planets at every q^{th} conjunction librates slowly about a direction determined by the lines of apsides and nodes of the planetary orbits. In terms of the action-angle variables for the Keplerian Hamiltonian, this geometry is naturally described by the libration of a so-called *resonant angle* which is a linear combination of the angular variables. For example, for the 2:1 mean motion resonance between a pair of planets, two possible resonant angles are

$$\phi_1 = 2\lambda_2 - \lambda_1 - \varpi_1, \quad \phi_2 = 2\lambda_2 - \lambda_1 - \varpi_2. \quad (13)$$

Close to the 2:1 mean motion resonance, both these angles have very slow variation (slow in comparison with the mean motions). The planet pair is said to be *in resonance* if at least one resonant angle exhibits a libration; in this case, the long term average rate of the resonant angle vanishes, and we speak instead of its ‘libration frequency’. If a resonant angle does not librate but rather varies over the entire range 0 to 2π cyclically, we speak of its ‘circulation frequency’.

How close does the mean motion ratio need to be for a planet pair to be considered resonant? There is not a precise answer to this question. A rough answer is provided by an estimate of the range, Δn , of orbital mean motion over which it is possible for the resonant angle to librate. For nearly circular orbits, and for $0 \leq q \leq 2$, this estimate is given by

$$\frac{\Delta n}{n} \simeq \mu^{\frac{q+1}{3}} \quad (14)$$

where μ is the planet-Sun mass ratio.

Amongst the major planets of the solar system, no planet pair exhibits a resonant angle libration, although several are close to resonance: Jupiter and Saturn are within 1% of a 5:2 resonance, Saturn and Uranus are within 5% of a 3:1 resonance, and Uranus and Neptune are within 2% of a 2:1 resonance. In the first extra-solar planetary system to be discovered, the three-planet system PSR B1257+12, the outer two planets are within 2% of a 3:2 resonance. None of these is close enough to exact resonance to exhibit a resonant angle libration. Amongst the several hundred extra-solar multiple planet systems detected by the *Kepler* space mission recently, it is estimated that at least ~30% harbor near-resonant pairs. One extra-solar planetary system, GJ 876, with four planets, appears to have at least two pairwise 2:1 resonances close enough to be in libration. In some of these cases, the nearness to resonance causes orbital perturbations large enough to be detectable, and has allowed measurements of the planetary masses and orbital inclinations.

Somewhat in contrast with the planets, several pairs of satellites of the solar system’s giant planets exhibit librations of resonant angles; these include the Galilean satellites Io, Europa and Ganymede of Jupiter, and the Saturnian satellite pairs Janus and Epimetheus, Mima and Tethys, Enceladus and Dione, Titan and Hyperion. The existence of these near-exact commensurabilities, as evidenced by the librating resonant angles, in the satellite systems has been a subject of much study over the past few decades. These are now generally understood to be the consequence of very small dissipative effects which alter the orbital semimajor axes sufficiently over very long timescales so much so that initially well separated non-resonant orbits evolve into an exact resonance state characterized by a librating resonant angle. Once a resonant libration is established, it is generally stable to further adiabatic changes in the individual orbits due to continuing dissipative effects. This hypothesis provides a plausible explanation for the most prominent cases of mean motion resonances amongst

the Jovian and Saturnian satellites. However, the Uranian satellites present a challenge to this view, as there are no exact resonances in this satellite system, and it is unsatisfactory to argue that somehow tidal dissipation is vastly different in this system. An interesting resolution to this puzzle was achieved when the dynamics of orbital resonances was analyzed carefully and the role of the small but significant splitting of mean motion resonances and the interaction of neighboring resonances was recognized. Such interactions can destabilize a previously established resonance, so that mean motion resonance lifetimes can be much shorter than the age of the solar system. Studies of the Jovian satellites, Io, Europa and Ganymede, also suggest a dynamic, evolving resonant orbital configuration over the history of the solar system.

Another notable example of resonance in the solar system is the dwarf planet Pluto whose orbit is resonant with the planet Neptune, and exhibits a libration of a 3:2 resonant angle; the origin of this mean motion commensurability is now understood to be due to the orbital migration of Neptune driven by interactions with the disk of planetesimals left over from the planet formation era. Studies of this mechanism have led to new insights into the early orbital migration history of the solar system's giant planets, and it is a very active area of current research.

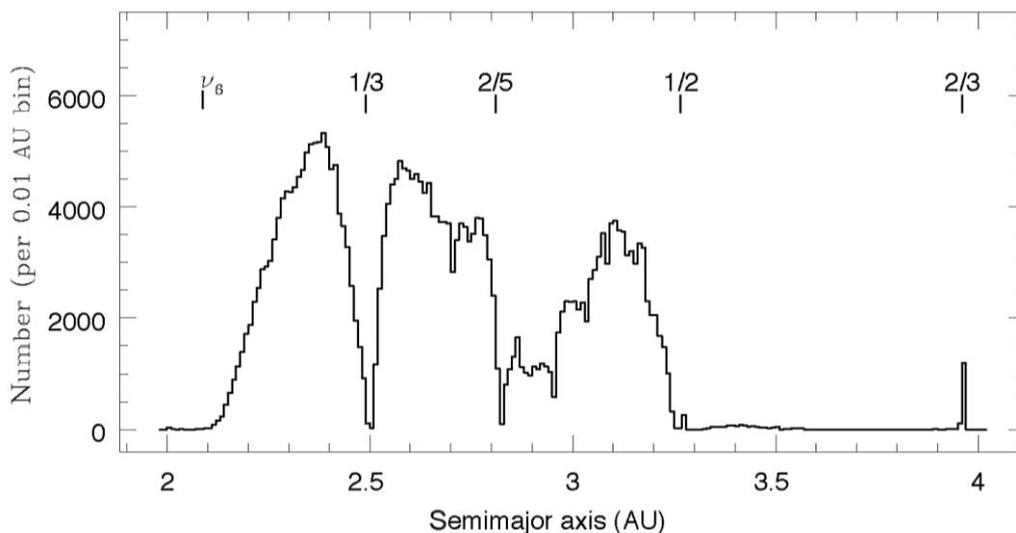


Figure 2. The semimajor axis distribution of asteroids in the main asteroid belt. The locations of several resonances are indicated near the top. (Data for all numbered asteroids from <http://hamilton.dm.unipi.it/astdys/>; synthetic proper elements computed numerically.)

The population of minor planets in the main asteroid belt in the solar system offers one of the most well-studied examples of the role of orbital resonances in shaping the distribution of orbits. Figure 2 plots the distribution of semimajor axis of asteroids in the main asteroid belt. (Note that some of the non-uniformities in the number distribution are attributable to observational selection effects: astronomical surveys for faint bodies in the solar system remain quite incomplete, so that many smaller and more distant objects remain undiscovered.) The inner edge of the asteroid belt is defined by a secular resonance, known as the ν_6 secular resonance, in which the apsidal secular precession

rate of an asteroid is nearly equal to the apsidal precession rate of Saturn. There are several prominent deficits coinciding with the locations of mean motion resonances with Jupiter; this correlation was first noted by Daniel Kirkwood (1814–1895) and the deficits are known as the Kirkwood Gaps. Interestingly, these gaps are significantly wider than would be anticipated by simple estimates of the resonant widths, such as in Eq. (14). Deeper analyses have revealed that chaotic dynamics owed to the small secular variations of the orbit of Jupiter are very important in widening the Kirkwood gaps, and, beyond that, even the early orbital migration history of Jupiter and Saturn is recorded in the widths and shapes of these gaps.

There also exist orbital resonances that do not neatly fall into the categories of ‘mean motion resonance’ or ‘secular resonance’. For example, the angular velocity of the apsidal precession rate of a ringlet within the C-ring of Saturn is commensurate with the orbital mean motion of Titan, the so-called Titan 1:0 apsidal resonance. Two retrograde moons of Jupiter, Pasiphae and Sinope, exhibit a 1:1 commensurability of their perijove apsidal precession rate with Jupiter’s heliocentric apsidal precession rate. So-called three-body resonances which involve a sequence of commensurable mean motions of a test particle with two planets have been identified as a source of weak chaos and orbital instability on Gigayear timescales; these may explain the absence of asteroids in some regions of the solar system that otherwise appear to be stable. A class of resonances known as ‘super resonances’ or ‘secondary resonances’ have been identified in the very long term evolution of planetary and satellite orbits; these are defined by small integer ratio commensurabilities between the libration frequency of a resonant angle and the circulation frequency of a different resonant angle. Pluto’s orbit and the Uranian satellite system provide two well-studied examples of this type of resonance.

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Biographical Sketch

Renu Malhotra (born in 1961 in New Delhi, India) earned her M.S. in Physics from the Indian Institute of Technology in Delhi in 1983, and her Ph.D. in Physics from Cornell University in 1988. She did post-doctoral research at Cornell and at Caltech, and worked as a staff scientist at the Lunar and Planetary Institute in Houston. In 2000 she joined the faculty of The University of Arizona in Tucson, where she is currently Professor and Chair of the Theoretical Astrophysics Program. Her research in orbital mechanics has spanned a wide variety of topics, including extra-solar planets and debris disks around stars, the formation and evolution of the Kuiper belt and the asteroid belt, the orbital resonances amongst the moons of the giant planets, and the meteoritic bombardment history of the planets. She has revolutionized our understanding of the early history of the solar system by using the orbital resonance between Pluto and Neptune to infer large-scale orbital migration of the giant planets and to predict the existence of the "Plutinos" and other small planets in resonance with Neptune. She has been the recipient of honors and awards from the American Astronomical Society (Harold C. Urey Prize, 1997), the International Astronomical Union (asteroid 6698 named 'Malhotra', 1997), the IIT-Delhi (Distinguished Alumna, 2006) and The University of Arizona (Galileo Fellow, 2010).