GEOMAGNETISM AND GEOELECTRICITY

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Contents

Summary

Earth's magnetism has been a subject of interest since the first century AD We give a brief summary of developments leading to the discovery of the inextricable link between magnetism and electricity by Oersted in 1820, and the definition of the unit of measurement for the field by Gauss, in terms of the three basic units of measurement (length, time, and mass) in 1832. These discoveries, together with those of Faraday and

Ohm, eventually led to the Maxwell equations governing the evolution of the magnetic and electrical fields in an electrically conducting medium.

The study of Earth's magnetic field both theoretically and observationally is discussed. The main features of the magnetic field as depicted by Gauss spherical harmonic analysis are summarized. The crustal field in the form of continental and oceanic anomalies is examined, and the temporal variations of the field due to ionospheric currents are discussed. The electric fields and currents associated with the temporal field are now influencing communication and power systems on the ground as well as air transport systems.

The origin of the main magnetic field is now firmly believed to be the dynamo produced by the interaction of the fluid motions and the magnetic field in the outer core. The development of the progress in dynamo theory, the energy source for the dynamo (precession, rotation, thermal convection, and compositional convection), and the interaction between the core and its boundary are discussed.
The study of the ancient field relies heavily on the physics of rocks and their magnetic properties. The advances made in this area have permitted the study of the behavior of the magnetic field variations and reversals over millions of years.

The direct influence of geomagnetism and geoelectricity on life is briefly discussed.

1. Introduction

Geomagnetism and Geoelectricity are the result of the interest in studying the Earth's magnetism, which has been known for a very long time. The inextricable link between magnetism and electricity discovered in the eighteenth century introduced electricity to the studies of Earth. The interaction of electromagnetic forces with motions in the outer fluid core of Earth is now believed to be responsible for maintaining Earth's magnetic field over geological time. Nowadays electrical effects in Earth's atmosphere are known to influence communications and therefore a thorough understanding of such effects is essential for technological advances in communications worldwide as well as in space exploration.

This article gives a summary of the developments and progress in our understanding of geomagnetism and Geoelectricity and their usage.

2. The History of Studies of Magnetism

There is no known date for the discovery of magnetism, but it is believed to have been recognized from time immemorial. It is, however, safe to assume that it came about as a natural observation that magnetic rocks, or lodestones as they are called, attract metals. The information available about early observations and experiments with magnetic objects are sketchy, but it is known that the Chinese observed that a spoon made from lodestone and spun on a smooth surface came to rest pointing towards the north in the first century AD. The discovery that an elongated magnetic object spun on a smooth surface always came to rest pointing towards the north was perhaps the main reason for the subsequent interest in studies of magnetism, because it provided an aid to navigation on land and sea. The earliest known attempt to construct a compass was made by the Chinese in 1088 AD, and later by Europeans in 1190. The expansion in trade and the quest for discovering the world in the fifteenth century focused attention in Europe on the use of Earth's magnetic field as an effective tool for navigation, and led to a number of investigations into the nature and properties of magnetic objects. At this time, it was found that the needle of a compass does not point to the geographic North Pole, but to a point close to it.

The angle between the horizontal direction of the needle and the geographic north, termed the declination, varied from one place to another. The variation in declination was confirmed by measurements made by Christopher Columbus during his voyage to the New World (that is, America). In 1581, R. Norman constructed his dip-circle experiment and demonstrated the existence of inclination, which is the angle between the direction of the magnetic field and the horizontal. In 1600, William Gilbert published "De Magnete," the first book on magnetism. The book contained the description of a number of carefully constructed experiments on the properties of magnetism. The book not only disproved a number of superstitious beliefs about magnetism (like the belief that magnetic substances lose their magnetism when rubbed with garlic), but it also presented a novel scientific approach to studying the subject, and advocated new ideas about Earth's magnetic field, albeit without proof, which were later found to be true. He made the assertion that Earth is a magnet, and that Earth's magnetic field has an internal origin. These ideas were to have a profound effect on research in geomagnetism for centuries to follow.

The discovery of the variation of the magnetic field both in direction and in magnitude with position motivated a number of projects to design magnetic charts in order to use them for traveling across the oceans of the world. A number of charts were constructed for the Atlantic Ocean, but it was soon realized that such charts had to be adjusted regularly, because Gellilbrand discovered in 1635 that the magnetic field at one location changes direction with time, a phenomenon now known as secular variation. Halley in 1693 used variations in the declination D from different places recorded at different times to infer that the magnetic field drifts slowly towards the west. The westward drift is still known to occur for part of the secular variation.

By the seventeenth century, most of the general observational properties of Earth's magnetic field were known, but there were still no known methods for measuring the strength (that is, the intensity) of the field. This required the definition of a unit of magnetic field in terms of the three known basic units of measurement: mass, length and time. Gauss in 1832 designed an experiment that used the fact that a magnetic needle will oscillate about its equilibrium position when disturbed slightly from that position. The strength of the field varies as the square of the frequency of the oscillation. The experiment showed that the unit of magnetic field has the dimension $m\ g^{1/2}\ m\ m^{-1/2}\ s^{-1}$. This unit is known as the centimeter-gram-second unit and $1\ m\ m^{-1/2}\ m\ g^{1/2}\ s^{-1} = 0.1$ G (where G is the unit Gauss). The unit of Gauss is still in use, but the unit of tesla is preferred (see Table 1).

Gauss was also responsible for showing that Earth's magnetic field has its origin within Earth. By introducing a magnetic potential he expressed the magnetic field as an infinite sum of spherical harmonics. Some of these harmonics have an internal origin, and the others have an external origin. He determined the first 24 coefficients of these harmonics to conclude that the external part of Earth's magnetic field is very small, and the field is almost entirely of internal origin. The potential representation of Earth's magnetic field outside the Earth is still used widely.

During the eighteenth century, considerable work on the variations of Earth's magnetic field was conducted using observational data. In addition to the long-term, or secular, variations, it was observed that variations of much shorter periods, of a day or so duration, are also present. One of the remarkable discoveries, originally by Celcius and Hiorter of Uppsala in Sweden in 1747, was that of the magnetic storm, which is a disturbance of relatively large magnitude, with a deflection of more than a degree on the magnetic needle, lasting only a few hours.

The second half of the eighteenth century witnessed the discovery of the relationship between electricity and magnetism by Oersted, who in 1820 showed that an electric current flowing in a copper wire deflected a pivoted magnetic needle. This work was developed further by Biot (1774–1862), Savart (1791–1841), Ampere (1775–1836), Faraday (1791–1867), Ohm (1787–1854), and Maxwell (1831–1879). These discoveries led to the set of equations now known as the Maxwell equations governing the evolution of magnetic and electrical fields in a moving medium. They are, respectively, Ampere's law, solenoidal condition, Gauss' law, and Faraday's law given by:

$$\nabla \wedge \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad (1)$$

$$\nabla . \boldsymbol{B} = 0 \qquad (2)$$

$$\nabla . \boldsymbol{D} = \mathrm{C} \qquad (3)$$

$$\nabla \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \mathrm{t}} \qquad (4)$$

Here ***B*** and ***E*** are, respectively, the magnetic and electric fields, ***H*** the magnetizing force, ***D*** the displacement current, ***J*** the electric current density, and *C* the electric charge density. These equations include too many variables to provide a unique solution, and must be supplemented with constitutive relations relating ***B***, ***D***, ***H***, ***E***, and ***J*.** These are:

$$\boldsymbol{B} = \mu \boldsymbol{H}\ ,\ \boldsymbol{D} = \varepsilon \boldsymbol{E} \qquad (5)$$

$$\boldsymbol{J} = \sigma \bar{\boldsymbol{E}} + C\boldsymbol{v} \qquad (6)$$

in which μ, ε, and σ are, respectively, the magnetic permeability, the electrical permittivity, and the electrical conductivity, which are all properties of the medium under study. The equation involving ***J*** is known as Ohm's law. ***v*** is the velocity of the flow in the medium, and $\bar{\boldsymbol{E}}$ is measured in a frame of reference moving with the flow velocity, so that:

$$\bar{\boldsymbol{E}} = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \tag{7}$$

The elimination of ***J*** and ***E*** from Eqs. (1), (4), (5), and (6) in favor of ***B*** and neglecting ***D*** and *C* leads to the famous induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}\,,\ \eta = 1/\mu\sigma \tag{8}$$

which governs magnetic induction in electrically conducting materials. η is known as the magnetic diffusivity. This equation has been used extensively over the period 1950–1980 to examine the kinematic dynamo problem.

It is now known that Earth's magnetic field can be represented generally in the form:

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_r + \boldsymbol{B}_c + \boldsymbol{B}_T \tag{9}$$

The field $\boldsymbol{B}_0$ is the field produced by a hypothetical dipole situated at the center of the earth and inclined at an angle θ_0 to the axis of rotation. This angle had the value θ_0 = 11.2º in 1980, and has remained almost constant over the recent past, although it can change over geological time scales. The field $\boldsymbol{B}_r$ is that part which varies on a long-time scale, of the order of hundreds of years, and is referred to as the secular variation field. The fields $\boldsymbol{B}_0$ and $\boldsymbol{B}_r$ both have their origin within the earth's core, and they are maintained by a dynamo mechanism. We will consider this in Section 4 below. $\boldsymbol{B}_c$ is the crustal field, which appears as anomalous patches on the general field (see Section 5 below), while $\boldsymbol{B}_T$ is the temporal field of external origin arising from currents flowing in Earth's ionosphere (see Section 6 below).

2.1The International Association of Geomagnetism and Aeronomy (IAGA)

The International Association of Terrestrial Magnetism was established as a forum for presentation of research in geomagnetism before World War II. In 1948, the association changed its name to the International Association of Geomagnetism and Aeronomy (IAGA), and became an active member of the International Union of Geodesy and Geophysics (IUGG). IAGA has played a very crucial role in the development of research in geomagnetism. It has not only provided regular meetings (every two years) for the presentation and discussion of research in all aspects of geomagnetism and aeronomy, but it has also initiated and coordinated a number of activities that have provided authentic information about Earth's magnetic field. In particular, IAGA has been instrumental in the establishment of an International Geomagnetic Reference Field (IGRF), which was adopted in 1968. This is a reference for the main field of Earth based on spherical harmonic coefficients. It has since been improved and reproduced every five years. A full description of this is given in chapter *The Earth's Magnetic Field*.

2.2Magnetic Units

The development of electromagnetism after the discovery of the relationship between electric currents and magnetic fields necessitated a system of units for magnetic and electric variables. Starting from Gauss' unit of magnetic field, the c.g.s. system of units was used. It adopted the basic units of centimeter (cm), gram (g), and second (s). This system of units, also called the electromagnetic system of units (e.m.u.), was used for measurement as well as for theoretical studies until the second half of the twentieth century, when the International System of Units (Systeme International, SI) was adopted. Many books are still available in c.g.s. units, and it is useful to include a comparison between the two systems of units. This is given in Table 1. The unit of Gauss was found to be rather large for most magnetic field measurements, and a more convenient unit, called γ, was used. Here 1 γ = 1 nanotesla = 10^{-5} G. According to the SI system of units, magnetic fields are measured in nanotesla, which is the same as γ. The nanotesla is related to the unit of tesla by 1 tesla = 10^9 nanotesla.

Quantity and symbol	SI units	c.g.s. units
Magnetic induction *B*	1 T (tesla) ≡ kg A s^{-2}	10^4 gauss
Magnetic field strength *H*	1 A m^{-1}	4×10^{-3} Oe (oersted)
Magnetization ***M***	1 A m^{-1}	10^{-3} emu cm^{-3}
Electric field *E*	1 N C^{-1} (Vm^{-1})	10^{-1} emu
Electric current *J*	1 A (ampere) = C s^{-1}	2.75 $\times10^9$ S C s^{-1}
Magnetic dipole moment *m*	1 A m^2	10^3 emu
Magnetic flux ϕ	1 Wb (weber)	10^4 maxwells
Magnetic pole strength ***p***	1 A m	10 emu
Permeability of free space μ_0	$4\pi\times10^{-7}$ H m^{-1} (nTA^{-2})	1
Relative permeability $\mu_r = \mu / \mu_0$	1	1
Permitivity of free space ε_0	8.85×10^{-12} F m^{-1}	1
Volume magnetic susceptibility χ	4π	1
Electrical conductivity σ	S (Siemens) = Mho m^{-1} = kg^{-1} m^{-1} A^{-1}	10^{-11} emu

Table 1. The main variables of electromagnetic theory and their units of measurement in both SI and c.g.s. units. The abbreviations used have the following meanings: ampere (A), centimeter (c), coulomb (C), faraday (F), gram (g), henry (H), kilogram (kg), meter (m), newton (N), second (s), siemens (S), volt (V).

3. Measurement and Analysis of the Main Magnetic Field of Earth

3.1 Measurement

Once it was established that the geomagnetic field is a vector quantity directed towards the magnetic North Pole, and a unit of measurement for its magnitude was devised by Gauss, the question of measuring it at different locations was addressed. Now, at every locality on Earth's surface, the usual reference system is north–south, east–west, and upwards–downwards. However, these directions are related to the geographic North Pole, which is different from the magnetic North Pole. At every location P on Earth, we take a Cartesian system of coordinates $P(x,y,z)$, with P_x eastwards, P_y northwards, and P_z vertically downwards. The magnetic field intensity, $\boldsymbol{F}$, at P makes an angle I, called the inclination, with the vertical. The magnetic field will have a component Z along the vertical and a component H lying in the horizontal plane at P. Since the magnetic north pole does not coincide with the geographic North Pole, the line of action of H will make an angle D, called the declination, with P_y. (See Figure 1) The magnetic field can be identified (in both magnitude and direction) in a number of ways. If we resolve H along P_x (eastwards) and P_y (northwards), and denote these components by X and Y, respectively, then the field is determined if we can measure X, Y, and Z. Equally, the field is determined if we can measure H, D, and I or H, D, and Z. This is because the quantities X, Y, Z, H, D, and I are related by:

$$
\begin{aligned}
Z &= F \sin I \\
H &= F \cos I \\
X &= H \cos D \\
Y &= H \sin D \\
\tan I &= Z / H \\
\tan D &= Y / X \\
H^2 &= X^2 + Y^2 \\
F^2 &= X^2 + Y^2 + Z^2 = H^2 + Z^2
\end{aligned}
\qquad (10)
$$

The magnetic field as observed on the surface of Earth has been measured and analyzed extensively. Although magnetic charts do not remain valid for very long, their construction continues today.

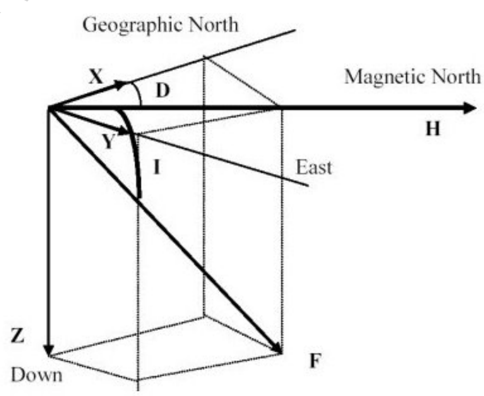

Figure 1. The definition of the coordinate system and the components of the magnetic field, together with the definition of angles of declination and inclination.

3.1.1 Presentation of Magnetic Data

Magnetic data are very difficult to present in a form that makes it easy to understand and appreciate the field properties. This is because there are many variables to examine and many points on the globe to consider. It is now accepted that the best presentation of magnetic data for the field on the surface of Earth is by magnetic charts. A chart is used to illustrate one property of the field at a time. In order to prepare a magnetic chart, enough data about the relevant property of the field must be acquired. This requires the availability of magnetic data over the relevant area at different intervals so that the mean value can be taken. These are obtained from magnetic observatories and repeat stations, as well as from local and aerial surveys on both land and sea. (See chapter *The Earth's Magnetic Field*)

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Biographical Sketch

Ibrahim A. Eltayeb, studied in government schools in the Sudan. He obtained his bachelor degree from the University of London and his Ph.D. from the University of Newcastle in 1972. He joined the University of Khartoum as a lecturer in the same year, and became a full professor in 1980. He maintained his interest in the study of rotating magnetic fluids with applications to Earth's core, and extended his interest to other applications of mathematics to the geophysical sciences like the motion of dust storms and sand dunes. In 1986, he moved to the newly established Sultan Qaboos University and established the Department of Mathematics and Computing. Professor Eltayeb has maintained an active international role. He has visited a number of universities in Europe and North America and collaborated with a number of scientists across the world. His collaborations with Professor David Loper at Tallahassee, Florida, have included publications on the compositional plumes, the thermal layer at the core–mantle boundary, and helical dynamos. Another major collaboration with Professor Mohamed Hassan at Trieste, Italy, concerned the motion of sand dunes and diffusion of dust. He has also organized, convened, and attended international conferences. He has been associated with IAGA since 1975. He was chairman of the working group on dynamo theory (1981–1985), member of the Executive Committee (1991–1999) and at present is chairman of Division I on Internal Magnetic Fields. Professor Eltayeb's contributions have been acknowledged internationally. He became Fellow of the African Academy of Sciences in1986, and was elected Fellow of the Royal Astronomical Society of London in 1987. In 1995, he was awarded the mathematics prize of the Third World Academy of Sciences (TWAS) for his contributions to mathematical modeling in the geophysical and environmental sciences. He became a fellow of TWAS in 1996.