HOMOGENEOUS RANDOM FIELDS AND THEIR EVALUATION

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Summary

Many phenomena in continuous media, e.g., in geophysical problems, can be interpreted as a sequence of realizations of random (stochastic) fields (scalar or vector-valued). Often the field can be reduced to ones that are invariant with respect to a Lie group's action. Correlation function (or the same - spectral density) of such field is its important characteristic. It can be used for interpolation of realizations of the field. Correlation functions must be semi-positive definite, however, in the process of its estimation the property may be lost. A regularization of such estimation (small perturbation that provides the positive definiteness) as well as meteorological applications is considered.

1. Introduction

The representation of spatial fields (scalar or vector-valued), that are time dependent as realizations of a random field can be useful for various applications. It means an exchange (*The exchange can be interpreted also as so-called "ergodic hypothesis"*.) of averages with respect to time (sometimes with respect to spatial variables, too) on the average with respect to a "random argument" ω. The representation describes our informal knowledge about mean values, variances, and the connection between values of the field in different spatial points. The connection can be described by the correlation function (CF) of the random field. Certainly, the description cannot be full; it is a suitable compromise between a desirable statistical description of a physical field and our measurement and computational possibilities.

CF's evaluation will be considered below for meteorological fields for Earth's troposphere and lower stratosphere. The methods that are used for the concrete evaluation problem can be considered as typical for various geophysical applications.

The typical horizontal scale for the problem is about 10^2 km as well as the vertical scale is about 1 km. The scales are the result of a compromise between

i. an understandable desire to know a "best" evaluation of CF;
ii. ii) the precision of CF, that is necessary for next applications, is not unlimited (the level "several % " is sufficient);
iii. homogeneity and isotropy of the large-scale random fields can be fulfilled along the horizontal arguments, only;
iv. a difference between the "true CF" and its best approximation under the homogeneity and isotropy hypotheses is not vanishing – there is a lot of extra-atmospheric phenomena with anisotropic influence on the Earth's atmosphere;
v. an available archive of the measurement data include errors and cannot be full.

Otherwise, to evaluate the CF, one can use a property of CF: it must be semi-positive definite. It gives a way to regularize the ill-posed computational problem of CF's evaluation.

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Biographical Sketch

Vladimir Alexander Gordin, was born on May 18, 1949, Leningrad, USSR. He left special mathematical Moscow school N 2 in 1966, and he graduated in 1972 in Moscow Institute of Electronic Machines as M. Sci. in Applied Mathematics. 1972- : collaboration in Hydrometeorological Center of USSR (later of Russia); presently leader fellow scientist. 1999- teaching at Independent Moscow

University his own "Applied Mathematics" course. Also he taught his own mathematical courses in special Moscow schools N2 and N1313 as well as in mathematical groups and summer school for students and undergraduates. PhD in Physics and Mathematics in 1979: Hydrometeorological Center of USSR; title of thesis: "The Study of the Finite Difference Approximations and the Boundary Conditions for Systems of Forecasting Equations". Dr.Sci. degree in Physics and Mathematics in 2000: Moscow Institute of Physics and Technology (MFTI). The title of the Dr.Sci. thesis: "Mathematical Problems and Methods in Hydrodynamical Weather Forecasting". He published the following books: "Mathematical Problems of the Hydrodynamical Weather Forecasting. Analytical Aspects". Gidrometeoizdat, Leningrad, 256p. (1987, Russian); "Mathematical Problems of the Hydrodynamical Weather Forecasting. Numerical Aspects". ibid, 264p. (1987, Russian); "Mathematics, Computer, Weather Forecasting". . ibid, 224p. (1991, Russian); "Mathematical Problems and Methods in Hydrodynamical Weather Forecasting". Gordon & Breach, 2000, 842p. (English); "How It Should Be Computed ?" To appear, 2003 (Russian) and about 70 articles. Member of the Moscow and American Mathematical Society. For his taking part in August 1991 events he was decorated with the medal "Defender of Free Russia". B.A. in Jewish Sciences (Touro College, Moscow, 1996).