MATHEMATICAL MODELS IN METEOROLOGY AND WEATHER FORECASTING

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1. Introduction

The accuracy of present day weather forecasts and the remarkable improvement in skill that has taken place over the last few decades can be directly attributed to improvements in the computer forecasts used as guidance by human weather forecasters. In general, the public is not aware that our daily weather forecasts start out from a best estimate of present conditions, i.e., as *initial-value problems*, on the major National Weather Services supercomputers. For example, in the US, computer weather forecasts issued by the National Centers for Environmental Prediction (NCEP) in Washington, D.C., guide the forecasts from the U.S. National Weather Service (NWS) as well as those in the private sector. These forecasts are performed by "running" (integrating in time using a computer) mathematical models of the atmosphere that can simulate, given today's weather observations, the evolution of the atmosphere in the next few days. Because the time integration of an atmospheric model *is an initial-value problem*, the ability to make a skillful forecast requires both that *the computer model be a realistic representation of the atmosphere,* and that *the initial conditions be accurately estimated*. We will give an introduction to how mathematical models of the atmosphere are designed and discretized into computer models, a science known as *numerical weather prediction*, and how observations are combined with forecasts to generate accurate initial conditions, a science known as *data assimilation*. The accuracy of the forecasts decays with the length of the forecast, but varies from day to day and from region to region. This is because the atmospheric flow is sometimes much more predictable than at other times. *Ensemble forecasts* that allow the forecasters to assess the reliability of the forecasts will also be briefly discussed. A detailed description of these subjects is available in specialized journals and in some texts (e.g., Kalnay, 2003).

NCEP (formerly the National Meteorological Center or NMC) has performed operational computer weather forecasts since the 1950s. From 1955 to 1973, the forecasts included only the Northern Hemisphere; they have been global since 1973. The quality of the models, methods for using atmospheric observations, and computer power has been upgraded throughout the world, resulting in major forecast improvements. Figure 1 shows the longest available record of the skill of numerical weather prediction. The "S1" score used in this figure measures the relative error in the horizontal pressure gradient in the middle of the atmosphere (on the surface of 500 hPa also referred to as 500 millibar). This score shows the relative error of the operational 36-hour forecasts over North America. Empirical experience in the late 1940's indicated that an S1 value of 70% or more corresponds to a useless forecast, and a score of 20% or less corresponds to an essentially perfect forecast. Both at 500 hPa and near the surface (around 1000 hPa, not shown) where the forecasts are not as accurate, the forecast scores indicate that the skill has more than doubled in the last two decades: a 3-day forecast at the present time is more accurate than a 36 hour forecast was 20 years ago. Similar improvements are observed for all weather parameters: surface temperature, precipitation, winds, etc., since they are all derived from the same evolving 4-dimensional model forecasts of the weather waves.

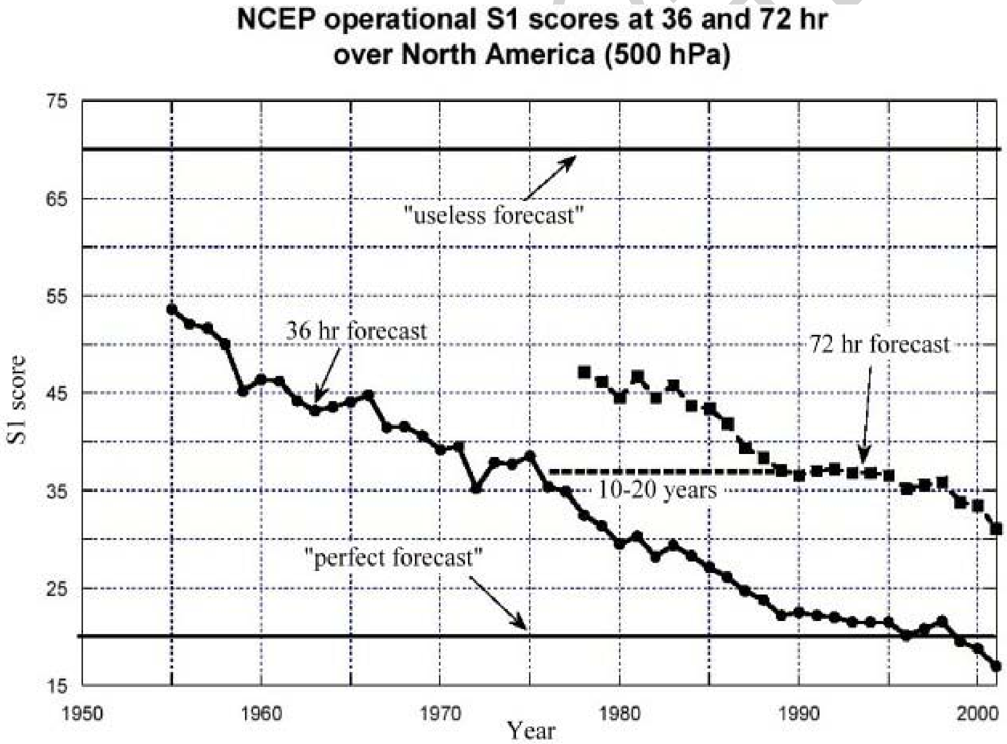

Figure 1: Evolution of the operational forecast skill of the National Weather Service computer models to forecast the weather. The score shown, known as S1 score, measures the relative error in the horizontal pressure gradient. The scores are for 500hPa forecasts, about 5 kilometers above the surface.

2. History of Numerical Weather Prediction

Vilhelm Bjerknes in 1904 demonstrated that the future state if the atmosphere is, in principle, completely determined by the fluid dynamic equations together with detailed initial and boundary conditions. In addition he proposed an observational program, and suggested possible graphical methods for the solution of the governing equations. The next major milestone was by Lewis Fry Richardson who proposed in 1922 to integrate the equations numerically rather than graphically. He performed a remarkably comprehensive experiment with a model that used 4 vertical layers of 200 hPa thickness and a horizontal grid of 200km side to compute the time derivative of the surface pressure over Germany. His experiment was actually a failure because the presence of high frequency gravity waves in the solution combined with observations containing random errors resulted in a forecast of a change of 146 hPa in 6 hours, whereas the observed change was less than 1 hPa. This result discouraged further experiments for many years, until Jule G. Charney, Ragmar Fjortoft and John von Neumann in 1950 used a barotropic (one level) vorticity equation model that filtered out gravity waves to perform a 1-day forecast. In terms of the S1 score of Figure 1, their 24-hour forecast would probably have had score of about 60, i.e., a somewhat useful forecast. Their forecast was quite successful in qualitatively predicting the 1-day change in the atmosphere and was the stimulus that led to operational numerical weather prediction. Soon after this success with a filtered model, Charney (1951) wrote an NWP primer in which he predicted that modelers would return to the more accurate primitive equations used by Richardson, and develop better methods to estimate initial conditions and to avoid the lack of balance between pressure and winds that ruined Richardson's experiment. He also envisioned the need to include parameterizations of some physical processes, such as radiation, condensation and moist vertical convection that take place at scales too small to be resolved by the numerical grids ("subgrid-scale processes"). Inspired by the success of Charney et al (1950), the first operational NWP system was started in Sweden in September of 1954, and 6 months later in the US.

We briefly describe the evolution of operational NWP in the US National Weather Service, which is typical of that in other major national and international weather centers in the world, such as the United Kingdom Meteorological Office, the Japan Meteorological Agency, the Canadian Meteorological Center, the Australian Bureau of Meteorology, and the international European Centre for Medium-range Weather Forecasts (ECMWF), located in Reading, England. In the US, the first *filtered* (quasi-geostrophic) model was implemented in 1955, and was replaced in 1966 with the first 6-layers *primitive equations* model for the Northern Hemisphere. A global model based on a spherical harmonics discretization (rather than finite differences) was implemented in 1980, with 12 layers and a rhomboidal spectral wave truncation of 30 waves (R30). A *data assimilation* system based on *statistical interpolation* replaced in 1978 the previous empirical interpolation scheme, and in 1991 it was replaced with a variational data assimilation system. In 1992 an operational *ensemble forecasting system was* implemented. The US currently (early 2002) runs an operational suite of models that includes a global model with triangular spectral wave truncation of 170 waves (T170) and 48 vertical levels and an ensemble system with 17 members integrated to 15 days every day. The initial conditions are obtained with a data assimilation system based on 3-dimensional variational data assimilation including the assimilation of satellite radiances. For short-range forecasts (less than 3-days), a regional model with 12km horizontal resolution and 50 levels is run four times a day over North America. This regional model is nested into the global model, i.e., it uses the global model forecast for its lateral boundary conditions.

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Biographical Sketch

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