MATHEMATICAL SOIL EROSION MODELING

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Summary

It is becoming increasingly clear that the transport of eroded material from land to water by overland flow is an important environmental problem, promoting the eutrophication of surface waters, damaging freshwater ecosystems and causing microbial contamination of surface water sources. Sediment derived from the soil is a pollutant in its own right: reducing light penetration and physically damaging freshwater ecosystems; it is a carrier of pollutants such as pesticides and phosphorus and many contaminants are associated with soil particle surfaces. As point-sources of pollutants are controlled there is an urgent need to provide the scientific understanding to underpin operational decisions being made with respect to diffuse pollutants. This Chapter reviews the major developments in mathematical soil erosion modeling over the past two decades. In particular, we review progress in finding solutions to the Rose-Hairsine model and their application to experimental data. Because of its unique ability of the Rose-Hairsine model to explicitly recognize the differential behavior of the various sediment particle size classes which comprise natural soils, not only can it provide greater insight into the movement of sediment across both farmlands and other contaminated land, but it is better positioned than any other model to estimate the impact of eroded sediment on water quality of surrounding rivers or streams. Such differential behavior results in the preferential movement of fine sediment with attached compounds such as nutrients, fertilizers and pollutants. Neglecting the size selectivity in the sediment transport and deposition process results in a significant underestimation of the downstream impact of suspended sediment enriched with absorbed chemicals.

1. Introduction

Sculpturing of the land surface by erosion, transport and deposition processes has always played a major role in shaping the land surface of the earth. Geomorphologists have long recognized that glaciation is a major erosion agent in cold climates, mass movement is common in steep humid regions, and in many regions both wind and water can play dominant roles in sediment transport.

When human activity substantially reduces the cover provided by vegetation or litter, and when soil is disturbed and loosened, these natural erosion processes can be greatly accelerated. Land management practices found to be necessary or beneficial to the development of agriculture were developed in many countries. Such practices were developed in temperate climatic regions, such as Europe, and typically involved forest clearing and subsequent cultivation. These practices were transferred to other regions of the world which were colonized or conquered, without realizing that the direct transfer of such land management methods may be inappropriate, or at least require modification for sustainable land use in quite different soil and climatic contexts. The scale and rate of expansion of such transferred land management practices was vastly increased by the rapid adoption and power increase in mechanized forms of cultivation.

Thus, early in the last century, especially in countries such as the USA and Australia, European-based agricultural practices were rapidly extended into regions where the soil and climatic contexts were quite different from their European origins. The resultant extensive and accelerated rates of soil erosion which occurred in such countries provided a major incentive for research into soil erosion, especially in the USA (Hudson, 1981). This is not to infer that land degradation due to water and wind erosion is restricted to such countries (Pimental, 1976; Oldeman, 1994). However, a brief history of water erosion research which follows will be restricted to the USA.

Early development of soil erosion research in the USA

The United States Department of Agriculture (USDA) declared a policy of land protection in 1907, and from 1915 onwards a number of agencies commenced investigation of the effect of different treatments on runoff and soil erosion from defined plots (Bennett, 1939). This early applied research was expanded and accelerated with the establishment of Federal and State Experiment Stations, and from 1928 to 1953 a period of intensive collection and tabulation of runoff and soil loss data occurred. This work included experiments on mechanical ways of controlling soil loss and runoff from small watersheds. In later years, data using artificial rainfall simulators added to the very large body of collected data.

This substantial empirical database provided guidance on the role of many factors and agronomic treatments in controlling soil loss (Ayres, 1936). However, very few plots were equipped to measure the rate of runoff; only the total runoff and soil loss were recorded. Since rate measurement technology was not the limiting factor, this measurement choice may have come from the mental model held by soil scientists concerned with soil erosion at the plot scale. This model appears to be that “raindrops detach soil and overland flow simply transports this previously removed sediment over the soil surface” (Rose, 1993).

This early emphasis on the role of raindrop impact, and relative neglect of the role of overland flow in soil erosion, appears to have been strengthened by the studies of raindrops and erosion by Laws (1940), Ellison (1947), Ekern (1951), and Hudson (1957).

The very large body of data collected by the USDA and collaborators called for some kind of synthesis, condensation, or generalization. For example, Zingg (1940) developed an empirical equation relating soil erosion to slope and slope length. Also important to subsequent development was Musgrave’s (1947) parametric equation which incorporated a rainfall erosivity index as well as other factors. This type of equation was revised and expanded several times to form the Universal Soil Loss Equation (or USLE) of Wischmeier and Smith (1978).

The USLE was developed by applying statistical multivariate regression techniques to the large data bases collected by the USDA Agricultural Research Service, its collaborators and predecessors. The data base included the results of long-term studies of factors believed to affect soil erosion in areas of agricultural significance east of the Rocky Mountains in the USA. Whilst large in size, the data base was for a restricted ecological range, covered slopes of only up to about 7%, and to soils with a low percentage of montmorillonite clay (Morgan and Davidson, 1986).

The factor-product form of data summary provided by the USLE is given by (Wischmeier and Smith, 1978)

$$A = R_f K_f L S_o C_f P_f \,, \qquad (1)$$

where A is the mass of soil lost from unit area per year, averaged over as many years as is appropriate. R_f, the rainfall erosivity factor, is calculated using data on both the kinetic energy and intensity of rainfall. The soil erodibility, K_f, is in practice calculated as the unknown in Equation (1), given values for the slope length L, slope S_o, the crop management factor, C_f, and P_f, the factor describing any erosion control practice which might be adopted. Experience with calculated values of K_f for agriculturally important soils in the USA has been summarized in the form of a nomogram, which can be used predictively for such soils (Wischmeier and Smith, 1978).

Wischmeier (1976) took pains to emphasize the limitations of the USLE, stressing that it was particularly designed to address objectives such as the following:

- Give estimates of long-term average annual soil loss from a particular field slope, and with a particular land use and management.
- Provide guidance on the selection of cropping, management systems and conservation practices for specific soils and slopes.
- Provide soil loss estimates for conservationists to use for determining soil conservation needs.

Wischmeier (1976) warned against using the USLE beyond the regions where the basic information was obtained, or to make soil loss estimates for individual erosion events. The USLE applies only to situations where net deposition does not occur. The USLE is based on correlations. Since there is no inclusion in the USLE of factors directly representing physical parameters such as infiltration or overland flow velocity, some factors will be influenced by correlations with effects due to these processes.

Especially in less humid environments, there is a practical limitation in developing locally relevant parameters for use in the USLE methodology. This limitation is that a long time period, possibly several decades, may be required in order to experience an adequate number of erosion events (Edwards, 1987) to reliably estimate these parameter values.

Many modifications have been made to the USLE designed to overcome some of its limitations. Perhaps the most widely accepted modification is RUSLE (Revised USLE) described by Renard *et al.* (1994).

Soil erosion and conservation developments beyond the USLE

It is clear that the origin and purpose of the USLE was not to describe the *processes* affecting soil erosion. The objective of more recent research on soil erosion has been to describe such processes so that more effective identification and predictability of parameters involved can be achieved. This objective has not been readily obtained, and research to support this objective is still actively in train.

The first general area of advance has been to recognize that a vital role in erosion is played, not only by rainfall, but also by overland flow. Thus, the currently accepted conceptual model, replacing that of earlier researchers previously given, is that: "Raindrops detach soil, and overland flow *both erodes* and transports eroded soil over the land surface" (Marshall, Holmes and Rose, 1996). The common presence of rills in erosion events provides evidence for this statement.

The need to predict excess rainfall from rainfall characteristics requires a robust model of the infiltration process. Especially as scale increases, there is increasing evidence that spatial variability in infiltration rate is common, so that infiltration equations that include this behavior have advantages over one-dimensional infiltration models (Yu *et al.*, 1997, Yu, 1999).

A second major area of development has been recognition of the general importance of the role of sediment deposition as an ongoing process which dynamically accompanies whatever mix of erosion processes is at work. The rate of sediment deposition depends on the settling-velocity characteristic of the sediment involved. Interaction in settling between sediments of quite different size (and therefore settling velocity) appears to be an important factor (Lovell and Rose, 1991).

The range of models developed to describe the series of dynamic processes involved in soil erosion, deposition and transport will be reviewed in subsequent sections. Although the form of description of these processes is not always in complete agreement, Figure 1 illustrates the dynamic form of interaction widely accepted.

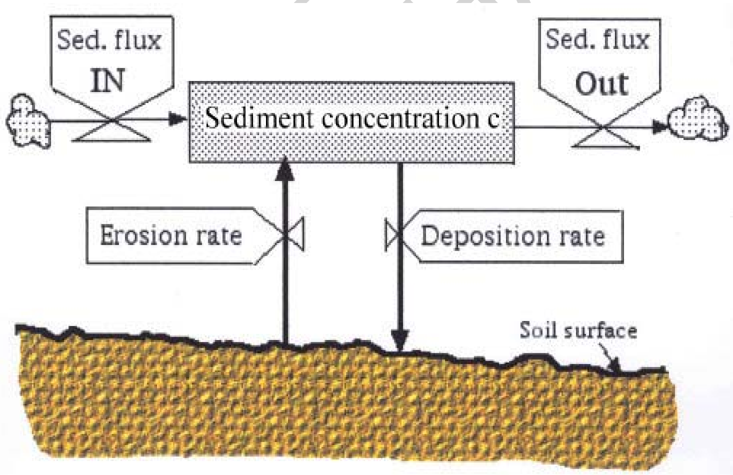

Figure 1: Flow diagram describing the interaction of erosion processes between the sediment flux and the soil surface. Rates of processes exchanging sediment are shown by valve symbols

A common experimental finding is that in any given erosion situation there is an upper limit to the resulting sediment concentration. For flow-driven erosion, Foster (1982) introduced the term 'transport limit' to describe this limiting value. A theoretical expression for the transport limit has been derived by Rose and Hairsine (1988), and a corresponding limit for rainfall-driven erosion by Hairsine and Rose (1991).

2. Surface Hydrology

Since it is overland flow which transports the suspended sediment, any model of soil erosion processes must first begin with a description of the surface hydrology. The governing equations, obtained from conservation of both mass and momentum, for unsteady one-dimensional non-uniform flow of water down a planar surface of unit width are given by

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = R \ , \qquad (2)$$

and

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = g(S_o - S_f) - \frac{Ru}{h} \ . \qquad (3)$$

In (2) and (3), which are usually referred to as the St Venant equations, h is the mean depth of flow, u is depth averaged velocity, R is the lateral inflow per unit length, g is gravity, S_o is the bed slope, S_f is the friction slope, t is time and x is distance downslope.

In general the St Venant equations need to be solved numerically, however under flow conditions where friction and gravity effects dominate those due to inertial and pressure effects, then (3) has the simple solution $S_o = S_f$. Consequently (2) and (3) reduce to the kinematic wave model for overland flow or,

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = R \ , \qquad (4)$$

where

$$q = Kh^m \ , \qquad (5)$$

$$K = \frac{S_o^{1/2}}{n} \ . \qquad (6)$$

In (4), (5) and (6), q is the volumetric flux per unit width, n is the Manning's roughness coefficient and m is an exponent having a value of approximately 5/3 for turbulent flow and 3 for laminar flow. Woolhiser and Liggett (1967) have shown that the kinematic wave model is a good approximation to the St Venant equations provided that the kinematic number k_e, where $k_e = S_o L / h_n F_r^2$ (h_n is the normal depth at $x = L$ and F_r is the Froude number based on normal flow) is greater than 20. This was however later modified by Morris and Woolhiser (1980) to $k_e F_r^2 > 5$ when $F_r < 0.5$ and $k_e > 20$ when $F_r > 0.5$.

The later inflow or excess rainfall rate R is defined from

$$R = P - I_{\mathrm{r}} \quad , \tag{7}$$

where P is the rainfall rate and I_{r} is the infiltration rate. Due to spatial and temporal variability in P and I_{r}, R in general depends on both x and t and only numerical solutions to (4) and (5) are possible (Sherman and Singh, 1976). Under the simplifying yet still physical realistic assumptions of a constant rainfall rate or a time varying rainfall rate, analytical solutions to (4) and (5) are possible and can be found by using the method of characteristics.

2.1 Analytical Solutions

(a) R = constant

The solution for a constant excess rainfall rate was first given by Henderson and Wooding (1964) as

$$h = \left(\frac{Rx}{K}\right)^{\frac{1}{m}} \qquad x \le K R^{m-1} t^m \quad , \tag{8}$$

$$h = Rt \qquad x \ge K R^{m-1} t^m \quad , \tag{9}$$

for the initial and boundary conditions

$$\begin{aligned} t &= 0, \quad x > 0, \; h = 0 \quad , \\ t &> 0, \quad x = 0, \; h = 0 \quad . \end{aligned} \tag{10}$$

(b) $R = R(t)$ with $R(t) \ge 0, \; t \ge 0$.

The solution of Henderson and Wooding (1964) was generalized by Parlange et al (1981) for a positive time dependence of R on t. For the initial and boundary conditions of (10) the solution is given parametrically by

$$h = \int_{t_0}^{t} R(t')\, dt' \qquad x \le x_{\mathrm{c}} \quad , \tag{11}$$

$$x = Km \int_{t_0}^{t} \left[\int_{t_0}^{\bar{t}} R(t')\, dt' \right]^{m-1} d\bar{t} \qquad x \le x_{\mathrm{c}} \quad , \tag{12}$$

with the parameter t_0 in the range $0 \le \mathrm{t}_0 \le \mathrm{t}$. The boundary condition (10) is given by $t_0 = t$ while the initial condition is satisfied by $t_0 = t = 0$. For values of x greater than x_{c} where

$$x_c = Km\int_0^t \left[\int_0^{\bar{t}} R(t')\,dt'\right]^{m-1} d\bar{t} \quad , \tag{13}$$

($t_0 = 0$ in (12)), then h is independent of x and given by (11) with $t_0 = 0$.

(c) $0 \le t \le t^*,\ R(t) \ge 0,$ and $t > t^*,\ R(t) < 0$.

When the rainfall rate falls below the infiltration rate then R becomes negative and neither of the solutions presented in (a) or (b) above apply. Two quite specific solutions for $R < 0$ have been given in the literature by Cundy and Tento (1985) and Giraldez and Woolhiser (1996). These were for a constant rainfall rate of finite duration t^* and a modified Philip infiltration equation (Cundy and Tento, 1985) or a Smith and Parlange (1978) infiltration equation (Giraldez and Woolhiser). In Sander et al (1990) though, a solution was developed for essentially an arbitrary *R(t)* function subject only to the constraint $0 \le t \le t^*,\ R(t) \ge 0,$ and $t > t^*,\ R(t) < 0$. This solution incorporates both the Cundy and Tento (1985) and Giraldez and Woolhiser (1996) solutions.

Since $R(t) \ge 0$ for $0 \le t \le t^*$, then the solution for this time period is still given by that of Parlange et al (1981) or (11), (12) and (13). For $t > t^*$ a drying free surface is formed and begins to move downslope from $x = 0$ so that the $h = 0$ boundary condition no longer occurs at $x = 0$, but at $x = x_d(t)$ where

$$x_d(t) = Km\int_{t_1}^t \left[\int_{t_1}^{\bar{t}} R(t')\,dt'\right]^{m-1} d\bar{t} \quad , \tag{14}$$

with $t_1 \le t^* \le t$, and t_1 defined from

$$\int_{t_1}^t R(t')\,dt' = 0 \ . \tag{15}$$

Equations (14) and (15) give the time dependence of the edge of the free surface for $t > t^*$. In the region $x > x_d$, the solution is still given by (11) and (12) but with t_o restricted to the range $0 \le t_0 \le t_1$.

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Bibliography

Alberts, E.E., Moldenhauer, W.C. and Foster, G.R., (1980). Soil aggregates and primary particles transported in rill and interrill flow. *Soil Science Society America Journal* **44**, 590-595. [Looks at the difference in the distribution of eroded sediment occurring from inter-rill or rill areas.]

Alberts, E.E., Wendt, R.C. and Priest, R.F., (1983). Physical and chemical properties of eroded soil aggregates. Transactions ASAE. 26, 465-471. [Provides experimental data demonstrating that sediment leaving an eroding slope is finer than the original soil.]

Ayres, Q.C. (1936). *Soil erosion and its control*. 365 pp. McGraw-Hill, New York, USA. [Discusses the role of the many different factors and agronomic treatments that effect soil loss.]

Bagnold, R.A. (1966). An approach to the sediment transport problem for general physics. *U.S. Geological Survey Professional Paper*, 422-I. [Develops an alternative formulation to sediment transport based on entrainment being proportional to stream power as opposed to bed shear stress.]

Bennett, H.H. (1939). Soil Conservation. McGraw-Hill Book Company, Inc., New York, NY. [Discusses the importance and role of soil conservation for the present and future.]

Beuselinck, L. (1999). Sediment deposition by overland flow: an experimental and modelling approach. *Ph.D. Thesis Katholieke Universiteit, Leuven, Belgium*. [New experiments are performed to obtain data for multi-size class soils under net deposition conditions with or without the influence of rainfall. Different net deposition routines used in various erosion models are then tested against this data.]

Beuselinck, L., Govers, G., Hairsine, P.B., Sander, G.C. and Breynaert, M. (2002c). The influence of rainfall on sediment transport by overland flow over areas of net deposition. *Journal of Hydrology* **257**, 145 - 163. [This paper uses the Hairsine-Rose erosion theory to model multi-size class sediment transport through net deposition zones during rainfall.]

Beuselinck, L., Govers, Steegen, G and Quine, T.A,. (1999). Sediment transport by overland flow over an area of net deposition. *Hydrological Processes* **13**, 2769 - 2782. [This paper obtains experimental data for sediment transport across net deposition zones.]

Beuselinck, L., Hairsine, P.B. and Govers, G. (2002a). Evaluating a single class net deposition equations across a range of conditions. *Water Resources Research*, **38**,14-1 – 14-11, 10.1029/2001WR000250. [Tests the applicability of single size class erosion models for describing sediment transport across net deposition zones.]

Beuselinck, L., Hairsine, P.B., Sander, G.C. and Govers, G. (2002b). Evaluating a multi-class net deposition equation in overland flow conditions. *Water Resources Research*. **38**,15-1 – 15-10, 10.1029/2001WR000248. [Tests the applicability of the Hairsine-Rose multi-size class erosion model for describing sediment transport across net deposition zones.]

Beven, K.J., and Kirby, M.J. (1979). A physically-based, variable contributing area model of basin hydrology. *Hydrological Sciences Journal* **24**, 43-69. [This paper considers that spatial variability in infiltration rate is directly related to the partial-area concept in hydrology and recognises the dynamically changing areas of a watershed which contribute to overland flow.]

Bricquet, J.P., and Claude, J. (1998). Latest developments in the design of hydrological studies in watersheds. *In* Soil erosion at multiple scales: Principles and methods for assessing causes and impacts, eds. F.W.T. Penning de Vries, F. Agus, and J. Kerr. pp. 175-191. CABI Publishing in association with IBSRAM, Wallingford, U.K. [Discusses some of the latest research tools for studying watershed hydrology, in particular digital terrain models.]

Campbell, I. A. (1985). The partial area concept and its application to the problem of sediment source area. *In soil erosion and conservation*. Eds. S. A. El-Sweify, W. C. Moldenhaver, and A. Lo. pp128-138. Soil Conservation Society of America, Auking, Iowa, USA. [Considers the role that the contributing partial area concept in overland flow has on sediment transport.]

Coughlan, K.J., and Rose, C.W. (1997). A New Soil Conservation Methodology and Application to Cropping Systems in Tropical Steeplands. *ACIAR Technical Report* **40**. Australian Centre for International Agricultural Research, Canberra, ACT. pp. 147. [Uses physical process erosion models at the field-plot scale to evaluate soil erodibility and depositability and the effect of various land management practices on soil erosion.]

Crowley, T.E., (1982). Unsteady Overland Sedimentation. *Journal of Hydrology* **56**, 325-346. [Analytical solutions using the method of characteristics are developed for erosion in a single rill under constant uniform excess rainfall.]

Crowley, T.E., and Foster, G.E., (1984). Unsteady sedimentation in nonuniform rills. *Journal of Hydrology* **70**, 101-122. [Analytical solutions using the method of characteristics are developed for erosion in a single rill under constant uniform excess rainfall.]

Cundy, T.W. and Tento, S.W. (1985). Solution to the kinematic wave approach to overland flow routing with rainfall excess given by Philip's equation. *Water Resources Research* **21**, 1132 - 1140. [Analytical solution using the method of characteristics is developed for shallow overland flow over a plane land element which includes time dependent infiltration.]

Dietrich, W.E, (1982). Settling velocity of natural particles. *Water Resources Research* **18**, 1615-1626. [Provides an algorithm that converts from particle diameter to settling velocity.]

De Vantier, B.A., and Feldman, A.D. (1993). Review of GIS applications in hydrological modelling. *Journal of Water Resources Planning and Management* **119**, 246-261. [Reviews the applications of GIS to hydrological modeling, including erosion prediction and control]

Edwards, K. (1987). Runoff and soil loss studies in New South Wales. Technical Handbook No. 10. Soil Conservation Service of NSW, and Macquarie University, NSW, Australia. [Discusses runoff and soil loss studies in New South Wales.]

Einstein, H. (1937). Der Geschiebebetrieb als Wahrscheinlichkeitsproblem, Verlag Rascher, Zurich, (Engl. Trans: in Shen H, (ed) 1972. Sedimentation. Symp to honour H.A. Eisnstein. H.W. Shen, Fort Collins CO). [This paper develops a stochastic model for bedload sediment transport in rivers.]

Ekern, P.C. (1951). Raindrop impact as a force initiating soil erosion. *Soil Science Society of America Proceedings* **15**, 7-10. [Presents a study on the role of raindrop impact on soil erosion.]

Ellison, W.D. (1947). Soil erosion studies. *Agricultural Engineering* **28**, Part I, 145-146. [A study of early soil erosion mechanisms.]

Evans, K. (2000). Methods for assessing mine site rehabilitation design for erosion impact. *Australian Journal of Soil Research* **38**, 231-247. [Considers how mine rehabilitation design can be assessed using erosion and hydrology models calibrated to mine site conditions.]

Everaert, W., (1991). Empirical relations for the sediment transport capacity of interrill flows. *Earth Surface Processes and Landforms* **16**, 513 - 532. [Compares empirical relations for the sediment transport capacity of interrill flows.]

Foster, G.R. (1982). Modelling the erosion process. In *Hydrologic modelling of small watersheds*, ed. C.T. Hahn. American Society of Agric. Eng. Monograph No. 5, 297-379, St. Joseph, Michigan, USA. [A review article on physically based soil erosion modeling.]

Foster, G.R. and Meyer, L.D. (1975). Mathematical simulation of upland erosion using fundamental erosion mechanics. *Proceedings, sediment yield workshop,* Rep. RS-S-40, pp 190 - 207, USDA Sedimentation Lab., Oxford Miss. [Discusses the then current approach to physical based soil erosion modeling.]

Giraldez, J.V. and Woolhiser, D.A. (1996). Analytical integration of the kinematic equation for runoff on a plane under constant rainfall rate and Smith and Parlange infiltration. *Water Resources Research* **32**, 3385 - 3389. [Analytical solution using the method of characteristics is developed for shallow overland flow over a plane land element which includes time dependent infiltration.]

Govers, G. (1990). *Empirical relationships on the transporting capacity of overland flow*, International Association of Hydrological Sciences, Publication **189**, 45 - 63. [Experimental study of transport capacity of overland flow and comparison of empirical relations used for its prediction.]

Govindaraju, R.S., Kavvas, M.L. and Jones, S.E., (1990). Approximate analytical solutions for overland flows. *Water Resources Research* **26**, 2903-2912. [Compares simplified analytical solutions of the kinematic and diffusive wave approximations to the St Venant equations.]

Govindaraju, R.S. and Kavvas, M.L., (1991). Modelling the erosion process over steep slopes : approximate analytical solutions. *Journal of Hydrology* **127**, 279-305. [This papers uses the simplified analytic solutions for overland flow and combines these with a soil erosion model for a single size class soil.]

Hairsine, P.B., Beuselinck, L. and Sander, G.C. (2002). Sediment transport through on area of net deposition, *Water Resources. Research* **38**, 10.1029/2001WR000265. [Presents theory for sediment transport through net deposition zones based on the Hairsine-Rose multi-size class erosion model.]

Hairsine, P.B., and Rose, C.W. (1991). Rainfall detachment and deposition: Sediment transport in the absence of flow-driven processes. *Soil Science Society of America Journal* **55**, 320-324. [Derives steady state solutions of the Hairsine-Rose soil erosion model when rainfall and deposition are the only erosion mechanisms.]

Hairsine, P.B., and Rose, C.W. (1992). Modelling water erosion due to overland flow using physical principles: 1. Uniform flow. *Water Resources Research* **28**, 237-243. [Develops the theory for multi-size class sediment transport due to entrainment and deposition processes. Provides a formal derivation of the transport capacity.]

Hairsine, P.B., Sander, G.C., Rose, C.W., Parlange, J.Y., Hogarth, W.L., Lisle, I. and Roukipow, M., (1999). Unsteady soil erosion due to rainfall impact : a model of sediment sorting on the hillslope. *Journal of Hydrology* **220**, 115-128. [Uses the Hairsine-Rose model to explain observed trends in sediment enrichment of fine sediment.]

Hancock, G., Evans, K., Willgoose, G., Moliere, D., Saynor, M., and Loch, R. (2000). Medium-term erosion simulation of an abandoned mine site using the SIBERIA Landscape Evolution Model. *Australian Journal of Soil Research* **38**, 249-263. [This paper evaluates a catchment evolution model that can simulate the evolution of landforms resulting from runoff and erosion over many years]

Heilig, A., De Bruyn, D., Walter, M.T., Rose, C.W., Parlange, J.Y., Steenhuis, T.S., Sander, G.C., Hairsine, P.B., Hogarth, W.L. and Walker, L.P., (2001). Testing a mechanistic soil erosion model with a simple experiment. *Journal of Hydrology* **244**, 9-16. [This paper provides the first clear experimental evidence for the development of a deposited surface layer of previously eroded sediment during an erosion event.]

Henderson, F.M. and Wooding, R.A. (1964). Overland flow and groundwater flow from a steady rainfall of finite duration. *Journal of Geophysical Research* **69**, 1531 - 1540. [Develops an analytical solution to the kinmatic overland flow equation for a constant rainfall excess.]

Hogarth, W.L., Rose, C.W., Parlange, J.Y., Sander, G.C. and Carey, G., (2004a). Soil erosion due to rainfall impact with no inflow : A numerical solution. Journal of Hydrology 294, 229-240. [A numerical study of the Hairsine-Rose model on the spatial and time dependence of suspended sediment subject to rainfall driven erosion.]

Hogarth, W.L., Parlange, J.Y., Rose, C.W., Sander, G.C., Steenhuis, T.S. and Barry, D.A., (2004b). Soil erosion due to rainfall impact with inflow : An analytical solution with spatial and temporal effects. *Journal of Hydrology* **295**, 140-148. [An approximate analytical solution is derived which gives the sediment concentration as a function of space and time when there is inflowing water at the upstream boundary.]

Hudson, N. (1981). *Soil conservation*, 2nd edn. Cornell University Press, Ithaca, N.Y., USA. [Soil erosion textbook.]

Hudson, N.W. (1957). Erosion control research. *Rhodesia Agr. J.* **54**, 297 – 323. [Studies the role of raindrop impact of soil erosion.]

Kirby, M.J. (ed). (1978). Hillslope hydrology, Wiley, New York. [Hydrology textbook]

Laflen, J.M., Elliott, W.J., Flanagan, D.C., Meyer, C.R., and Nearing, M.A. (1997). WEPP - Predicting water erosion using a process-based model. *Journal of Soil and Water Conservation* **52**, 96-102. [An evaluation of the WEPP model.]

Lane, L.J., and Nearing, M.A. (1989). USDA Water Erosion Prediction Project: Hillslope Profile Model Documentation, *NSERL Report No. 2*, National Soil Erosion Laboratory, USDA-ARS, W. Lafayette, IN. [This paper presents the development of the WEPP model.]

Lane, L.J., Renard, K.G., Foster, G.R., and Laflen, J.M. (1992). Development and application of modern soil erosion prediction technology - The USDA experience. *Australian Journal of Soil Research* **30**, 893 - 912. [Reviews the experience of the USDA in the development and application of erosion models to soil conservation and management.]

Lane, L. J, and Shirley, E. D. (1982). Modelling erosion in overload flow. *In estimating erosion and sediment field on rangelands*. USDA, AR5, Agricultural Reviews and Manuals. ARM-W-26/June 1982. [Numerical solution of a simple process based erosion model.]

Laws, J.O. (1940). Recent studies in raindrops and erosion. *Agr. Eng.* **21**, 431 – 433. [A study on the effect of rainfall on soil erosion.]

Lisle, I.G., C.W. Rose, W.L. Hogarth, P.B. Hairsine, G.C. Sander and J.-Y. Parlange. (1998). Stochastic sediment transport in soil erosion. *Journal of Hydrology* **204**, 217 – 230. [This paper extends the Einstein bedload transport model to account for the time particles spend in suspension and shows that this reduces to the Hairsine-Rose erosion model.]

Liu, B.Y., Nearing, M.A., Baffaut, C., and Ascough II J.C. (1997). The WEPP watershed model: III Comparison to measured data from small watersheds. *Transactions of the ASAE* **40**, 945-952. [Applies the WEPP model to sediment transport within small watersheds]

Lovell, C.J., and Rose, C.W. (1991). Wake capture effects observed in a comparison of methods to measure particle settling velocity beyond Stokes' range. *Journal of Sedimentary Petrology* **61**, 575-582. [This paper shows that the interaction in settling between sediments of quite different size (and therefore settling velocity) appears to be an important effect.]

Marshall, T.J., Holmes, J.W., and Rose, C.W. (1996). *Soil physics*, 3rd edn. Cambridge University Press, Cambridge, UK. [Popular soil physics textbook.]

Meyer, L.D., Foster, G.R. and Römkens, M.J.M., (1975). Source of soil eroded by water from upland slopes. In : Present and Prospective Technology for Predicting Sediment Yields and Sources, Proc. Sediment Yield Workshop, Oxford, MS, 28-30 November 1972. USDA-ARS-40, pp177-189. [This paper supports previous experimental work indicating that sediment leaving an eroding slope is finer than the original soil.]

Morgan, R.P.C. (1994). The European soil erosion model: An update on its structure and research base. In R.J. Rickson (ed). *Conserving Soil Resources, European Perspectives.* CAD International, Wallingford. 428pp. [Describes the development of a new European soil erosion model.]

Morgan, R.P.C., and Davidson, D.A. (eds.) (1986). Soil erosion and conservation. Longman Scientific and Technical, Longman Group UK Ltd, Harlow, Essex, CM20 2JE, UK. [A collection of papers on soil erosion and conservation.]

Morgan, R.P.C., Quinton, J.N., and Rickson, R.J. (1992). *EUROSEM: Documentation Manual*. Silsoe College, Silsoe, U.K. [Documentation manual for the European soil erosion model.]

Morgan, R.P.C., J.N. Quinton, R.E. Smith, G. Govers, J.W.A. Poesen, K. Auerswald, G. Chisci, D. Torri, and M.E. Styczen. (1998). The European Soil Erosion Model (EUROSEM): a dynamic approach for predicting sediment transport from fields and small catchments, *Earth Surf. Process. Landforms* **23**, 527-544. [This paper fully describes the complete theory behind the EUROSEM model.]

Morris, E.M. and Woolhiser, D.A. (1980). Unsteady one-dimensional flow over a plane: partial equilibrium and recession hydrographs. *Water Resources Research* **16**, 355 - 360. [This paper provides a flow criteria which determines when the kinematic flow equation is a suitable approximation to the St Venant equations for overland flow.]

Musgrave, G.W. (1947). The quantitative evaluation of factors in water erosion - a first approximation. *J. Soil and Water Conservation* **2**, 133-138, 170. [Develops a parametric equation which incorporates a rainfall erosivity index as well as other factors in estimating soil erosion.]

Nearing, M.A., Foster, G.R., Lane, L.J., and Finkner, S.C. (1989). A process based erosion model for USDA Water Erosion Prediction Project technology. *Transactions of the ASAE* **32**, 1587-1593. [Overviews the development of the WEPP model.]

Nearing, M.A., Lane, L.J. and Lopes, V.L. (1994). Modelling soil erosion, in R. Lal (ed) *Soil Erosion Research Methods 2nd ed*, St Lucia Press, 127-156. [Overviews soil erosion modeling and its application to watersheds.]

Oldeman, L.R. (1994). The global extent of soil degradation. In *Soil resilience and sustainable land use*, eds. D.J. Greenland and I. Szaboles. pp. 99-118. CAB International, Wallingford, UK. [Reviews the global extent and problems of soil degradation.]

Parlange, J.Y., Rose, C. W. and Sander, G. (1981). Kinematic flow approximation of runoff on a plane: an exact analytical solution. *Journal of Hydrology* **52**, 171-176. [This paper provides an analytical solution to the kinematic overland flow equation for a positive time dependent excess rainfall rate.]

Parlange, J.Y., Hogarth, W.L., Rose, C.W., Sander, G.C., Hairsine, P., and Lisle, I., (1999). Addendum to unsteady soil erosion model. *Journal of Hydrology* **217**, 149-156. [Develops a simple analytical approximate solution to the Hairsine-Rose model for rainfall driven erosion.]

Pimental, D. (1976). Land degradation: Effects on food and energy resources. *Science* **194**, 149-155. [Discusses the consequences of the need for food and energy resources on the degradation of agricultural land.]

Proffitt, A.P.B, Rose, C.W. and Hairsine, P.B. (1991). Rainfall detachment and deposition: Experiments with low slopes and significant water depths. *Soil Science Society of America Journal* **55**, 325 - 332. [These experiments measure the particle size distribution of the suspended sediment through time. The role of the deposited layer in determining the dynamic and steady state sediment concentration is clearly shown.]

Proffitt, A.P.B, Hairsine, P.B. and Rose, C.W. (1993). Modelling soil erosion by overland flow: application over a range of hydraulic conditions. *Transactions of the American Society of Agricultural Engineers* **36**, 1743 - 1753. [This paper examines how varying hydraulic conditions affect sediment transport.]

Renard, K.G., Laflen, J.M., Foster, G.R., and D.K. McCool (1994). The revised universal soil loss equation. In *Soil Erosion Research Methods*, R. Lal (ed.). pp. 105-124. Soil and Water Conservation Society, Ankeny, Iowa, USA. [This article describes the differences between the USLE and the revised universal soil loss equation - RUSLE.]

Rose, C.W. (1985). Developments in soil erosion and deposition models. *Advances in Soil Science* **2,** 1-63. [A review article which covers the developments in sediment transport models at that time.]

Rose, C.W. (1993). Erosion and sedimentation. In *Hydrology and water management in the humid tropics - Hydrological research issues and strategies for water management*, eds. M. Bonnell, M.M. Hufschmidt, and J.S. Gladwell. pp. 301-343. Cambridge University Press, Cambridge, UK. [This article outlines some of the issues involved in erosion and sedimentation and the major approaches adopted in coping with them The development of models and how they are used in practice to assess sediment transport is illustrated.]

Rose, C.W. (ed.) (1995). Soil erosion and conservation. *Soil Technology* **8**(3) (Special Issue). pp. 241. [Special journal issue devoted to soil erosion and conservation.]

Rose, C.W., and Hairsine, P.B. (1988). Processes of water erosion. In *Flow and transport in the natural environment*, eds. W.L. Steffen, O.T. Denmead. pp. 312-316. Springer-Verlag, Berlin, Germany. [Covers the early development of the Hairsine-Rose erosion model and compares this to alternative models.]

Rose, C.W., J.-Y. Parlange, I.G. Lisle, W.L. Hogarth, P.B. Hairsine and G.C. Sander. (1998). Unsteady soil erosion due to rainfall impact and sediment transport. *Trends in Hydrology*, **18**, 245 – 258. [This paper reviews progress in the development of the understanding of soil erosion resulting from the multi-size class Hairsine-Rose model.]

Rose, C.W., Parlange, J.Y., Sander, G.C., Campbell, S.Y., and Barry, D.A., (1983). Kinematic flow approximation to runoff on a plane: an approximate analytical solution. *Journal of Hydrology* **62**, 363-369. [This paper develops a very simple but reasonable accurate approximate solution to the kinematic overland flow equation .]

Rose, C.W. and Dalal, R.C., (1988). Erosion and runoff of nitrogen. In : Wilson, J.R. (Ed.) Advances in Nitrogen Cycling in Agricultural Ecosystems. CAB International Wallingford, U.K., pp 212-235. [This paper shows how the role of preferential sediment transport of fine material is a key mechanism in the enrichment of soil-sorbed nutrients leaving eroding slopes.]

Sander, G.C., Hairsine, P.B., Beuselinck, L. and Govers, G. (2002). Steady state sediment transport through an area of net deposition: multi-size class solutions. *Water Resources Research* **38**, 23-1 – 23-7, 10.1029/2001WR000249. [An analytical solution is developed for the steady state transport of sediment distributions through a net deposition zone.]

Sander, G.C., Parlange, J.Y., Hogarth, W.L., Rose, C.W. and Haverkamp, R., (1990). Kinematic flow approximation to runoff on a plane: solution for infiltration rate exceeding rainfall rate. *Journal of Hydrology* **113**, 193-206. [This paper develops a fully analytic solution to kinematic flow for an arbitrary time dependent excess rainfall rate.]

Sander, G.C., Hairsine, P.B., Rose, C.W., Cassidy, D., Parlange, J.Y., Hogarth, W.L. and Lisle, I., (1996). Unsteady soil erosion model, analytical solutions and comparison with experimental results. *Journal of Hydrology* **178**, 351-367. [Analytic solutions are developed for time dependent soil erosion which are shown to agree very well with experimental data obtained over a range of flow conditions.]

Sherman, B. and Singh, V.P. (1976). A distributed converging overland flow model 2. Effect of infiltration. *Water Resources Research* **12**, 897 - 901. [This paper develops solutions for overland flow on an infiltrating converging surface. For constant infiltration and rainfall, analytical solutions are found but for more realistic relationships, numerical solutions are required.]

Singh, V.P., and Regl, R.R., (1983). Analytical solutions of kinematic equations for erosion on a plane I. Rainfall of indefinite duration. *Advances in Water Research* **6**, 2-10. [The method of characteristics is used to find solutions to a simplified model of combined water and sediment transport for constant rainfall.]

Singh, V.P., (1983). Analytical solutions of kinematic equations for erosion on a plane II. Rainfall of finite duration. *Advances in Water Research* **6**, 88-95. [The method of characteristics is used to find solutions to a simplified model of combined water and sediment transport for a constant rainfall of finite duration.]

Smith, R.E. and J.-Y. Parlange. (1978). A parameter-efficient hydrologic infiltration model. *Water Resources Research* **14**, 533 - 538. [This paper develops a very accurate model for both infiltration rate and cumulative infiltration for time dependent surface fluxes.]

Veihe, A., Rey, J., Quinton, J.N., Strauss, P., Sancho, F.M. and Somarriba, M. (2001). Modelling of event based soil erosion in Costa Rica, Nicaragua and Mexico: evaluation of the EUROSEM model. *Cantena*, **44**, 187 – 203. [The EUROSEM model is evaluated for both single event and yearly soil loss estimations using plot and rainfall simulation data.]

Walling, D.E. (1990). Linking the field to the river: Sediment delivery from agricultural land, In *Soil Erosion on Agricultural Land*, eds J. Boardman, I.D.L. Foster and J.A. Dearing, pp129 - 152. Wiley, Chichester, UK. [This research carried out on two rivers in the UK very clearly demonstrated the enrichment of fine suspended material in these waterways as compared to the size distribution of the source material.]

Wischmeier, W.H. (1976). Use and misuse of the University Soil Loss Equation. *J. Soil and Water Conservation* **31**, 5-9. [Discusses both the limitations of the USLE and the particular objectives it was designed for.]

Wischmeier, W.H., and Smith, D.D. (1978). *Predicting rainfall-erosion losses - a guide to conservation planning*. Agricultural Handbook No. 537, U.S. Department of Agriculture, Washington, DC, USA. [An agricultural handbook on soil conservation planning.]

Woolhizer, D.A., and Liggett, J.A. (1967). Unsteady one-dimensional flow over a place. The rising hydrograph. *Water Resources Research* **3**, 753-771. [This paper looks at developing flow criteria which determines when the kinematic flow equation is a suitable approximation for overland flow.]

Yu, B. (1998). Theoretical justification of the SCS method for runoff estimation. *Journal of Irrigation and Drainage Engineering* **124**, 306-310. [This paper shows that if both the spatial variation in the infiltration capacity and the temporal variation in the rainfall rate are exponentially distributed, a theoretical basis for the SCS method exists.]

Yu, B. (1999). A comparison of the Green-Ampt and a spatially variable infiltration model for natural storm events. *Transactions of the American Society of Agricultural Engineers* **42**, 89-97. [This paper shows how spatial variability clearly needs to be accommodated in infiltration models, and that with a simple formulation of the infiltration rate as a function of rainfall intensity to address this spatial variability, good agreement with experimental plot data is found.]

Yu, B., and Rose, C.W. (1999). Application of a physically based soil erosion model, GUEST, in the absence of data on runoff rates. I. Theory and methodology. *Australian Journal of Soil Research* **37**, 1-11. [This paper describes methods that can be used to overcome lack of data on runoff rates, as distinct from the amount of runoff during an event, to determine soil erodibility parameters or to predict the rate of soil loss.]

Yu, B., Rose, C.W., Coughlan, K.J., and Fentie, B. (1997). Plot-scale rainfall-runoff characteristics and modelling at six sites in Australia and South East Asia. *Transactions of the American Society of Agricultural Engineers* **40**, 1295-303. [Using an exponential distribution to describe the spatial variation in the maximum infiltration rate and a linear storage formulation to model the lag between runoff and rainfall, a satisfactory three-parameter model is developed for the runoff rate at 1-min intervals within a storm event.]

Yu, B., and Rosewell, C.J. (2001). Evaluation of WEPP for runoff and soil loss prediction at Gunnedah, NSW, Australia. *Australian Journal of Soil Research* **39**, 1131-1145. [Historical data sets were used to test both a physically based runoff and soil erosion model and the method used to estimate the model parameters. WEPP was then validated for bare fallow and annual wheat treatments at Gunnedah, New South Wales, Australia.]

Yu, B., Sajjapongse, A., Yin, D., Eusof, Z., Anecksamphant, C., Rose, C.W., and Cakurs, U. (1999). Application of a physically based soil erosion model, GUEST, in the absence of data on runoff rates. II. Four case studies from China, Malaysia and Thailand. *Australian Journal of Soil Research* **37**, 13 - 31. [In this paper runoff rates were estimated from rainfall rates and runoff amounts for 4 experimental sites in China, Malaysia, and Thailand. The GUEST erosion model was then evaluated for its potential to predict event based soil losses.]

Yu, B., Sombatpanit, S., Rose, C.W., Ciesiolka, C. A. A., and Coughlan, K.J. (2000). Characteristics and modelling of runoff hydrographs for different tillage treatments. *Soil Science Society of America Journal,* **64**, 1763-1770. [A three parameter runoff model is tested across a range of different tillage treatments and it was found to perform very well for large storm events.]

Zingg, A.W. (1940). Degree and length of land slope as it affects soil loss in runoff. *Agricultural Engineering* **21**, 59-64. [An empirical equation relating soil erosion to slope and slope length is developed.]

Biographical Sketches

Graham Sander is a Reader in Hydrology in the Department of Civil and Building Engineering at Loughborough University, England. Prior to his current position he was in the Faculty of Science and the Faculty of Environmental Sciences at Griffith University, Australia. His research and teaching interests are in environmental science and engineering and cover predominantly soil erosion modeling, water and solute transport in porous media and unsaturated two-phase flow.

Professor Calvin Rose is an Emeritus Professor and founding Dean in the Faculty of Environmental Sciences at Griffith University, Brisbane, Australia. Jointly with a research team he is working on both fundamental and applied aspects of soil and water management. His early book "Agricultural Physics" (1966, Pergamon Press) provided a basic link between physics and the range on agricultural concerns. He has directed research programs involving Australia and Asian countries on soil erosion and the development of sustainable cropping systems for tropical environments. His recent text, "An Introduction to the Environmental Physics of Soil Water and Watersheds" (2004, Cambridge University Press) provides the basic physical knowledge required to understand processes involved in the sustainable use of the earth's land and water resources

Bill Hogarth is Pro Vice Chancellor Teaching and Learning and Head of the Faculty of Science and Information Technology at the University of Newcastle, Australia. He was previously Dean of the Faculty of Environmental Sciences at Griffith University, Australia. His teaching and research interests are in environmental modeling with a strong emphasis on the numerical aspects. The particular focus of his research has been on soil processes concentrating on soil infiltration, soil erosion and more recently wind erosion.

Jean-Yves Parlange is a Professor of Agricultural and Biological Engineering at Cornell University having previously been a Professor of Applied Mathematics at Griffith University, in Brisbane, Australia. Yves Parlange has been a Fellow of the American Geophysical Union since 1996, received the Hydrology award in 1996, Horton medal in 2002 and elected to the U.S. National Academy of Engineering in 2006. His research interests are many and varied but centre on problems in environmental science and include water movement in porous media, solute transport in soils, surface and subsurface hydrology and erosion and sediment transport.

Ian Lisle lectures in Mathematics at the University of Canberra in Australia, having previously worked and studied at Griffith University and the University of British Columbia. His research interests include numerical and analytical solution of problems in soil and water, including solutions based on Lie group methods. Ian is also active in mathematical problems arising in Lie symmetry analysis of differential equations, and algorithmic solution of these problems using computer algebra.