

# NUMERICAL METHODS FOR WEATHER FORECASTING PROBLEMS

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## Summary

This chapter presents the basic aspects of numerical methods for weather forecasting problems. The spectrum of models and some additional questions near this problem are described. The parameterization schemes for models and also the use of numerical weather forecasting products is considered. The ways of development of numerical weather prediction are pointed out.

## 1. Introduction

Weather forecasting is one of the most complicated and exceedingly important problems of modern science. Despite evident progress in the last few decades and transition from manual forecasting methods to numerical ones, there are still some

specific problems that are being solved either by manual methods or methods based on direct man-computer interaction. An ever increasing need in more detailed information on the actual meteorological conditions and problems related to the use of manual labor are responsible for intensive development of numerical weather prediction (NWP).

A wealth of experience in mathematical modeling based on the achievements in the development of theoretical principles of dynamic meteorology and computational mathematics has been gained in recent decades. It is this experience that enables us today to create powerful systems for assimilation meteorological and oceanographic data with high spatial-temporal resolution, to develop high-quality efficient technologies of NWP and to do investigate on mathematical modeling of climate. And it all started a long time ago.

V. Bjerknes might have been the first who pointed out that a forthcoming state of the atmosphere could be predicted by solving a specific set of differential equations. This is what NWP represents - integration of the set by using a current state of the atmosphere as initial data and applying numerical methods. L.E. Richardson took the first step in numerical prediction in the twenties of the 20<sup>th</sup> century. He was the first to give a closed set of equations describing atmosphere dynamics and to apply a numerical method to solve them. This attempt was not successful for a few reasons one of which implied that at that time there were no developed numerical methods to solve similar problems. In addition, the physical laws of atmospheric processes were not then sufficiently studied, therefore, the initial set of equations proved to be inadequate to cope with the task of predicting meteorological components. Moreover, there was an obvious lack of information on the distribution of meteorological values, which adversely affected the calculation results. Finally, the absence of computing equipment led to a situation when solving the problem required several man-years. Nevertheless, his findings were published and played a positive role in pursuing subsequent scientific investigations.

Inasmuch as the development of numerical prediction was impossible until the middle of the century because of the absence of computers, the scientists followed another path. It involved the identification of main weather-forming factors in the initial set of equations of hydrothermodynamics followed by their simplification. With electronic computers coming into play, the models developed in this way were primarily used for NWP. Besides, the models themselves, though being significantly simplified, appeared to be a good tool for carrying out scientific investigations and understanding the basic physical mechanisms responsible for the processes taking place in the atmosphere. A mere list of the authors who contributed to these investigations would take quite a lot of space. More details are given in the reference at the end of the article, where in a number of monographs there is a comprehensive review of the above-mentioned studies.

It may be safely suggested that a major breakthrough in NWP started at the end of the forties, when the first electronic computer ENIAC, Electronic Numerical Integrator and Computer, was introduced. It was at that time that J.G. Charney, R. Fjortoft and J. Von Neumann got the first successful result in numerical prediction based on the eddy equation solution.

At present a full set of hydrothermodynamic equations is used for NWP. The derivation of this set is based on the fundamental laws of conservation including the following ones:

1. Newton's second law should be applied in a rotating coordinate system. The mathematical representation of this law is a projection of acceleration on three directions of the corresponding coordinate axes (equations of motion).

$$\frac{d\mathbf{V}}{dt} + 2\boldsymbol{\Omega} \times \mathbf{V} = \mathbf{F} + \mathbf{G} + \mathbf{P}. \quad (1)$$

Here  $\mathbf{V}$  – vector of velocity,  $\boldsymbol{\Omega}$  - angular velocity of the Earth's rotation,  $\mathbf{F}$  – viscosity forces,  $\mathbf{G}$  –gravitational force,  $\mathbf{P}$  – pressure forces, per unit mass for force terms.

2. Law of conservation of mass as applied to motion of fluid or gas and mathematically expressed as a continuity equation

$$\frac{d\rho}{dt} + \rho \nabla \mathbf{V} = 0. \quad (2)$$

Here  $\rho$  is air density.

3. The first law of thermodynamics or law of conservation of energy as applied to thermal energy and expressed as a heat fluxes equation.

$$\frac{dT}{dt} - \frac{R}{c_p} \frac{T}{p} \frac{dp}{dt} = \frac{\varepsilon}{c_p}, \quad (3)$$

where  $T$  – air temperature,  $R$  – specific gas constant for dry air,  $c_p$  - specific heat capacity,  $\varepsilon$  - non-adiabatic sources (sinks) of heat.

4. Equation of state, representing a one-valued coupling among pressure, air density and temperature and meaning that the atmospheric air can be considered as an ideal gas

$$p = \rho RT. \quad (4)$$

5. Law of conservation of specific humidity providing a total humidity balance in the atmosphere, which can be locally expressed as

$$\frac{dq}{dt} = S, \quad (5)$$

where  $q$  is specific air humidity,  $S$  - term, describes phase transformation of humidity in atmosphere, processes of evaporation and precipitation.

Thus, one can compile a set of seven equations relative to seven unknown quantities: three components of wind speed, air temperature, pressure, density, and specific

humidity. Depending on the goal the initial system can be transformed into any form. In particular, a geopotential instead of pressure can be considered as a prognostic variable and pressure rather than altitude above the underlying surface should be used as a vertical coordinate, respectively. By the way, this is precisely what a synoptic makes, and it is certainly associated with the fact that when probing with a radiometer, pressure, temperature, wind speed and some characteristic of humidity, a dew point in particular, are measured.

## **2. Data Assimilation System**

NWP based on numerical integration of the set of differential equations is a constituent part of general technology, which consists of the following stages:

1. Collection of data from weather stations, buoys, planes, satellites, remote sensing ground systems and so on.
2. Data reduction, which involves decoding of every message, quality control and preparation of databases for subsequent analysis.
3. Four-dimensional assimilation of data and their objective analysis aimed at converting prognostic fields by way of continuous or discrete assimilation of data and multi-level interpolation into the regular grid points of a typical coordinate system.
4. Data initialization aimed at suppressing high-frequency inertial-gravitational disturbances resulting from unbalanced initial fields.
5. Numerical integration of the equations describing the dynamics of the atmosphere.
6. Synoptic-statistical interpretation of the prognostic results obtained to derive additional information or specific weather characteristics, such as cloudiness, visibility, etc.
7. Postprocessing, which involves formatting digital messages into standard codes accepted in the global data processing system, their distribution along the network, graphic presentation of prediction products.

It is evident that weather analysis/prediction requires a host of data, as the quality of forecast depends on their quantity. It should be emphasized that over the years the data grow in quantity.

In order to efficiently process these data bulks, very powerful computers are required. At present the speed of the computers in use at the prognostic centers lies in the range of 10 to 1000 Mflops.

So, given above is a general scheme for data analysis and weather prediction that is known as the data assimilation system. It must be noted that this system can be applied both in a continuous assimilation mode when data are assimilated as they come in and in a discrete assimilation mode when incoming data are correlated with a discrete observation period, typically 6 hours. Implementation of this scheme is not an end in itself. In the long run it is the synoptic that makes the final forecast by using the numerical prediction results, his knowledge and experience. He gets the information in the form of baric topography charts, with additional data applied on these charts in accordance with the relevant rules. The charts should not need to be in paper copy.

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### Biographical Sketch

**A.A. Fomenko** was born in Zaporozhie, Ukraine. He completed his Diploma in Mathematics and Applied mathematics at the Novosibirsk State University, Russia in 1976. In 1984 he obtained the Russian degree of candidate in Physics and Mathematics at the Computing Center of Siberian Branch of the Russian Academy of Sciences, Novosibirsk. In 2000, Alexander A. Fomenko was awarded the Russian degree of Doctor in Physics and Mathematics from Institute of Computational Mathematics and Mathematical Geophysics (Computing Center) of Siberian Branch of the Russian Academy of Sciences, Novosibirsk with the thesis "Finite-difference modeling of large-scale dynamics of atmosphere". From 1976 to 1985

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