MATHEMATICS IN EGYPT AND MESOPOTAMIA

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Summary

Egypt and Mesopotamia were the first civilizations to develop mathematical cultures for which we still have written evidence today. For both civilizations, mathematics constituted an essential tool to administer available resources; many of their developments seem to have happened at the same time. However, while Mesopotamia and Egypt even used mathematics to handle similar administrative and organization tasks, their mathematical cultures show significant differences, clearly indicating that mathematics is a cultural product, dependent on the humans who create, use and change it. To be fully understood, the development of mathematics in Egypt and Mesopotamia has to be seen in light of their respective historical and cultural backgrounds (for introductory overviews of Egyptian history cf. Shaw 2000, for the history of Mesopotamia cf. Roux 1993 and van de Mieroop 2006.).

1. The Beginnings: Invention of Script, Numbers, and Metrological Systems

1.1. Egypt

Around the end of the 3rd millennium BCE, a social stratification of Egypt’s population can be traced, based on evidence from cemeteries. As in later periods, evidence from Egypt usually originates from cemeteries (or temples), because these happened to be in desert areas, where excellent conditions existed for the preservation of artifacts. In

contrast, only little evidence from cities has survived. Then as in modern times, Egyptian towns and cities were located mostly in direct proximity to the Nile, i.e. in proximity to water and humidity. As a consequence, modern settlements have been built on top of the ancient settlements, and have rendered the latter unavailable for archaeological excavations. But even if this were possible, it is uncertain how much would have survived the moist conditions.
Cemeteries from the 3rd millennium show tombs of different sizes. There exist a large number of smallish-sized one-room tombs, fewer larger tombs with several rooms, and then some outstanding large tombs with even more rooms. A good example of this variety of tomb sizes can be found at cemetery U at Abydos. And it is in this cemetery that the earliest evidence for writing in Egypt has been discovered (Dreyer 1998). It seems that from the outset, writing was a tool which the elite managed to restrict to themselves. Early evidence suggests that writing was used to represent power as well as to administer goods when their amounts became too large to be overseen without a written record. Consequently, it is not surprising that numbers appeared with the first evidence of writing. From the beginning, two systems seem to have been in use, a hieroglyphic script, where signs were incised into stone objects, but also a script written with ink and a brush on pottery and other suitable surfaces. Throughout later Egyptian history, hieroglyphs were used for monumental inscriptions in stone, whereas hieratic was used for the kind of writing needed in daily life, mostly on papyrus or ostraka (stone or pottery sherds).

The number system used in Ancient Egypt can be described in modern terminology as a decimal system without positional (place-value) notation. It used different signs for 1, 10, 100, 1000, 10.000, 100.000 and 1.000.000. Why the individual signs were attached to their specific numerical values is to some degree arbitrary, however, some may have been the result of practice or observations. The number 1 is represented by a simple stroke, the sign for 10 is supposed to be the hieroglyph of a hobble for cattle. 100 is represented by a rope: from later periods, 100 cubits = 1 xt was used as a measure to determine the size of fields, a process which was executed with a rope. 1000 is represented by the hieroglyph of the lotus flower, 10,000 by the hieroglyph of a finger – again, one could think of 10,000 as 10 times 1000 and thereby create the association of our ten fingers. 100,000 was represented by the hieroglyph of a tadpole – probably from the observation that these appear in rather large numbers. The largest Egyptian sign for a number was that for 1,000,000, which was represented by the seated god ¡H. Each of these signs was written as often as required to represent the specific number, e.g. 205 would be written by writing the sign for 100 twice and the sign for 1 five times. Like in hieroglyphic script, the individual signs would be grouped appropriately. There was no sign for the zero – since the Egyptian number system did not use the place-value characteristic, it sufficed to leave out the symbol for ten to indicate the absence of tens in a number like 205.

The Egyptian number system was fully developed at the time of king Narmer, as is documented by his ritual mace head, which was found at temple of the god Horus at Hierakonpolis, the most important predynastic site in the south of Egypt. The object originated from a ceremonial context and the scenes represent King Narmer receiving a tribute consisting of bulls, goats, and captive prisoners. The numbers of each are indicated below them: 400,000 bulls, 1,422,000 goats, 120,000 captives. The symbols and method used to express these rather high numbers are the same as those employed later in hieroglyphic inscriptions. Thus, this mace head gives evidence for a fully developed number system before the First Dynasty. It also demonstrates that the number system was not only used for obvious administrative purposes, but it was also important in representational spheres of Egyptian culture.

The period of the Old Kingdom is usually considered to be the first cultural peak in the history of ancient Egypt, which is documented by its output in art, architecture and literature. It is hard to imagine that this could have taken place without the further development of mathematical techniques. The Great Pyramid of Giza, for example, which required several million limestone blocks in its construction, must have been a major logistic enterprise, and it is hard to grasp without the use of mathematics for architectural calculations (e.g. determining the relation of base, height and inclination or the amount of stones required). Furthermore, the logistics of a project of this scale (number of workers, rations of the workers and others) also obviously required the use of mathematics. Unfortunately, practically no information about mathematical techniques used at that time has survived. Despite this lack of direct evidence, however, there are a number of signs that mathematics indeed did play a significant role during the Old Kingdom. Depictions in tombs regularly show scribes as administrators taking inventory of various goods. From these depictions it is obvious that literacy and numeracy were essential prerequisites for a bureaucratic career, as is also expressed in the scribal statues, a statue type found in the tombs of high ranking officials. This evidence documents the importance of scribes in the achievements of the Egyptian state, and at the same time their own awareness of their role. So called- market scenes, found in a number of graves, inscriptions about land-ownership found in another tomb, and the papyrus archives from two temples of the Old Kingdom prove the existence of several metrological systems, to measure lengths (e.g. of a piece of cloth), areas (of land), volumes (e.g. of grain, grain products or beer) and weights.

Another part of mathematics can be traced back at least as far as the Old Kingdom: fractions. The Egyptian concept of fractions, i.e. parts of a whole, was fundamentally different from our modern understanding. This difference is so elementary that it has often led to a distorted analysis of Egyptian fraction reckoning viewed solely through the eyes of modern mathematicians, who marveled at the Egyptian inability to understand fractions in the same way we do. However, the Egyptian system of fractional notations and their handling in calculations becomes understandable if the evolution of the Egyptian concept of fractions is traced from its beginnings. The first Egyptian fractions consisted of a small group of specific fractions designated by individual special signs. These fractions are first attested within the context of metrological systems, i.e. 3/4 in 3/4 of a finger (an Egyptian length measure), 1/4 in 1/4 of a `sTAt` (an Egyptian area measure), but they retain their notation in later times as abstract fractions (for a detailed account of these early fractional notations in Egypt and Mesopotamia cf. Ritter 1992). From this set of earliest fractions, a general concept of fractions seems to have developed. The list of earliest fractions comprises 1/2, 1/3 and 1/4, and it may be inferred that from these first fractions, fractions came to be understood as the inverses of integers. As a consequence, the Egyptian notation of fractions did not consist of numerator and denominator, but rather of the respective integer of which it was the inverse and a symbol to designate it as an inverse, i.e. a fraction. The hieroglyphic sign that came to be used for this purpose was the sign for part, which was already used in the writing of 2/3 and 3/4. In order to designate a "general fraction" (always an inverse), it was placed above the integer of which the fraction was the respective inverse. In hieratic, this fraction-marker was abbreviated to a mere dot. One of the first researchers of Egyptian fraction reckoning, Otto Neugebauer, devised a notational system, which imitates this Egyptian notation (Neugebauer 1926). Fractions (i.e. inverses of integers) are rendered by the value of the integer and an over-bar to mark them as fractions. This notational system is close to the Egyptian concept and remains the best way of rendering Egyptian fractions. Following the concept of fractions as inverses of integers, the next step would have been to express parts that consist of more than one of these inverses. This was done by (additive) juxtaposition of different inverses, e.g. what we write as 5/6 was written in the Egyptian system as 1/2 1/3. The writing of any fractional number thus consisted of one or more different inverses written according to their size in descending order. Note that this notation enables one to be as accurate as necessary by considering only elements up to a certain size. It also allows for an easy comparison of the size of several fractions, e.g. while the immediate comparison of 5/8 and 4/7 only yields that they are both a little more than 1/2, their representation as Egyptian-style fractions 1/2 1/8 and 1/2 1/14 allows for immediate comparison.

1.2. Mesopotamia

As will be the case throughout this comparison of the first mathematical cultures, evidence for the invention of numbers and writing is much more plentiful from Mesopotamia than from Egypt. While we can trace little or no development of script before the first evidence from tomb Uj, precursors of writing in Mesopotamia appear as early as 8000 BC. Little clay objects of a variety of specific shapes like disks, cylinders, cones, spheres were found throughout the Near East. Denise Schmandt-Besserat identified these objects as counters, and has developed a theory for how writing evolved from their use (e.g. Schmandt-Besserat 1996). Individual shapes represented individual measuring units of specific goods. During the Late Uruk Period (3600-3150 BC), tokens appear to have been placed into envelopes, hollow clay spheres, which were sealed with the seal of the owner of the goods recorded by those tokens. This indicates the transition from a time when exchange of goods was based on trust and personal guarantees to a time of increasing complexity of economic structures, when personal guarantee is no longer sufficient and has to be secured by an impersonal control mechanism (Nissen 1999, p. 41). To control a transaction, the sphere needed to be broken to enable access to the enclosed tokens. The next step in this development is marked by the appearance of clay envelopes that show impressions of the enclosed tokens, thereby enabling traders to keep the object intact. This system of recording quantities was simplified in the later part of the fourth millennium by using flattened pieces of clay, i.e. clay tablets, to impress the shapes of the counters and sealing this tablet. These earliest tablets only include the notation of quantities; there is no indication about the objects or any other information related to the individual transactions. During the period of Uruk IV (c. 3300-3100), first tablets occur that show numerical notations as well as a small number of ideographic signs. While the ideographic signs cannot always be assigned a specific meaning, it is clear that these tablets come from an administrative context, and that the signs most likely refer to the object counted or its recipient (for examples see

Nissen/Damerow/Englund 1993, chapter 5).

The earliest metrological texts originate from late fourth millennium Uruk (Robson 2008, p. 75). They show a variety of metrological systems according to the type of object to be measured (for examples see Nissen/Damerow/Englund 1993, p. 28-29), e.g. discrete objects, grain products, rations, areas, weights and others. Prompted by an administrative organization that was large and complex, the sexagesimal place value system, one of the key foundations of Mesopotamian mathematics, was probably developed at the beginning of the Ur III period (Robson 2008, p. 77). Only two basic signs were used to write numbers, a vertical wedge and a corner wedge. Ten vertical wedges equaled one corner wedge. Multiples of these two signs were used to write numbers up to 59, e.g. three vertical wedges to write the number 3, or two corner wedges and five vertical wedges to write the number 25. The number 60 was again written by a vertical wedge, with its position indicating its value. Thus the number 75 would be written by a vertical wedge (= 60), followed by a corner wedge and five vertical wedges (=15). The actual value of a vertical wedge (60^0, 60^1, 60^2, and so on) or a corner wedge (10×60^0, 10×60^1, 10×60^2, and so on) was determined by its place. Sexagesimal fractions were written using the same signs, a vertical wedge (60^{-1}, 60^{-2}, 60^{-3}, and so on) and a corner wedge (10×60^{-1}, 10×60^{-2}, 10×60^{-3}, and so on), their actual value again determined by their position in the representation of a number. Unlike our modern decimal place value system, however, the Mesopotamian sexagesimal system did not have a symbol for zero, nor a decimal point, and therefore the absolute value of a number-representation without any context could be ambiguous.

While the bulk of mathematical texts from Mesopotamia originates from the Old Babylonian period, finds of tablets with mathematical contents begin as early as the Late Fourth Millennium (for an overview of published cuneiform mathematical tablets, see Robson 2008, p.299-344 (Appendix B)). Several examples from the Uruk IV phase are discussed in Friberg 1997/1998. They include field-area and field-side texts. Field-area texts indicate numbers representing areas, field-side texts record the lengths of the sides of a quadrilateral field (Friberg 1997/1998, p. 8). Each of the numeric values is written in sexagesimal notation into a segment of the tablet, indicating by a horizontal or vertical line whether it is a length or width – with these lines, supposedly opposite sides can be assigned to each other. For example the tablet W20 044,35 (FS U4) indicates two horizontal dimensions, each marked by a horizontal line – with numeric values 1 15 (rods) and 1 00 (rods) and two vertical dimensions of 2 00 (rods) by 1 40 (rods). Friberg 1997/1998, p. 11 proposes that the field-side texts provided the necessary data to compute the area (A) of the respective field (with sides a, b, c, d) according to the approximation rule $A = ½\ (a+c) \times ½\ (b+d)$, with a and c, b and d designating opposite sides of the quadrilateral field.

2. Mathematical Texts: Education and Mathematical Practices

Our main sources for our knowledge of Egyptian and Mesopotamian mathematics are the so-called mathematical texts from each culture. Not all texts that involve numbers are what we call "mathematical texts". Using a definition from Eleanor Robson, mathematical texts are texts that "have been written for the purpose of communicating or recording a mathematical technique or aiding a mathematical procedure to be carried out" (Robson 1999, p.7). Egypt and Mesopotamia produced a major corpus of mathematical texts at roughly the same time, about 1800 BCE. In Egypt this coincides with the Middle Kingdom; in Mesopotamia, with the Old Babylonian period. The choice of writing material in each culture, papyrus in Egypt and clay tablets in Mesopotamia, has led to the situation that, apart from a few chance finds, the Egyptian mathematical papyri have been lost, whereas thousands of mathematical clay tablets from Mesopotamia survive. This should not be seen as an indicator that the output of Mesopotamia was of a much higher quantity; rather, the modern reader needs to be aware that the few mathematical texts extant from ancient Egypt are unlikely to give us as detailed a picture of mathematics in Egypt as the one we obtain from the much richer evidence from Mesopotamia (for editions of mathematical texts from Egypt see Peet 1923, Struve 1930 and Imhausen/Ritter 2004; Clagett 2000 presents a collection of sources in one volume; for editions of Mesopotamian mathematical texts see Neugebauer 1935-1937, Neugebauer/Sachs 1945, Robson 1999, Robson 2004, Friberg 2007 (tablets from a private collection) and Proust 2007 and Proust 2008). Note that the earlier editions originate from a time when it was common practice to "translate" ancient mathematics into modern mathematical notation and terminology, a practice that has since been recognized as misleading. For modern approaches cf. Ritter 1995, Ritter 2004 and Høyrup 2002; for an overview of the historiography of Mesopotamian mathematics see also Høyrup 1996)

The context of the mathematical texts in each culture was that of education, more precisely education of scribes who would need mathematical abilities in their daily work when administering all kinds of goods. The mathematical texts are the most important sources for our knowledge of Egyptian and Mesopotamian mathematics. They inform us not only about the types of problems, but indicate also how they were solved, and (in the case of Egypt) how some arithmetic operations were solved in written form. The aim of any Egyptian or Mesopotamian mathematical problem was the numeric solution of a given problem. Although some problems include geometric objects, a geometric construction is never the topic. Drawings, which can be found within the group of geometric problems, are not to scale, but have to be read with their annotations, which indicate the numerical values of their specific parts.

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