MONEY IN ECONOMIC ANALYSIS

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Summary

In this chapter, we survey the fundamental topics of ‘money in economic analysis’, especially money in standard microeconomic and macroeconomic theories. In Section 1, we make some introductory remarks on the definition and the functions of money as well as an earlier quantitative theory on money. Section 2 is devoted to the role of money in the mainstream microeconomic analysis, the Walrasian general equilibrium theory. In Section 3 we take up the theoretical treatment of demand and supply of money in Keynesian Macroeconomics. In Sections 4, 5, and 6, we investigate some fundamental topics on monetary analysis in the Keynesian and classical traditions. Section 7 is devoted to some concluding remarks.

1. Introduction

We are destined to fail if we try to define ‘money’ from the viewpoint of materials or forms. Money is sometimes considered to be some commodity such as gold or silver, and it is sometimes considered to be the paper on which some numbers and figures are printed. Furthermore, it is often considered to be the only abstract number recorded in the computers used by the banks. Money changed its materials and its forms in the course of the development of economic society. As Hicks (1967) pointed out correctly, therefore, we must define ‘money’ from the viewpoint of its function. Usually, the
economists define ‘money’ as the ‘generally accepted means of payments’, and as a result it is said that ‘money’ must have the following three functions.

- Means of payments (or means of exchange)
- Measure of value (or unit of calculation)
- Means of store of value

This is the conventional definition of ‘money’ in Economics. As Hicks (1967) noted, this definition has somewhat paradoxical nature, because it means that ‘money’ is what is considered to be money by a lot of people in a society. It may be worth noting that the first function is primary, and other two functions are derived from the first function. That is to say, money is used as the measure of value and the means of store of value because it is generally accepted as the means of payments or exchange. Since ancient times, the enigmatic properties of money have fascinated philosophers, and the philosophical and metaphysical speculations on money abound. In this chapter, however, we confine ourselves to some fundamental topics concerning ‘money in economic analysis’, especially money in standard microeconomic and macroeconomic theories.

In standard economic theory, the necessity of money is usually explained by using Figure 1, which is an illustration of the so called ‘Wicksell’s problem’ due to Wicksell (1934). By the way, Figure 1 is an adaptation from Niehans (1978) Chap. 6. Suppose that there are three economic agents, D (Denmark) who has the commodity \( w \) (wheat), S (Sweden) who has the commodity \( t \) (timber), and N (Norway) who has the commodity \( f \) (fishes). Suppose, furthermore, that D wants \( t \), S wants \( f \), and N wants \( w \). In this case, any exchange in barter is impossible because there is no ‘double coincidence of the wants’. However, the indirect exchanges become possible if one of the commodities, for example, \( w \), is used as a ‘generally accepted means of payments’, that is, money. In this case, S receives \( w \) from D in exchange for \( t \), and then S receives \( f \) from N in exchange for \( w \). In this example, a commodity \( w \) became the ‘commodity money’. But, what kind of commodity is likely to become commodity money? Menger (1892) considered this problem, and his answer was as follows (cf. Negishi1985 Chap. 13). “The commodity with the highest salability or marketability will be accepted as money by the society.” Historically such a commodity was gold or...
silver. This is a semi theoretical/semi historical consideration of the origin of money. Needless to say, in modern society money is not commodity money but paper money and/or credit money. But, it is true that they are still the commodities with the highest salability/marketability in modern society.

One of the oldest quantitative theories on money will be the ‘quantity theory of money’, which asserts that the price levels of the commodities will be proportional to the existing quantity of money at least approximately. Although this notion was already explicitly stated by some economic writers in the 18th century, the ‘quantity equation of exchange’ by Fisher (1918), which is written as $MV = PT$ ($M =$ nominal quantity of money, $V =$ income velocity of money, $P =$ price level, $T =$ real quantity of transaction), will be the clearest expression of such a notion. This equation is the tautological equation which is noting but the definition of the income velocity of money ($VP T = M$), but it is transformed to the equation that determines $P$ by means of $M$ if we assume that $V$ and $T$ are constant and $M$ is determined exogenously by the existing quantity of the gold or by the monetary policy by the central bank. Another famous equation with the similar implication is the so called ‘Cambridge equation’ due to Marshall (1923), which is written as $M = kPY$, where $Y$ is the real national income and $k$ is called ‘Marshallian $k’$. Marshall’s equation is considered to be the equilibrium condition for the money market. Namely, the left hand side of this equation is nominal money supply, and its right hand side is interpreted as nominal demand for money. If we can assume that $k$ and $Y$ are constant, $P$ becomes proportional to $M$ again. Obviously, the above two equations become identical if we can suppose that $1/kV = YT$.

If money does not affect the real variables such as real national income and labor employment but it only affects the general price level as the quantity theory of money asserts, money is said to be ‘neutral’. On the other hand, money is not neutral if it affects the real variables as well as the general price level. In what follows, we shall consider both of the situations in which money becomes neutral and those in which money does not become neutral. First, we consider the role of money in the mainstream microeconomic theory (Walrasian general equilibrium theory), and then we proceed to the monetary analysis in macroeconomic theory, especially Keynesian Macroeconomics. Finally, we investigate the implication of the paradoxical dynamics in the ‘classical’ monetary model with full employment and perfect foresight.

2. Money in Walrasian General Equilibrium Theory

In this section, we take up a simplified version of Walrasian general equilibrium theory originated in Walras (1900), which is still a symbol of the mainstream approach to economic analysis. We concentrate on the static pure exchange model without production and capital formation for simplicity of the exposition, although it is not difficult to introduce production explicitly to this analytical framework at the cost of the complexity of notation. For the expositions of such models with and without production, see Hicks (1939), Hansen (1970), Nagatani (1977), and Niehans (1978).

Let us consider the following typical optimization problem of the $j$-th agent ($j = 1, 2, \cdots, m$).
Maximize $U^j(x_1^j, x_2^j, \cdots, x_n^j)$ subject to $\sum_{i=1}^{n} p_i x_i^j = \sum_{i=1}^{n} p_i \bar{x}_i^j$ (1)

where $U^j$ is the utility of the $j$-th agent, $x_i^j$ and $\bar{x}_i^j$ are the demand and the initial holding quantity of the $i$-th good by the $j$-th agent respectively, and $p_i$ is the price of the $i$-th good ($i=1,2,\cdots,n$). It is assumed that each agent acts as a price taker, and the quantities of the demand for the goods are the control variables by each agent. We can solve this optimization problem by introducing the following Lagrangian function.

$L^j = U^j(x_1^j, x_2^j, \cdots, x_n^j) - \lambda^j(\sum_{i=1}^{n} p_i x_i^j - \sum_{i=1}^{n} p_i \bar{x}_i^j)$ (2)

where $\lambda^j$ is the Lagrangian multiplier. We can write the first order conditions for this constrained maximization as follows.

\[
\frac{\partial L^j}{\partial x_i^j} = \frac{\partial U^j}{\partial x_i^j} - \lambda p_i \\
= MU^j_i(x_1^j, x_2^j, \cdots, x_n^j) - \lambda p_i = 0 \quad (i=1,2,\cdots,n) (3)
\]

\[
\frac{\partial L^j}{\partial \lambda^j} = \sum_{i=1}^{n} p_i \bar{x}_i^j - \sum_{i=1}^{n} p_i x_i^j = 0 \quad (4)
\]

where $MU^j_i \equiv \frac{\partial U^j}{\partial x_i^j}$ is the marginal utility of the $i$-th good by the $j$-th agent, which is assumed to be positive. For an exposition of such a mathematical method of static optimization, see, for example, Intriligator (1971). We assume that the usual second order conditions for the constrained maximization are satisfied. Eq. (4) is nothing but the budget constraint in Eq. (1), which means that the value of total expenditure by an agent is equal to the value of his/her initial holdings. Eq. (3) can be rewritten as

\[
MRS^j_{ni} \equiv \frac{MU^j_n(x_1^j, x_2^j, \cdots, x_n^j)}{MU^j_i(x_1^j, x_2^j, \cdots, x_n^j)} = \frac{p_i}{p_n} \quad (i=1,2,\cdots,n-1) (5)
\]

where $MRS^j_{ni}$ is the $j$-th agent’s marginal rate of substitution(ratio of marginal utilities) between the $i$-th good and the $n$-th good.

Equations (4) and (5) are $n$ independent equations with $n$ unknowns $(x_1^j, x_2^j, \cdots, x_n^j)$ and $(n-1)$ parameters $\left(\frac{p_1}{p_n}, \frac{p_2}{p_n}, \cdots, \frac{p_{n-1}}{p_n}\right)$ as relative prices. Therefore, we can derive the following demand functions for the goods of $j$-th agent as functions of relative prices solving simultaneous equations (4) and (5).
In this case, the excess demand function for the \( i \)-th good becomes as follows.

\[
ED_i \left( \frac{p_1}{p_n}, \frac{p_2}{p_n}, \ldots, \frac{p_{n-1}}{p_n} \right) \equiv \sum_{j=1}^{m} x_i^j \left( \frac{p_1}{p_n}, \frac{p_2}{p_n}, \ldots, \frac{p_{n-1}}{p_n} \right) - \sum_{j=1}^{m} \bar{x}_i^j \equiv 0 \quad (i = 1, 2, \ldots, n)
\]  

(7)

Then, the general equilibrium conditions of this Walrasian competitive economy can be expressed by the following system of \( n \) simultaneous equations with \( (n-1) \) unknowns as relative prices.

\[
ED_i \left( \frac{p_1}{p_n}, \frac{p_2}{p_n}, \ldots, \frac{p_{n-1}}{p_n} \right) = 0 \quad (i = 1, 2, \ldots, n)
\]  

(8)

At first glance, it seems that this system does not work well because we have more equations than unknowns. In fact, however, that is not the case because of the following reason. If we consider the budget constraint (4), we can derive the following identity, which is called the ‘Walras law’.

\[
\sum_{i=1}^{n} p_i ED_i \equiv \sum_{i=1}^{n} \sum_{j=1}^{m} p_i x_i^j - \sum_{i=1}^{n} \sum_{j=1}^{m} p_i \bar{x}_i^j \equiv \sum_{i=1}^{n} \left( \sum_{j=1}^{m} p_i x_i^j - \sum_{i=1}^{n} p_i \bar{x}_i^j \right) = 0
\]  

(9)

Therefore, in fact we have only \( (n-1) \) independent equations with \( (n-1) \) unknowns (relative prices), so that in principle this system of equations is solvable. Counting the numbers of the independent equations and unknowns is Walras (1900)’s method. Obviously, this method is insufficient to ensure the existence of the economically meaningful solutions such as positive equilibrium prices. The rigorous investigation of such an existence problem requires a much advanced mathematical method. Another problem is the stability of the general equilibrium solution. Even if the economically meaningful equilibrium solution exists, this solution may be unstable under reasonable adjustment mechanism. For an exposition of such problems by means of the advanced mathematical method, see, for example, Nikaido (1968). Even if all of such difficult problems are solved, however, this system of equations can determine only relative prices. How the absolute levels of prices are determined?

At this point, the following disaggregated version of Fisher (1918)’s quantity equation of exchange enters the stage.

\[
MV = \sum_{i=1}^{n} p_i \bar{x}_i \equiv \sum_{i=1}^{n} p_i \left( \sum_{j=1}^{m} \bar{x}_i^j \right) \equiv p_n \sum_{i=1}^{n} \left( \frac{p_i}{P_n} \right) \left( \sum_{j=1}^{m} \bar{x}_i^j \right)
\]  

(10)
where $M$ is the exogenously given nominal money supply, which is supposed to be determined by the central bank, and $V$ is the income velocity of money, which is also assumed to be constant that is determined by the social custom.

Let us introduce the following (somewhat arbitrary) measure of the general price level $P$.

$$P = \sum_{i=1}^{n} \alpha_i p_i \equiv p_n \left( \sum_{i=1}^{n} \alpha_i \frac{p_i}{p_n} \right)$$  \hspace{1cm} (11)

where $\alpha_i$ is an arbitrary constant such that $0 < \alpha_i < 1$, and $\sum_{i=1}^{n} \alpha_i = 1$.

The relative prices $p_i / p_n$ $(i = 1, 2, \cdots, n - 1)$ were already determined by the equilibrium conditions for the goods market through Eq. (8). This means that the only unknown in Eq. (10) is $p_n$, and it is determined by the level of money supply. In this case, $P$ is also determined. Furthermore, $p_n$ and $P$ are proportional to the money supply $M$. For example, if the money supply doubles, the price level also doubles, and the relative prices and the demand for each good are unaffected. In other words, money becomes neutral in this system in the sense that the quantity of money does not affect the conditions for the goods market (real sectors) except the general price level, so that the classical dichotomy between money and real sectors applies in this system. In fact, we can write Eq. (10) as $M V = P T$, where $T \equiv \left( \sum_{i=1}^{n} p_i \bar{x}_i \right) / P$ is the measure of the aggregate real quantity of transaction, which is completely determined by the relative prices independent of money supply. This is nothing but Fisher’s classical quantity theory of money.

At first glance, the above ‘solution’ seems to be logically flawless. But, where does the quantity equation (10) come from? It seems as if this equation is deus ex machina or the manna from the heaven. We can rewrite this equation and we have the following disaggregated version of the ‘Cambridge equation’.

$$M = k \left( \sum_{i=1}^{n} p_i \bar{x}_i \right)$$  \hspace{1cm} (12)

where $k \equiv 1/V$ is the ‘Marshallian k’. Usually, Eq. (12) is interpreted as the equilibrium condition for the money market, where the right hand side of this equation is considered to be the demand function for money. But, we can not derive such a demand function for money from Eq. (1), which is the utility maximization problem of a typical agent. This is nothing but contradiction. In fact, this is the criticism raised by Patinkin (1965).

Patinkin (1965) tried to reformulate the model to provide a consistent solution to the above problem. Next, we shall present a simplified version of Patinkin (1965)’s model. First, we reformulate the utility maximization problem of the $j$-th agent as follows.
Maximize \( U^j(x_1^j, x_2^j, \ldots, x_n^j, M_d^j/P) \)

subject to \( \sum_{i=1}^n p_i x_i^j + M_d^j = \sum_{i=1}^n p_i x_i^j + \beta_j M \)  \( (13) \)

where \( M_d^j \) is the nominal money demand of the \( j \)-th agent, \( M \) is the exogenously determined nominal money supply, \( P \) is a measure of the price level that is expressed by Eq. (11), and \( \beta_j \) is a constant such that \( 0 < \beta_j < 1, \sum_{j=1}^n \beta_j = 1 \). In this formulation, the central bank determines nominal money supply \( M \), and each agent receives a share of money stock as the initial holdings according to the pre-determined distribution scheme. Furthermore, the real money balance enters into the utility function of the agents, so that this approach is called the ‘money in utility approach’. In other words, money is one of the utility-generating goods in this model. Other examples of the money in utility approach in the static and dynamic contexts are Sidrausky (1967), Allingham and Morishima (1973), and Ono (1994).

In this case, the Lagrangian function becomes as

\[
L^j = U^j(x_1^j, x_2^j, \ldots, x_n^j, M_d^j/P) - \lambda^j \left( \sum_{i=1}^n \frac{p_i}{P} x_i^j - \frac{M_d^j}{P} \right)
- \sum_{i=1}^n \left( \frac{p_i}{P} x_i^j - \frac{M_d^j}{P} \right)
\]

where \( \lambda^j \) is the Lagrangian multiplier. A set of the first order conditions for this constrained maximization can be expressed as follows.

\[
\frac{\partial L^j}{\partial x_i^j} = \frac{\partial U^j}{\partial x_i^j} - \lambda^j \left( \frac{p_i}{P} \right)
\]

\( (i = 1, 2, \ldots, n) \) \( (15) \)

\[
\frac{\partial L^j}{\partial \left( \frac{M_d^j}{P} \right)} = \frac{\partial U^j}{\partial \left( \frac{M_d^j}{P} \right)} - \lambda^j
\]

\( = MU^j_d(x_1^j, x_2^j, \ldots, x_n^j, \frac{M_d^j}{P}) - \lambda^j = 0 \) \( (16) \)

\[
\frac{\partial L^j}{\partial \lambda^j} = \sum_{i=1}^n \left( \frac{p_i}{P} x_i^j \right) - \sum_{i=1}^n \left( \frac{p_i}{P} x_i^j \right) + \frac{\beta_j M}{P} - \frac{M_d^j}{P} = 0
\]

It follows from equations (15) and (16) that
Equations (17) and (18) are \((n+1)\) independent equations with \((n+1)\) unknowns \((x_1^i, x_2^i, \ldots, x_n^i, \frac{M_i^j}{P})\) and \((n+1)\) parameters \((\frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P})\). Solving this set of equations, we can derive the following demand functions of the \(j\)-th agent for goods and money.

\[
x_i^j = x_i^j \left( \frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P} \right) \quad (i = 1, 2, \ldots, n \ ; \ j = 1, 2, \ldots, m)
\]

\[
\frac{M_i^j}{P} = f^j \left( \frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P} \right) \quad (j = 1, 2, \ldots, m)
\]

In this case, the equilibrium conditions for the goods markets and the money market become as follows.

\[
ED_i \left( \frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P} \right) \equiv \sum_{j=1}^{n} x_i^j \left( \frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P} \right) - \sum_{j=1}^{n} \bar{x}_i^j = 0 \quad (i = 1, 2, \ldots, n)
\]

\[
ED_M \left( \frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P} \right) \equiv \sum_{j=1}^{n} f^j \left( \frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P} \right) - \frac{M}{P} = 0
\]

Furthermore, it follows from Eq. (11), which is the definition of the price level, that

\[
1 = \sum_{i=1}^{n} \alpha_i \left( \frac{p_i}{P} \right).
\]

Equations (21), (22), and (23) are \((n+2)\) equations with \((n+1)\) unknowns \((\frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{M}{P})\). However, from the budget constraint (17) we have the following identity, which is an extended version of the Walras law.

\[
\sum_{i=1}^{n} p_i ED_i + ED_M = \sum_{i=1}^{n} \sum_{j=1}^{m} p_i x_i^j - \sum_{j=1}^{m} \sum_{i=1}^{n} p_i \bar{x}_i^j + \sum_{j=1}^{m} M_i^j - M = 0.
\]
This means that the number of the independent equations is \((n+1)\), so that the system is determinate. Equations (21) and (23) can determine \(\left(\frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P}\right)\), and in this case Eq. (22) is automatically satisfied. Then, the price level \(P\) is determined by the following equation.

\[
P = \frac{M}{\sum_{j=1}^{m} f'(\frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P})}
\]  

(25)

In this expression, the relative prices \(\frac{p_j}{P}\) and the real money supply \(\frac{M}{P}\) are already determined by other equations, so that the price level \(P\) is proportional to the nominal money supply \(M\). The changes of the nominal money supply only induces the proportional change of the price level, and the relative prices and the real money supply are unaffected. This means that the classical property of the neutrality of money also applies to this reformulated system in spite of the fact that the demand for goods is not independent of the real money balance (so called ‘real balance effect’ exists).

By the way, we can rewrite Eq. (25) as

\[
MV = PT \quad ; \quad T = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\frac{p_i}{P}\right)\bar{x}_i^j,
\]

\[
V = \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \left(\frac{p_i}{P}\right)\bar{x}_i^j\right] / \left[\sum_{j=1}^{m} f'(\frac{p_1}{P}, \frac{p_2}{P}, \ldots, \frac{p_n}{P}, \frac{M}{P})\right],
\]

(26)

which is nothing but Fisher’s quantity equation of exchange. Note that \(T\) and \(V\) in this equation only depend on pre-determined relative prices and real money balance, which are independent of the value of the nominal money supply \(M\).

Treatment of money in Patinkin’s reformulated model is more satisfactory than that in the traditional Walrasian approach. Hence, such a reformulated model may be called ‘neo-Walrasian’ model. As Rogers (1989) pointed out correctly, however, there is no rationale for the agents to hold money in the Walrasian general equilibrium model. In such an economy, money is not required for transactions, and there is no incentive to store money because there is no uncertainty. Therefore, it is difficult to interpret why the real money balance enters into the utility function of the agents. This is the fundamental flaw of the ‘money in utility’ approach. Another approach in the neo-Walrasian setting is the ‘money in advance’ or ‘liquidity constraint’ approach by Clower (1967) and Granmond and Younes (1972). This approach simply assumes that the agents are required institutionally to use money for transaction, so that there is an incentive to hold money. This approach is far from convincing, and as Ostroy (1973) correctly observed, “this step generated the paradoxical conclusion that the introduction of money had led to a loss in efficiency” (Rogers 1989 p. 63). We have no choice but to agree with the following statement by Rogers. “Both Patinkin’s real balance effect and Clower’s finance constraint are examples of analysis which reflect the failure to note the
inessential nature of ‘money’ in neo-Walrasian models. That is to say, they illustrate that ‘money’ can be added to a neo-Walrasian model but that such a step is unnecessary because none of the perfect barter results are thereby altered.”(Rogers 1989 pp. 58 – 59).

In the next section, we shall turn to the investigation of the monetary analysis in Keynesian Macroeconomics.

Bibliography


Davidson P.(1994). *Post Keynesian Macroeconomic Theory*. 309 pp. Edward Elgar, Brookfield, Vermont, USA. [This is a typical textbook of Post Keynesian Macroeconomics.]


Fisher I. (1930). *The Theory of Interest*, 566 pp. Macmillan, New York, USA. [This presents an original theory of nominal rate of interest and real rate of interest.]


Hayashi F. (1982). *Tobin’s Marginal q and Average q: A Neoclassical Interpretation*. *Econometrica* 50, 213 – 224. [This is a theoretical and empirical study of the investment function by means of Tobin’s q theory.]

Hicks J. R. (1937). Mr. Keynes and the Classics: A Suggested Interpretation. *Econometrica* 5, 147 – 159. (reprinted in Hicks 1967) [This is the original paper of the IS-LM analysis.]


Kaldor N. (1982). *The Scourge of Monetarism*, 114 pp. Oxford University Press, Oxford, UK. [This is a criticism of quantity theory of money from the viewpoint of the endogenous money and exogenous rate of interest.]


Moore B. J. (1979). The Endogenous Money Stock. *Journal of Post Keynesian Economics* 2, 49 – 70. [This is an interpretation of the theory of endogenous money supply.]


**Bibliographical Sketch**
