

MODELS OF INTERNATIONAL ECONOMICS

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Summary

The importance of international economics is increasing, owing to the increasing openness of the single national economic systems: on average, at the world level more than 28% of national income is spent on foreign commodities and services. A feature of current international exchanges with respect to those of the past is that exchanges of financial assets have been growing much more rapidly than exchanges of commodities. Moreover, these financial exchanges, once ancillary to commodity trade, have taken on an autonomous importance and are far greater than the value of exchanges of commodities. This situation is also reflected in the theoretical models, which -once mostly concerned with the theory of commercial flows- have had to cope with the theory of financial and macroeconomic flows in an open macroeconomy. This chapter deals with both aspects, namely with the theory and policy of international trade, and with international monetary economics. A non-mathematical treatment of international economics is already contained in EOLSS (see *International Economics, Finance and Trade*). Hence the present chapter, after a brief general introduction, will concentrate on the mathematical models.

1. Introduction

A feature of current international exchanges with respect to those of the past is that exchanges of financial assets have been growing much more rapidly than exchanges of commodities. Moreover, these financial exchanges, once ancillary to commodity trade, have taken on an autonomous importance and are far greater than the value of exchanges of commodities. This situation is also reflected in the theoretical models, which -once mostly concerned with the theory of commercial flows- have had to cope with the theory of financial flows in an open macroeconomy.

International finance (also called *international monetary economics*) is often identified with *open-economy macroeconomics* or *international macroeconomics* because it deals with the monetary and macroeconomic relations between countries. Although there are nuances in the meaning of the various labels, we shall ignore them and take it that our field deals with the problems deriving from balance-of-payments disequilibria in a monetary economy, and in particular with the automatic adjustment mechanisms and the adjustment policies concerning the balance of payments; with the relationships between the balance of payments and other macroeconomic variables; with the various exchange-rate regimes and exchange-rate determination; with international financial markets and the problems of the international monetary systems such as currency crises, debt problems, international policy coordination, international monetary integration.

In the theory of international monetary economics we can distinguish two different views. On the one hand there is the “old” or traditional view, which considers the balance of payments as a phenomenon to be studied as such, by studying the specific determinants of trade and financial flows. On the other hand there is the “new” or modern view that considers trade and financial flows as the outcome of intertemporally optimal saving-investment decisions by forward-looking agents. More precisely, since the excess of national saving over national investment equals the country’s current account, the idea is to concentrate on the determination of such an excess via an intertemporal optimization; the current account (and the matching capital flows) will be a consequence.

In treating the traditional approaches to balance-of-payments adjustment we shall distinguish between *flow* approaches (which include the elasticity and multiplier approaches, and the Mundell-Fleming model) and stock approaches (which include the monetary approach to the balance of payments and the portfolio-balance model), because strikingly different results may occur according as we take the macroeconomic variables involved as being pure flows, or as flows only deriving from stock adjustments.

To clarify this distinction let us consider, for example, the flow of imports of consumption commodities, which is part of the flow of national consumption. Suppose that the agent decides how much to spend for current consumption by simply looking at the current flow of income, *ceteris paribus*. This determines imports as a pure flow. Suppose, on the contrary, that the agent first calculates the desired stock of wealth (based on *current* values of interest rates, income, etc.) and then, looking at the existing stock, decides to adjust the latter toward the former, thus determining a flow of saving

(or dissaving) and consequently the flow of consumption. This determines imports as a flow deriving from a stock adjustment.

The *theory of international trade* is concerned with the causes of international trade, and has an essentially microeconomic nature, because –unlike international monetary theory– it does not deal with aggregates, but with the *structure* of international trade, namely which goods are exported, and which are imported, and why, by each country. It also deals with the gains from international trade and how these gains are distributed; with the determination of the relative prices of goods in the world economy; with international specialization; with the effects of tariffs, quotas and other impediments to trade; with the effects of international trade on the domestic structure of production and consumption; with the effects of domestic economic growth on international trade and vice versa; and so on. The distinctive feature of the theory of international trade is the assumption that trade takes place in the form of *barter* (or that money, if present, is only a veil having no influence on the underlying real variables but serving only as a reference unit, the *numéraire*). A by-no-means secondary consequence of this assumption is that the international accounts of any country vis-à-vis all the others always balance: that is, no balance-of-payments problem exists.

In the traditional or *orthodox* theory of international trade it is possible to distinguish three main models aimed at explaining the determinants of international trade and specialization:

- 1) the classical (Ricardo-Torrens) model, according to which these determinants are to be found in technological differences between countries;
- 2) the Heckscher-Ohlin model, which stresses the differences in factor endowments between different countries;
- 3) the neoclassical model (which has had a longer gestation: traces can be found in J.S. Mill; A. Marshall takes it up again in depth, and numerous modern writers bring it to a high level of formal sophistication), according to which these determinants are to be found simultaneously in the differences between technologies, factor endowments, and tastes of different countries. The last element accounts for the possible presence of international trade, even if technologies and factor endowments were completely identical between countries.

The two fundamental assumptions of the orthodox theory are perfect competition and product homogeneity. The *new theories* of international trade drop these assumptions and analyze international trade in a context of imperfect competition and/or product differentiation.

This chapter will focus on the theoretical aspects of the main basic models: space limitations do not allow the treatment of the empirical tests.

2. Models of International Trade

2.1. The Orthodox Theory

2.1.1. The Classical Model

If we simplify to the utmost, we can assume that there are two countries (England and Portugal in the famous example of Ricardo's), two commodities (cloth and wine), that all factors of production can be reduced to a single one, labor, and that in both countries the production of the commodities is carried out according to fixed technical coefficients: as a consequence, the unit cost of production of each commodity (expressed in terms of labor) is constant.

It is clear that if one country is superior to the other in one line of production (where the superiority is measured by a lower unit cost) and inferior in the other line, the basis exists for a fruitful international exchange, as earlier writers, for example Adam Smith, had already shown. In this example we have reasoned in terms of *absolute* costs, as one country has an absolute advantage in the production of one commodity and the other country has an absolute advantage in the production of the other. That in such a situation international trade will take place and benefit all participating countries is obvious. Less so is the fact that international trade may equally well take place even if one country is superior to the other in the production of *both* commodities. The great contribution of the Ricardian theory was to show the conditions under which even in this case international trade is possible (and beneficial to both countries).

The basic proposition of this theory is: for international trade to take place there must be a difference in *comparative costs* (which depend on technology), and the international terms of trade must lie between the comparative costs without being equal to either. It will then be beneficial to each country to specialize in the production of the commodity in which it has the relatively greater advantage (or the relatively smaller disadvantage), and to obtain the other commodity through trade.

Comparative cost can be defined in two ways: as the ratio between the (absolute) unit costs of the two commodities in the same country, or as the ratio between the (absolute) unit costs of the same commodity in the two countries. Following common practice, we shall adopt the former, but they are totally equivalent.

In fact, if we denote the unit costs of production of a good in the two countries by a_1, a_2 (where the letter refers to the good and the numerical subscript to the country: this notation will be constantly followed here) and the unit costs of the other good by b_1, b_2 , then

$$(a_1/b_1 > a_2/b_2) \iff (a_1/a_2 > b_1/b_2),$$

$$(a_1/b_1 < a_2/b_2) \iff (a_1/a_2 < b_1/b_2),$$

$$(a_1/b_1 = a_2/b_2) \iff (a_1/a_2 = b_1/b_2)$$

We can now show a simple diagram to represent the theory of comparative costs, based on the concept of *transformation curve* (or *production-possibility frontier*). In our

simplified model, in which there is only one factor of production and the technical coefficients are fixed, the transformation curve is linear. It is in fact given, for country 1, by the equation

$$a_1x + b_1y = \bar{L}_1, \tag{1}$$

where x, y denote the two commodities (cloth and wine), and \bar{L}_1 is the total amount of labor existing in country 1. Equation (1) is the equation of a monotonically decreasing straight line in the (x, y) plane, since we can write it as

$$y = -\frac{a_1}{b_1}x + \frac{\bar{L}_1}{b_1}. \tag{2}$$

In a similar way, we obtain the transformation curve of country 2. Consider then Figure 1, where we have brought together the transformation curves of the two countries.

The line $A'B'$ is the transformation curve of country 1, i.e. the diagram of (2); in absolute value, $\tan \alpha$ equals the comparative cost of country 1. The line $A''B''$ is the transformation curve of country 2, rotated anticlockwise by 180° and placed so that point B'' coincides with point A' ; it goes without saying that $O''B''$ and $O'B'$ are parallel. The absolute value of $\tan \beta$ equals the comparative cost in country 2.

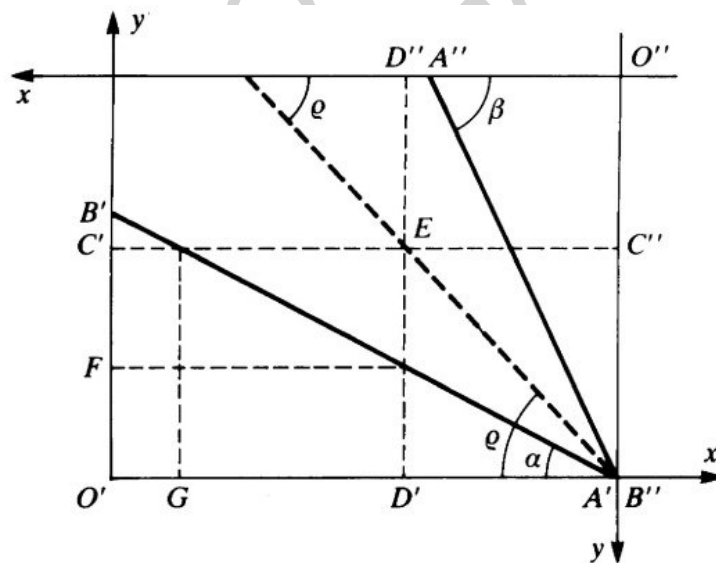


Figure 1: Transformation curve and comparative costs

Let us take an arbitrary admissible value of the terms of trade (that is, the ratio at which the two commodities are exchanged in the world market, or international relative price), say $\tan \rho$, where $\tan \alpha < \tan \rho < \tan \beta$, and assume that international trade occurs at point E , whose coordinates are the quantities exchanged. Country 1 specializes completely in the production of commodity x , of which it produces the amount $O'A'$; of this, a part is consumed domestically ($O'D'$), whilst the remaining part ($D'A'$) is

exported in exchange for the quantity $O'C' = ED' = C''B''$ of commodity y . Note that, since the terms of trade are measured by $\tan \varrho$, and since (by considering the right-angled triangle $ED'A'$) we have $ED' = D'A' \cdot \tan \varrho$, it follows that by giving $D'A'$ of x , ED' of y can be obtained, and vice versa. This means that the trade balance is necessarily in equilibrium. In fact, balance-of-trade equilibrium, or value of exports=value of imports, requires

$$p_x D'A' = p_y ED'$$

or

$$\frac{p_x}{p_y} D'A' = ED', \quad (3)$$

which is indeed true, since commodities are exchanged at a relative price (p_x/p_y) given by the terms of trade, namely $p_x/p_y = \tan \varrho$.

Similarly, country 2 completely specializes in y and produces the amount $O''B''$ of this commodity, consuming $O''C''$ domestically and exporting $C''B''$ in exchange for $O'D'' = D'A'$ of commodity x . This result (complete specialization in both countries) is the normal outcome of trade in the Ricardian model. This may not be the outcome when one country (say country 1) is small with respect to the other, so that this country's production of x is not sufficient to fully satisfy, in addition to its own domestic demand, also the demand for this commodity by country 2. In such a case country 2 will not specialize completely in commodity y and will continue to produce both y and x .

As can be seen, point E lies *beyond* both transformation curves, and so it represents a basket of goods that neither country could have obtained in autarky. Consider, for example, country 1. In autarky, together with $O'D'$ of x this country could have obtained $O'F$ of y (less than the amount $O'C'$ that it obtains through international trade). The gains from trade accruing to this country can be measured, in terms of y , by $C'F$ (in terms of x they are measured by GD'). The gains from trade accruing to country 2 can be found in a similar way.

It is also obvious from the diagram that the closer the terms-of-trade line is to a country's transformation curve, the smaller that country's share of the gains; this share drops to zero when the terms-of-trade line coincides with that country's transformation curve (and all the gains go to the other country).

2.1.2. The Neoclassical Model

For each country the given data are:

- (a) the total amounts of the two factors existing in the economy;
- (b) the distribution of these among the members of the economy, namely the amounts of the various factors owned by each member;

- (c) the tastes of consumers;
- (d) the state of technology, represented by well-behaved aggregate production functions (a “well-behaved” production function shows constant returns to scale and has positive but decreasing marginal productivities).

Perfect competition exists in all markets (commodities and factors).

If we consider the simple $2 \times 2 \times 2$ model (two countries, two commodities, two factors of production), the neoclassical model is a general equilibrium model where two countries trade owing to symmetric excess demands and supplies. While in a closed economy the only possible equilibrium point is where demand=supply for all commodities, in an open economy international equilibrium occurs when, at the equilibrium terms of trade, in a country (say, country 1) there exists a positive excess demand for a commodity (say, commodity A) matched by a negative excess demand (excess supply) for the same commodity by the other country. This means that country 1 will import commodity A , while country 2 will export it. Since, by Walras’s law (see below), within a country the excess demand for a commodity goes hand in hand with an excess supply of the other commodity, it follows that in country 2 there will be an excess supply of commodity B (exports) matched by an excess demand for B (imports) by country 2. Trade will be beneficial to both countries. Note that, since excess demand for a commodity within a country depends on tastes (that lie behind the demand function) as well as on technology and factor endowments (that lie behind the supply function), we conclude that the existence of commercial relations, the pattern and the volume of trade, and the terms of trade, are jointly determined in a general equilibrium setting by factor endowments, technology, and tastes, none of which can be in general said to be an exclusive or predominant causal agent.

The point of departure of the neoclassical model is Walras’s law, a fundamental property of all general equilibrium models with a budget constraint.

Let p_K and p_L indicate factor rewards, S_A and S_B the quantities of the two commodities supplied, K and L with a subscript A or B the quantities of the two factors allocated in the two sectors. Let us now recall that in each sector total factor rewards equal the value of output. This is true with constant returns to scale (first-degree homogeneous production functions: see Euler’s theorem), but is also true with any kind of production function provided that free entry and exit of competing firms obtain. Thus we have

$$p_K K_A + p_L L_A = p_A S_A,$$

$$p_K K_B + p_L L_B = p_B S_B,$$

from which

$$p_K (K_A + K_B) + p_L (L_A + L_B) = p_A S_A + p_B S_B. \quad (4)$$

The left-hand side of (4) is the total income of all the individuals in the economy (that they obtain by selling the services of the productive factors they own). Since in this model income is entirely spent in buying commodities A and B , we can write

$$p_K(K_A + K_B) + p_L(L_A + L_B) = p_A D_A + p_B D_B, \quad (5)$$

where D_A and D_B are the quantities demanded of the two commodities. Eq. (5) is the aggregate budget constraint. From Eqs. (4) and (5) it follows that the right-hand sides must be equal, as the left-hand sides are equal. Therefore

$$p_A D_A + p_B D_B = p_A S_A + p_B S_B, \quad (6)$$

whence

$$p_A(D_A - S_A) + p_B(D_B - S_B) = 0, \quad (7)$$

which is true for any admissible value of p_A and p_B . The form (6) states that the sum of the values of the quantities demanded must equal the sum of the values of the quantities supplied; the form (7) states that the sum of the values of the excess demands must be equal to zero. This relationship, whichever the form used, is known as *Walras' law*. In general, given n markets linked by a (budget) constraint, Walras' law implies that if $n-1$ markets are in equilibrium, the n th must also be in equilibrium. In our case there are only two markets, so that if one is in equilibrium the other must also be: for example, if $D_A = S_A$ then Eq. (7) implies $D_B = S_B$, and vice versa.

In a closed economy the only possible equilibrium is where $D_A = S_A, D_B = S_B$, but in an open economy the possibility arises of an equilibrium where $D_A \neq S_A, D_B \neq S_B$, provided that condition (7) is satisfied. Suppose that $D_A > S_A, D_B < S_B$. If the rest of the world is willing to supply to the country an amount $D_A - S_A$ of commodity A in exchange for an amount $S_B - D_B$ of commodity B , then an equilibrium will obtain. If we let the subscripts 1 and 2 refer to countries 1 and 2 respectively, and recall that Walras's law must hold for each country and so for the world as a whole, we have

$$\begin{aligned} p_A D_{1A} + p_B D_{1B} &= p_A S_{1A} + p_B S_{1B}, \\ p_A D_{2A} + p_B D_{2B} &= p_A S_{2A} + p_B S_{2B}. \end{aligned} \quad (8)$$

By addition we obtain

$$p_A(D_{1A} + D_{2A}) + p_B(D_{1B} + D_{2B}) = p_A(S_{1A} + S_{2A}) + p_B(S_{1B} + S_{2B}), \quad (9)$$

namely the total value of world demands equals the total value of world supplies.

Suppose now that, at a particular price ratio, the international market for commodity A

is in equilibrium, i.e.

$$D_{1A} + D_{2A} = S_{1A} + S_{2A}; \quad (10)$$

then it follows from (9) that

$$D_{1B} + D_{2B} = S_{1B} + S_{2B}, \quad (11)$$

namely that the international market for commodity B is also in equilibrium. From (10) and (11) it also follows that

$$\begin{aligned} D_{1A} - S_{1A} &= S_{2A} - D_{2A}, \\ S_{1B} - D_{1B} &= D_{2B} - S_{2B}, \end{aligned} \quad (12)$$

which state that excess demand for good A by country 1 (country 1's demand for imports) is equal to excess supply of the same good by country 2 (country 2's supply of exports) and that country 1's supply of exports of good B is equal to country 2's demand for imports of the same good.

It is also worth pointing out that conditions (8) imply that no country can be a net importer or exporter of *both* commodities. In fact, if we rewrite these conditions as

$$\begin{aligned} p_A(D_{1A} - S_{1A}) &= p_B(S_{1B} - D_{1B}), \\ p_A(D_{2A} - S_{2A}) &= p_B(S_{2B} - D_{2B}), \end{aligned} \quad (13)$$

we see that if $D_{1A} > S_{1A}$ (excess demand for commodity A by country 1, which thus imports this commodity), then $S_{1B} > D_{1B}$ (country 1 exports commodity B) and vice versa. This result is obvious if we think that in the model under consideration a country can obtain imports only by paying for them with exports. It should also be noticed that Eqs. (13) can be interpreted as the equality, for each country, between the value of its imports and the value of its exports when both are *evaluated at the given international prices*. Therefore, as is typical in the barter theory of international trade, the balance of trade always balances.

Since both demand and supply are ultimately functions of the relative price $p = p_B/p_A$, we define the excess demands

$$\begin{aligned} E_{1A}(p) &= D_{1A}(p) - S_{1A}(p), \\ E_{1B}(p) &= D_{1B}(p) - S_{1B}(p), \end{aligned} \quad (14)$$

Since $p_A E_{1A}(p) + p_B E_{1B}(p) = 0$ by Walras's law, we have

$$E_{1A}(p) + pE_{1B}(p) = 0,$$

$$E_{1B}(p) = -\frac{1}{p}E_{1A}(p) \tag{15}$$

Without loss of generality we can assume that country 1 wishes to import commodity A and to export commodity B, while the opposite is true for country 2. Thus

$$E_{1A}(p) > 0, \quad E_{1B}(p) < 0,$$

$$p = \frac{E_{1A}(p)}{-E_{1B}(p)}. \tag{16}$$

Equations (16) give rise to the (international) *offer curve* (also called *reciprocal demand curve* or *demand-and-supply curve*) of country 1. If we plot the demand for imports $E_{1A}(p)$ on the vertical axis and the corresponding supply of exports $-E_{1B}(p)$ on the horizontal axis, we obtain the quantity of exports that the country is willing to supply in exchange for any given quantity of imports. The offer curve of a country can be defined as the locus of all points which represent the quantity of the exported good that the country is willing to give in exchange for a given amount of the imported good (or, if we prefer, the quantity of the imported good that the country demands in exchange for a given amount of the exported good supplied). Equivalently, this curve indicates the various terms of trade at which the country is willing to trade. The terms of trade p , as defined in (16), are given by the slope of the straight line drawn from a point of the offer curve to the origin. For example, given $E_{1A}(p) = OH_A$, $-E_{1B}(p) = OH_B$, we obtain point Q of the offer curve, where $p = \tan \alpha$. This curve will have (in the normal case) the shape drawn in Figure 2 as OG_1 namely increasing but with a decreasing slope, and concave to the import axis.

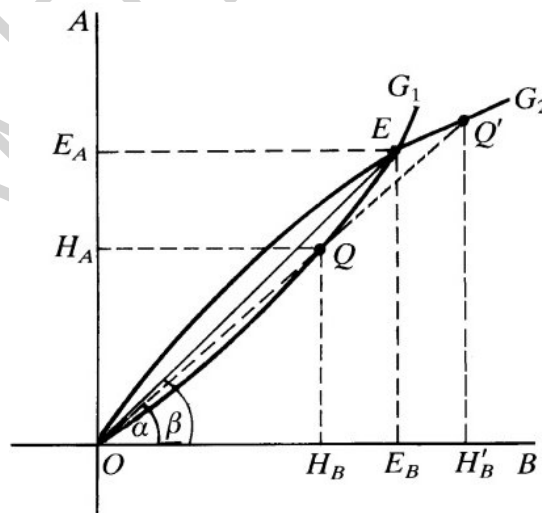


Figure 2: Offer curves and international equilibrium

Similarly we obtain OG_2 , the offer curve of country 2, where by assumption there is an

excess supply of commodity A and an excess demand for commodity B .

The only point where the demand and supplies of the two countries match is point E , which is the international equilibrium point. There the amount OE_A of commodity A will be imported by country 1 and exported by country 2, in exchange for the amount OE_B of commodity B , exported by country 1 and imported by country 2 at the terms of trade $p_E = \tan \beta = \text{slope of } OE$. The offer curves are widely used in international economics not only for determining international equilibrium but also for a number of other purposes.

The forces that bring the world economy to the equilibrium point E are dynamic forces acting on prices and excess demands. We make the usual Walrasian assumption that p moves according to the pressure of world excess demands, namely that the price increases when demand exceeds supply and vice versa. The mathematical counterpart of this assumption is the following differential equation

$$\frac{dp}{dt} = \psi [E_{1B}(p) + E_{2B}(p)] = \psi \left[E_{2B}(p) - \frac{1}{p} E_{1A}(p) \right], \quad (17)$$

where ψ is a sign-preserving function, namely $\psi [E_{1B}(p) + E_{2B}(p)]$ is positive, negative, or zero according as $E_{1B}(p) + E_{2B}(p)$ is positive, negative, or zero respectively. In the case of normal offer curves, the equilibrium point is globally stable. This can be shown by Liapunov's second method. Consider the Liapunov function

$$V = \frac{1}{2} [p(t) - p_E]^2, \quad (18)$$

that has the required properties. Then

$$\frac{dV}{dt} = [p(t) - p_E] \frac{dp}{dt} = [p(t) - p_E] \psi [E_{1B}(p) + E_{2B}(p)]. \quad (19)$$

A simple inspection of Figure 2 shows that $E_{1B}(p) + E_{2B}(p) > 0$ when $p(t) - p_E < 0$. Take, for example, $p(t) = \tan \alpha < \tan \beta = p_E$. Then, as we know, $-E_{1B}(p) = OH_B$, while $E_{2B}(p)$ is the abscissa of point Q' , where the straight line emanating from the origin and passing through Q intersects the OG_2 curve, namely $E_{2B}(p) = OH'_B$. Clearly, $OH'_B > OH_B$, hence $E_{1B}(p) + E_{2B}(p) > 0$. Similarly it can be shown that $E_{1B}(p) + E_{2B}(p) < 0$ when $p(t) - p_E > 0$. It follows that

$$\frac{dV}{dt} < 0 \quad (20)$$

everywhere except at the origin. This proves global stability.

Other behavior assumptions are possible, for which the reader can consult the relevant literature.

Let us now come to the gains from trade. We saw in the context of the classical theory that international trade is beneficial in so far as it enables a country to obtain a commodity at a lower cost than the domestic production cost. A similar conclusion holds in neoclassical theory.

Consider for example Figure 3 and suppose that the pre-trade closed-economy price ratio is represented by the slope of the straight line PP , whereas the terms of trade (post-trade open-economy price ratio) are represented by the slope of the straight line RR . Before trading started the country produced and consumed a commodity bundle given by the coordinates of point E . When trade is opened up, the country produces the commodity bundle given by the coordinates of point E' (*production point*). But it can now trade along the RR line, thus attaining previously unattainable points, outside its transformation curve.

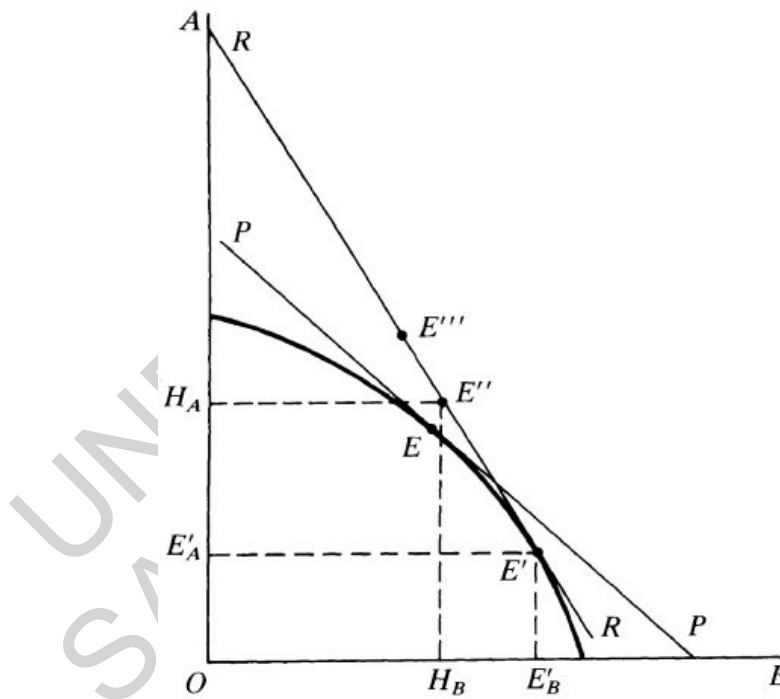


Figure 3: The gains from trade

For example, it can move to point E'' (*consumption point*) by trading $H_B E'_B$ of commodity B (exportables) for $H_A E'_A$ of commodity A (importables); point E'' is clearly better (excluding inferior commodities) than the pre-trade point E because the amounts of both commodities are greater at E'' than at E . It can also be seen that—since we have assumed that A is the imported, and B the exported, commodity—the opportunity cost of A in terms of B is greater in the closed economy situation (slope of PP referred to the vertical axis) than in the open economy situation (where the

additional amount of B that has to be given up to obtain an additional amount of A is measured by the appropriate terms of trade, namely by the slope of RR referred to the vertical axis).

But what if the post-trade situation is E''' ? This point is undoubtedly outside the transformation curve, and thus it could not be reached before trade, but since with respect to E it contains a greater amount of commodity A and a smaller amount of commodity B , it cannot be considered unambiguously better than E . It is however easy to observe that the value of national income at E''' is in any case greater than at E . This is true whether national income is calculated at the closed-economy (pre-trade) prices or at the new (post-trade) prices. Let us first consider the closed-economy prices. The value of national income at E is given by the position of the equal income line (which we call *isoincome*) PP , while at E''' it is given by the position of the isoincome line (not shown in the diagram) parallel to PP and passing through E''' , which is clearly more distant from the origin than PP . It follows that national income evaluated at the closed-economy prices is higher at E''' than at E . At the post-trade prices, the value of national income at E''' is given by the position of the isoincome line RR , while at E it is given by the isoincome line (not shown in the diagram) parallel to RR and passing through E , which is clearly nearer to the origin than RR (and hence represents a lower income).

2.1.3. The Heckscher-Ohlin Model

The neoclassical model is certainly more general than the two other orthodox models, but this generality entails a price, which is vagueness (international trade depends on everything). This explains the enduring success of the Heckscher-Ohlin model which, by concentrating on factor endowments, is able to give a simple answer to why a country exports a certain good.

This model, that can be seen as a particular case of the neoclassical model, stresses the differences in factor endowments as the cause of trade; more precisely, its basic proposition is that each *country exports the commodity which uses the country's more abundant factor more intensively* (the Heckscher-Ohlin theorem).

In addition to the usual basic assumptions (no transport costs, free trade, perfect competition, international immobility of factors) there are the following:

- 1) the production functions exhibit positive but decreasing returns to each factor (i.e., positive but decreasing marginal productivities) and constant returns to scale (i.e., first degree homogeneity). They are internationally identical, but, of course, different between the two goods, that is the production function of good A is the same in country 1 and country 2, and is different from that of good B (which is identical in the two countries).
- 2) The structure of demand, that is the proportions in which the two goods are consumed at any given relative price, is identical in both countries and independent of the level of income.
- 3) Factor-intensity reversals are excluded.

The first assumption, which embodies the usual properties of well-behaved production functions, and excludes the presence of international technological differences, is self-evident. The difference between the production functions of the two goods is of course necessary; otherwise it would not be possible to speak of two *different* goods.

The second assumption implies that tastes are internationally identical *and* represented by utility functions such that the income elasticity of demand is constant and equal to one for each good. Utility functions having this property belong to the class of *homothetic* utility functions. This assumption serves to exclude the possibility that, although tastes are internationally identical, the two goods are consumed in different proportions in the two countries because of possible differences in income levels.

It is then clear that the first two assumptions serve to exclude any difference between the countries as regards technology and demand, so that one can concentrate on the differences in factor endowments.

The third assumption is necessary to univocally determine the *relative factor intensities* of the two goods. In general, given two factors (capital K and labor L) and two commodities A and B , we say that a commodity (for example A) uses a factor more intensively or is more intensive in a factor (for example capital) relative to the other commodity if the (K/L) input ratio in the former commodity is greater than the (K/L) input ratio in the latter.

Now, if production of each good took place according to only one technique with fixed and constant technical coefficients (L -shaped isoquants), it would be an easy matter to determine the relative factor intensities once and for all. But since we are dealing with production functions with a continuum of techniques (smoothly continuous isoquants), different techniques will be chosen—in accordance with the standard cost minimization procedure—for each good at different factor-price ratios.

It follows that the classification of goods according to their factor intensities becomes ambiguous. To remove this ambiguity we add the requirement that the classification must remain the same for any (admissible) factor-price ratio, namely—in our example—that commodity A is more capital-intensive relative to commodity B if the (K/L) input ratio in the former commodity is greater than the (K/L) input ratio in the latter *for all factor-price ratios*.

The first step (a lemma) in our proof is to show that—at the same commodity-price ratio—a country abundant in one factor has a production bias in favor of the commodity which uses that factor more intensively. Let us define the (variable) technical coefficients a_{ij} ($i = K, L, j = A, B$) as the input of factor i per unit of good j , and consider the full-employment relations

$$\begin{aligned} a_{KA}A + a_{KB}B &= K, \\ a_{LA}A + a_{LB}B &= L. \end{aligned} \tag{21}$$

If we divide through by L , we obtain

$$\begin{aligned} a_{KA}A/L + a_{KB}B/L &= K/L, \\ a_{LA}A/L + a_{LB}B/L &= 1. \end{aligned} \quad (22)$$

By solving this linear system we can express A/L and B/L in terms of the remaining quantities, namely

$$A/L = \frac{a_{LB}K/L - a_{KB}}{a_{KA}a_{LB} - a_{KB}a_{LA}}, \quad B/L = \frac{a_{KA} - a_{LA}K/L}{a_{KA}a_{LB} - a_{KB}a_{LA}}, \quad (23)$$

whence

$$\frac{A/L}{B/L} = \frac{A}{B} = \frac{a_{LB}K/L - a_{KB}}{a_{KA} - a_{LA}K/L}. \quad (24)$$

Equation (24) expresses the output ratio (A/B) in terms of the factor endowment ratio (K/L), given the technical coefficients a_{ij} . These coefficients depend on the factor-price ratio but, given this ratio, are constant for any output level owing to the assumption of constant returns to scale. Therefore, for any factor-price ratio we can compute the derivative

$$\frac{d(A/B)}{d(K/L)} = \frac{a_{KA}a_{LB} - a_{LA}a_{KB}}{(a_{KA} - a_{LA}K/L)^2} = a_{LA}a_{LB} \frac{\varrho_A - \varrho_B}{(a_{KA} - a_{LA}K/L)^2}, \quad (25)$$

which will have an unambiguous sign thanks to the assumption of no factor intensity reversal; this assumption enables us to state that either $\varrho_A = a_{KA}/a_{LA}$ is always greater than $\varrho_B = a_{KB}/a_{LB}$ or ϱ_B is always greater than ϱ_A *independently* of the factor-price ratio. If we assume, for example, that commodity A is capital intensive, the derivative under consideration turns out to be positive, that is, the greater the factor endowment ratio (K/L) the higher the output of A relative to B , and vice versa. Since the production functions are assumed to be internationally identical, the above result holds for both countries; this proves the lemma.

It is now easy to show that *each country exports the commodity which uses the country's more abundant factor more intensively*. This follows from the lemma and from the assumption that the structure of demand is identical in both countries (and independent of the level of income). In fact, with free trade and no transport costs, the commodity-price ratio (terms of trade) is the same in both countries. Now, according to the lemma, at the same relative price of goods country 1 (the capital-abundant country) will produce relatively more A (the capital-intensive commodity) and country 2 (the labor-abundant country) will produce relatively more B (the labor-intensive commodity): the ratio A/B is greater in country 1 than in country 2. But, given the assumption as to the structure of demand, at the same relative price of goods both countries wish to consume A and B in the same proportion: it follows that country 1 will export A (and import B , which will be exported by country 2) so that after trade

the structure of the quantities of the goods available (the quantity available is given by domestic output plus imports or less exports) turns out to be identical in both countries and equal to the structure of demand. This completes the proof of the theorem.

2.1.4. The Four Core Theorems

Ricardian comparative-cost theory, neoclassical theory, and Heckscher-Ohlin theory together form the body of the orthodox theory of international trade. However, the factor-proportion theory is often identified with “the” orthodox theory, and the Heckscher-Ohlin theorem, together with the factor-price-equalization (FPE) theorem and two additional theorems (the Stolper-Samuelson theorem and the Rybczynski theorem), are said to constitute the four core theorems of the orthodox theory of international trade.

The *FPE theorem* states that international trade in commodities, under the assumptions of the Heckscher-Ohlin model and notwithstanding the international immobility of factors, equalizes factor prices across countries, $p_{1L} = p_{2L}$, $p_{1K} = p_{2K}$. To prove this theorem, we start from the relations

$$\begin{aligned} a_{LA}p_L + a_{KA}p_K &= p_A, \\ a_{LB}p_L + a_{KB}p_K &= p_B, \end{aligned} \quad (26)$$

where a_{ij} , $i = K, L$, $j = A, B$, denote the quantity of factor i required to produce a unit of commodity j . These relations derive from competitive equilibrium, where the price of a commodity equals the cost of the inputs used to produce a unit of the commodity. Given the assumption of internationally identical production functions, and since the prices of goods are internationally identical, Eqs. (26) are the same for both countries. This system can be solved for p_L, p_K , obtaining

$$p_L = \frac{a_{KB}p_A - a_{KA}p_B}{a_{LA}a_{KB} - a_{LB}a_{KA}}, \quad p_K = \frac{a_{LA}p_B - a_{LB}p_A}{a_{LA}a_{KB} - a_{LB}a_{KA}}. \quad (27)$$

This proves the equalization of factor prices. Note that when there is complete specialization these relations would not exist: in fact, with complete specialization, either $a_{LA} = a_{KA} = 0$ (complete specialization in commodity B) or $a_{LB} = a_{KB} = 0$ (complete specialization in commodity A).

The *Stolper-Samuelson theorem* states that the increase in the relative price of a commodity favors (in the sense that it raises the unit real reward of) the factor used intensively in the production of that commodity. Consider the total differential of Eqs. (26):

$$\begin{aligned} p_L da_{LA} + a_{LA} dp_L + p_K da_{KA} + a_{KA} dp_K &= dp_A, \\ p_L da_{LB} + a_{LB} dp_L + p_K da_{KB} + a_{KB} dp_K &= dp_B. \end{aligned} \quad (28)$$

The standard cost minimization procedure implies that, for any given output level, the

entrepreneur minimizes costs, treating factor prices as given. In other words, the entrepreneur chooses the input coefficients so as to minimize unit costs. The first order condition is, for commodity A

$$d(a_{LA}p_L + a_{KA}p_K) = p_L da_{LA} + p_K da_{KA} = 0, \quad (29)$$

and similarly for commodity B . Hence

$$\begin{aligned} a_{LA} dp_L + a_{KA} dp_K &= dp_A, \\ a_{LB} dp_L + a_{KB} dp_K &= dp_B, \end{aligned} \quad (30)$$

from which

$$dp_L = \frac{a_{KB} dp_A - a_{KA} dp_B}{a_{LA} a_{KB} - a_{LB} a_{KA}}, \quad dp_K = \frac{a_{LA} dp_B - a_{LB} dp_A}{a_{LA} a_{KB} - a_{LB} a_{KA}}. \quad (31)$$

We can assume, without loss of generality, that commodity A is the numéraire, so that $dp_A = 0$. A positive (negative) value of dp_B therefore means an increase (decrease) in the relative price p_B/p_A and, likewise, a positive (negative) value of dp_L means an increase (decrease) in the unit real reward (i.e., in terms of the numéraire) of labor.

Assume, for example, that commodity B is labor-intensive and that the relative price of this commodity increases. Given the definitions of the a 's, the greater relative labor intensity of B amounts to the inequality $a_{LB}/a_{KB} > a_{LA}/a_{KA}$ and, therefore, the denominator of the fractions in (31) is negative. As we have assumed $dp_B > 0$, it follows that $dp_L > 0$, $dp_K < 0$. The increase in the unit real reward of the factor used intensively in the industry producing the commodity whose relative price increases is thus proved.

The *Rybczynski theorem* states that the increase in the quantity of a factor (given the other) will cause an increase in the output of the commodity which is intensive in that factor and a decrease in the output of the other commodity, at unchanged commodity and factor prices. Consider the total differential of the full employment conditions (21), which is

$$\begin{aligned} A da_{LA} + a_{LA} dA + B da_{LB} + a_{LB} dB &= dL, \\ A da_{KA} + a_{KA} dA + B da_{KB} + a_{KB} dB &= dK. \end{aligned} \quad (32)$$

Since factor prices are given, the input coefficients (that depend solely on factor prices) do not change, hence $da_{ij} = 0$. Suppose that $dL > 0$, $dK = 0$. Then Eqs. (32) become

$$\begin{aligned} a_{LA} dA + a_{LB} dB &= dL, \\ a_{KA} dA + a_{KB} dB &= 0, \end{aligned} \quad (33)$$

whose solution is

$$dA = \frac{a_{kB}}{a_{LA}a_{kB} - a_{LB}a_{KA}} dL, dB = -\frac{a_{kA}}{a_{LA}a_{kB} - a_{LB}a_{KA}} dL. \quad (34)$$

Without loss of generality we assume that A is the labor intensive commodity, namely $a_{LA}/a_{KA} > a_{LB}/a_{KB}$. It follows that the denominator in (34) is positive, hence $dA > 0$, $dB < 0$. This proves the theorem.

2.2. Generalizations

Up to now we have considered the basic $2 \times 2 \times 2$ model of international trade, based on several simplifying assumptions: perfect competition, rigid supply of ubiquitous and internationally immobile productive factors, constant returns to scale, absence of intermediate goods, absence of transport costs, and so on.

Most of these assumptions can be relaxed while remaining in the context of the orthodox theory. Here is a partial list of the generalizations that have been made:

- 1) many countries, many factors, and many goods
- 2) increasing returns to scale
- 3) factor intensity reversals in the Heckscher-Ohlin model
- 4) non-identical structures of demand in the Heckscher-Ohlin model
- 5) transport costs
- 6) specific factors of production
- 7) intermediate goods
- 8) elastic factor supply
- 9) non-traded goods
- 10) international factor mobility and trade in factors

A treatment of these and other problems can be found in the textbooks and handbooks mentioned in the Bibliography.

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Biographical Sketch

Giancarlo Gandolfo began his career as an economist in the research department of the Bank of Italy. He then left the Bank for the academic career, first as assistant professor of economics, University of Rome, then as associate professor of mathematical economics at the University of Siena, full professor of economics there, and finally full professor of international economics at the Sapienza University of Rome, where he still teaches. He has given lectures, been visiting professor, and participated in PhD committees in several foreign universities. He is a member of the Accademia Nazionale dei Lincei, Rome, research fellow of the CESifo research network, Munich, honorary professor at Deakin University (Australia), and has been awarded a doctorate *honoris causa* by the Faculty of Economics of the University of Frankfurt. He is on the editorial board of several international journals (Review of International Economics, Journal of International Trade and Economic Development, Journal of Banking and Finance, Macroeconomic Dynamics, Studies in Nonlinear Dynamics and Econometrics, International Economics and Economic Policy), and is regularly consulted by leading international journals and publishers to act as referee for submitted articles and books. His main research interests concern international economics, mathematical methods and models in economic dynamics, and continuous-time econometrics, fields in which he has published about one hundred and fifty articles and about fifteen books, most of them in English. His books on international economics and economic dynamics have been translated into Chinese. He has received several research grants by national and international institutions. Among his books related to the present chapter, in addition to those already mentioned in the Bibliography, see *Elements of International Economics* (Springer, 2004) and *Economic Dynamics* (Springer, 1997). His CV and full list of publications can be found in his home page, <http://gandolfo.org>.