MATHEMATICAL MODELS IN REGIONAL ECONOMICS

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**Summary**

This chapter reviews the evolution of mathematical and statistical models that have been used for the development of theory and policy analysis in regional economics. The benefits of mathematical approaches are outlined. Theories and estimation techniques have developed in tandem with developments in the availability of geo-referenced data, new technologies and particularly computing power. Regional modeling was initially strongly influenced by macro-econometric and input-output models developed for national economies, but evolved into models that integrated these approaches into multi-regional settings that are often of a highly nonlinear nature and explicitly account for various forms of spatial interaction. New theoretical developments such as the new economic geography and endogenous growth theory have had a major impact on recent modeling trends and on the increasing popularity of spatial econometrics. Regional economic modeling has a long tradition of integrating approaches from different disciplines and in this paper we provide several examples of trans-disciplinary advances, notably entropy-interpretations of spatial interaction, computational neural networks and complexity theory. We conclude with outlining a range of areas of regional modeling in which promising developments are likely.

1. The Modeling Revolution in Economics

We cannot imagine modern economics without the use of mathematical and statistical tools. The past fifty years of economic science can indeed be characterized by the development of a plethora of quantitative methods for describing, analyzing and
predicting economic phenomena. There are very good reasons why such quantitative approaches have become almost universally adopted in economic research throughout the world:

- models offer a concise and compact – and hence efficient – representation of a multi-faceted economic reality;
- the use of models presupposes a clear and unambiguous definition of variables and symbols, so that a transparent discussion of economic phenomena can take place;
- models reduce a complex reality to a more simple but still insightful image that should encapsulate the most prominent economic forces and drivers, as well as their consequences;
- a formal quantitative approach is able to derive testable hypotheses, so that a consistent validation of economic theory can be ensured;
- a quantitative approach is not only able to test empirical statements about the economy in a statistically justified way, but is also able to infer statements on the same phenomenon in a different research domain (referred to as transferability);
- and, finally, models are an important communication tool, not only among economists, but also between scientists from other disciplines.

The quantitative revolution in economics has had far reaching impacts on the practice of economic research and has penetrated in almost all research domains of economics: macro-economics, industrial organization, finance, consumer behavior, international trade, labor market analysis, environmental policy, and so forth.

This trend has also manifested itself in regional and urban economics, where often similar tools and techniques are deployed. However, the nature of spatial processes is such that new techniques for data analysis and modeling have been warranted, and in recent decades realized with advances in computing and information availability, such as Geographical Information Systems (GIS), spatial interaction modeling, and spatial econometrics. In addition, regional and urban economics has a strong policy orientation and is therefore often at the interface of related disciplines, such as geography, planning, political science, environmental science, etc. The cross-fertilization with these disciplines has also had its own impact on modeling developments in regional and urban economics. A multi- and trans-disciplinary approach has been the genesis of regional science as a unifying framework for all social science phenomena with a spatial dimension. Commencing with Walter Isard’s 1956 book *Location and Space-Economy*, regional science emerged as the common ground for urban and regional policy analysis in the world’s developed economies and in recent years is increasingly spreading to less developed parts of the world. The 2004 special issue of *Papers in Regional Science* provides an excellent overview of influential developments and challenges for the future. The focus of this EOLSS chapter is specifically on the use of models in regional economics. Models for urban, resource, environmental and transportation issues are addressed in separate chapters of the EOLSS.

Against this background, we start in the next section with a brief history and typology of regional models that evolved in regional economic theory since the middle of the
20th century. Then we will address some issues of empirical regional modeling with an explicit spatial dimension. Next, we will take a cross-border view on modeling developments in relation to complementary disciplines. We will highlight – by way of interesting examples – a few areas where regional theoretical and empirical modeling has made a particularly important contribution. And finally, we will conclude with some speculative comments on the future of regional modeling.

2. The Evolution of Models in Regional Economic Research

Regional-economic models started in the tradition of macro-economic models as developed in particular in the 1950s by Nobel-Prize winner Jan Tinbergen. The main aim was to depict the interwoven spatial-economic mechanisms in quantitative terms so as to arrive at reliable predictions or adequate policy decisions. In the 1960s, mainly two classes of regional-economic models were developed, viz. (i) traditional Keynesian-type models for macro-economic policy (including private and public consumption and expenditures) estimated at a regional scale and (ii) regional input-output models in the spirit of the path-breaking work of W.W. Leontief.

The use of matrix algebra greatly condenses mathematical modeling into compact expressions. A very simple regional macro-economic model is of the form

\[ Ax + By = c \]  

which is a system of \( N \) linear equations in which \( y \) refers to a vector of \( N \) endogenous variables that are determined by the regional economy, \( c \) is a \( N \times 1 \) vector of constants and \( x \) refers to a vector of \( K \) exogenous variables. The latter are either determined outside the region or instruments of economic policy with values that can be changed by regional or national economic policy makers. The matrices \( A (N \times K) \) and \( B (N \times N) \) represent coefficients of behavioral equations and identities. Some of the \( y \) variables will have desired values that could be considered goals of economic policy. Hence, the effect of changes in policy instruments on regional economic outcomes can be calculated as

\[ \Delta y = B^{-1}A \Delta x \]  

in which \( \Delta \) refers to a change in one or more elements of a vector. Potential trade-offs between goals can be identified by considering various alternative changes to \( x \). Where specific goals are aimed for, the number of instruments would in general have to be at least equal to the number of desired goals.

In the case of the regional input-output model, the fundamental principle is that inputs of regional sectors may be outputs of other regional sectors, or imports from outside the region. Sales of regional sectors, which can be represented by a vector \( q \), are either to other regional sectors or are final demand by households or the public sector in the region, gross fixed capital formation, or exports to elsewhere. Final demand can be aggregated into the vector \( f \). Assuming proportionality between a sector’s sales and its
various input requirements (including imports) yields for each sector a vector of input requirements per unit of output, that can be combined in a matrix $A$ of technical production coefficients. Because total production in the regional economy is in equilibrium the sum of inter-industry sales and total final demand, the following equation describes the regional equilibrium:

$$\mathbf{x} = \mathbf{Ax} + \mathbf{f}$$

(3)

This equation can be used to assess how changes in regional final demand and/or exports would affect the production of each sector by solving for $\Delta \mathbf{x}$:

$$\Delta \mathbf{x} = [I - \mathbf{A}]^{-1} \Delta \mathbf{f}$$

(4)

These basic regional models were not explicitly spatial in that they tended to be concerned with a single region and not with a system of interacting regions. Instead, regional models largely replicated the mathematical equations describing the national economic structure and behavior. More emphasis, however, could be placed on the greater openness of a region vis-à-vis a country as reflected in flows of trade, capital and people. Other aspects, such as monetary policy and the prices of traded goods could be considered determined at the national or international level, exogenous in the model of the region.

At the same time, the potential was recognized by W. Isard and others of explicitly modeling a system of interacting regions through interregional input-output models that in the simplest form linked models of the form of Eq. (3) through assuming that regional imports were sourced in constant relative shares from supplying regions. This approach showed much promise but applications remained relatively limited in many countries due to an absence of data on flows of good and services between regions of a country. Interregional models were, however, at the cradle of current global multi-country modeling that is facilitated by the wealth of statistics on international trade and of which the Global Trade Analysis Project (GTAP) coordinated at Purdue University in the USA is a prime example.

In models that describe all trading that takes place in the economy, be it inter-regionally or internationally, equilibrium conditions must be imposed that ensure market clearing, at least in the long run, for the system as a whole. Such market clearing will generally require relative price changes. Models that embed input-output relations, but that also impose market clearing conditions that determine relative price changes, are referred to as Computable General Equilibrium (CGE) models. These have their origin in a multi-sectoral growth model of Norway published by L. Johansen in 1962. CGE models can be formulated for a single region, or for a system of regions (that may be sub-national or international).

One common feature of the first generation of regional models is that many applied ones tended to have a linear structure, as illustrated by the simple models described in Eqs. (1) to (4) above. This greatly facilitated the calculation of equilibria as solutions to a system of linear equations at a time when computing power and analytical tractability
of nonlinear systems was limited. While nonlinearity was not ruled out, linear approximation was seen as a pragmatic assumption that was valid provided changes to variables remained sufficiently small. However, where dynamic relationships were considered, linearity of dynamics limited the range of possible paths to either convergent or divergent and either non-oscillatory or oscillatory. For an analysis of long-run dynamics, regional economics resorted to popular models of national capital accumulation and output growth, namely the Keynesian Harrod-Domar model or the neoclassical Solow-Swan model.

In the 1970s Keynesian macro-economic and input-output models were increasingly integrated and used to identify policies that aimed to maximize welfare, witness the popularity of linear and nonlinear programming models at a regional scale. In their simplest form, such models combined linear structures such as discussed above with linear or quadratic objective functions. These served important policy purposes in the area of regional development policy and land use planning. These models were also extended to the domain of transportation science, infrastructure policy and urban land-use planning. Most of these models were taking regional economic growth for granted without any regard to limited resources (e.g., environment, safety).

Another regional modeling approach that was developed in the 1970s and applied extensively since then is a family of models that aimed to analyze and predict various types of spatial interaction. These models were commonly referred to as gravity models. This prominent class of spatial modeling has its roots in gravity theory from Newtonian physics, witness the statement of its founding father H.C. Carey, who claimed in 1858: “Man tends, of necessity, to gravitate towards his fellow man.” Spatial interaction models share with the gravity law of physics the property that flows tend to be positively related to some form of mass at origin and destination, while being inversely related to distance between origin and destination. W. Alonso provided a very general formulation of these models in the 1970s, which he referred to as a theory of movements. The Alonso model has been applied to regional issues, such as migration, trade, commuting and traffic flows, but also to less obvious spatial allocation problems such as that of patient flows to a set of hospitals.

The Alonso model can be represented by three equations:

\[
T_{ij} = A_i^{1-\alpha} V_i B_j^{1-\beta} W_j F_{ij} \quad (5)
\]

\[
A_i = \left( \sum_{j=1}^{N} B_j^{1-\beta} W_j F_{ij} \right)^{-1} \quad (6)
\]

\[
B_j = \left( \sum_{i=1}^{M} A_i^{1-\alpha} V_i F_{ij} \right)^{-1} \quad (7)
\]

in which \( T_{ij} \) represents the observed flow between origin \( i (=1,2,...,M) \) and destination \( j (=1,2,...,N) \), \( V_i \) is an index of observed push factors that generate flows out of the
origin, \( W_j \) is an index of observed pull factors that attract flows to destination \( j \) and \( F_{ij} \) is an observed measure of facilitation of flows between \( i \) and \( j \) (for example, \( F_{ij} \) is inversely related to distance between \( i \) and \( j \), or to barriers imposed by authorities). \( A_i \) and \( B_j \) are referred to as balancing factors that are artificial variables which can be calculated from Eqs. (6) and (7), for given observations and parameter values. However, they have an economic interpretation in that \( A_i \) will be large when there are relatively few attractive destinations in the vicinity of \( i \). Similarly, \( B_j \) will be large when there are relatively few unattractive origins near \( j \). It is easy to see that the model yields plausible mathematical expressions for the overall outflow from \( i \) and the overall inflow into \( j \), respectively

\[
\sum_{j=1}^{N} T_{ij} = A_i^{-\alpha} V_i
\]

(8)

\[
\sum_{i=1}^{M} T_{ij} = B_j^{-\beta} W_j
\]

(9)

The parameters \( \alpha \) and \( \beta \) can be calibrated based on goodness of fit criteria. They will generally be in the interval between 0 and 1. The case of \( \alpha = \beta = 1 \) is referred to as an unconstrained gravity model, while \( \alpha = \beta = 0 \) generates a doubly constrained gravity model. It should be noted that the model described above provides a static description of the generation and allocation of flows. The first reformulation with explicit dynamic feedbacks was developed by the present authors in 1987.

A concept related to gravity modeling, that also had its origins in physics and that became popular in the social sciences during the 1970s, is that of entropy. This refers to the idea that elements of a system tend towards an arrangement that can be organized in as many ways as possible, which is called the maximization of the entropy of a system. In information theory, entropy represents expected information. Entropy found many applications in regional systems. P. Nijkamp and J.P.H. Paelinck showed that entropy maximizing models are a sub-class of geometric programming models, and that such models possess a dual formulation that links closely to the gravity model. Further details are provided in Section 4 below.

In the 1980s, the first generation of regional economic models was enriched by incorporating also environmental constraints, energy availability, quality of life, as well as equity. This led to a new generation of regional-economic models which were more integrative in nature and which encapsulated also various distinct policy objectives, reflected in the popularity of multi-objective programming models and multi-criteria analyses.

However, enthusiasm for the use of large-scale regional econometric models diminished during the 1980s for two reasons. One was widespread acceptance of the so-called Lucas critique, which refers to the work of R.E. Lucas Jr, that pointed to the limited
predictability and effectiveness of policy intervention based on macro-economic models. Essentially, the problem is that rational economic agents might anticipate the impact of government policy guided by a macro-economic model and through their actions remove the stability of the underlying behavioral relationships.

The second reason for the diminished popularity of traditional macro-economic models was the advent of sustainability objectives that were popularized through the Brundtland report of 1987. A concern for sustainability required greater evaluation of environmental outcomes, the need to consider nonlinear feedback mechanisms in economic-environmental modeling, and the use of much longer planning horizons.

Overall, to gain better understanding of behavior of economic agents at the level of the firm and individual, greater emphasis was, from then on, placed on micro-foundations based theory and the analysis of micro-level behavioral data that could be obtained from a rapidly growing number of social science surveys. Consequently, a new strand of modeling literature focused attention more on specific socio-economic sectors at the regional level, such as the labor market, the housing market, migration, commuting, transportation and congestion, etc. Most of these models formed a blend of macro-meso analysis and micro-based discrete choice theory developed by D. McFadden and others that led to logit or probit-type econometric models. Surveys can also provide a pseudo-experimental approach to analyzing human behavior, such as stated preference techniques in environmental modeling, of which contingent valuation methods and conjoint analysis are common examples.

Two theoretical developments during the 1990s have had a major impact on the understanding and evolution of economic activity across regions. The first of these is a strand of literature referred to as the new economic geography. This literature commenced with a seminal article by P. Krugman in 1991 that describes a relatively simple core-periphery model with two sectors: agriculture and manufacturing. A range of models was developed subsequently by M. Fujita, P. Krugman, A.J. Venables and others. The strength of these models is that they use building blocks of theories of trade and imperfect competition developed since the 1970s to model centrifugal and centripetal forces in the spatial economy, thereby overcoming the ‘straightjacket’ of the assumption of perfectly competitive markets that was the cornerstone of much traditional general equilibrium modeling, but rather implausible in a spatial setting. The new economic geography models also generate a spatial configuration of economic activity that is defined by economic rather than administratively determined regions, although the existence of administrative borders will have an impact on cross-border interaction.

A second influential development has been that of theories of endogenous growth, following seminal work by P.M. Romer and R.E. Lucas Jr. Unlike the neoclassical growth model that considered the long-run rate of technological change as an exogenous force, endogenous growth models explicitly identify the impact on growth of regional endowments such as population scale, and the significance of behavior and policy with respect to education, research and development expenditure, etc. A 1998 survey by the present authors shows how such issues may matter for convergence or divergence of regional growth paths. Of particular significance is the role of innovation.
as a driver of technological change and growth, as elucidated by the work of Z. Acs, R. Stough and others. However, there are various other regional characteristics that may also impact on regional structure and growth (such as the presence of amenities, the composition of the labor force, etc.). W.-B. Zhang proposed several such models in a two-region setup. Thus, the new economic geography and new growth theories have re-oriented regional-economic modeling, and instilled a deep concern about growth drivers, entrepreneurship and innovation, not only from a theoretical but also from an applied perspective. This approach is based on both the new growth perspective at a meso-scale and a micro-economic Schumpeterian approach to understanding the influence of entrepreneurial motives.

In terms of input-output and computable general equilibrium modeling, new software and better data have led since the late 1980s to increasing popularity of CGE models vis-à-vis input-output models. Many examples of urban and regional CGE models can now be found. A promising development has been the construction of multi-regional CGE models, in which Australian researchers such as J.R. Madden and J.A. Giesecke have played a leading role. Such models, especially when formulated as dynamic systems, have traditionally been constrained by computational requirements but as computing power continues to expand, computing issues are becoming less restrictive than the still limited availability of high quality regional economic data.

However, input-output models continue to be useful to study the evolving structure of the regional economy, but also form the basis for the calculation of multipliers that are central to the assessment of the economic impact of specific regional investments or publicly-subsidized events. Such economic impact analysis at the regional level is widely practiced throughout the world. However, limitations due to paucity of available data or due to adoption of a rather partial approach are unfortunately often downplayed.

Ever increasing computer power has also since the 1990s led to a growing interest in introducing nonlinear dynamics in describing socio-economic systems. This has generated many interesting theoretical developments such as chaos theory, complexity theory and network analysis. Such theories are also increasingly applied to regional economic systems, as is elaborated in Section 4.

Similarly, the increase in computing power is showing much promise for microsimulation and/or agent based modeling. In the former, large-scale data sets on individuals are used to assess by means of simulation the impact of changes in conditions or policies on the individual outcomes, taking account of the probabilistic and transitional nature of outcomes over time. In the latter, the key aspect is that in making transitions, individuals are guided by the decisions of others and by aggregate outcomes. This may lead to simulations generating very complex behavior and outcomes despite fairly simple descriptions of reality and small datasets. While microsimulation and agent based modeling approaches developed in parallel but independently, a synthesis of these approaches is likely to emerge in the near future.

Having synoptically described the evolution of regional economic models during the last half century, we will briefly focus on issues of data availability and model estimation approaches in the next section.
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Biographical Sketches


Peter Nijkamp is Professor of Regional Economics, Faculty of Economics and Business Administration, Free University Amsterdam, since 1973. He has been Visiting Professor in several universities in Europe, North-America and Asia. He is a Fellow of the Royal Netherlands Academy of Arts and Sciences (KNAW) and of the Belgian Academy of Sciences. At present, he is President of the Governing Board of the Netherlands Organization for Scientific Research (NWO).

Peter Nijkamp’s main research interests cover plan evaluation, multicriteria analysis, regional and urban planning, transport systems analysis, mathematical modeling, technological innovation, and resource management. In the past years he has focused his research in particular on quantitative methods for policy analysis, as well as on behavioral analysis of economic agents. He has a broad expertise in the area of public policy, services planning, infrastructure management and environmental protection. In all these fields he has published many books and numerous articles.

Professor Nijkamp is a member of editorial boards of some 30 international journals in the field of regional and urban economics, environmental management and transportation policy.


Jacques Poot is Professor of Population Economics, Population Studies Centre, University of Waikato, New Zealand. He was previously Foreign Professor at the University of Tsukuba in Japan (1994-97 and 2002) and employed in various academic positions at Victoria University of Wellington between 1979 and 2003. He is an elected Corresponding Member (Honorary Fellow) of the Royal Netherlands Academy of Arts and Sciences and is an adjunct professor in the Department of Spatial Economics at the Free University of Amsterdam. He is a member of the Board of Management of the Building Research Capability in the Social Sciences (BRCSS) network, a government-funded New Zealand-wide initiative to enhance capabilities in social science research.

Jacques Poot’s research interests include all aspects of the economics of population (such as migration, fertility, labor force, and ageing) and especially the spatial dimension of these topics. He has also carried out research projects in a wide range of other areas, such as: meta-analysis of non-experimental social science research; monopsony in local labor markets; the impact of geography on economic growth; regional development; globalization; innovation and efficiency in the New Zealand construction sector; transportation policy and the environment; housing markets; and forecasting.

Professor Poot is a member of the editorial board of a number of international journals and was 1997-2006 Pacific Editor of *Papers in Regional Science*. 

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