

PHILOSOPHY OF SPACE AND TIME

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Summary

It is easy to think that our notions of space and time have barely changed since the ancient Greeks. With the proviso of certain relativistic effects, we more or less take them for granted as the backdrop in which all other Sciences take place. But is this correct?

We will see in this chapter that since the 1950s it has been clear that the assumption of a spacetime continuum is actually unjustified and could well be wrong. Only in the 1980s, however, were serious alternatives developed by mathematicians and since then a growing confluence of philosophical ideology, mathematical innovation and the possibility of experimental test has led to the emergence of a new science of space and time.

In principle there are as many approaches to spacetime as there are to the unsolved problem of a unified theory of quantum gravity. However, we will explain how a coherent new science has emerged in spite of that, based on the idea of ‘quantum spacetime’ as a common framework. The mathematics used will be noncommutative differential geometry and we will use a form of it coming out of ‘quantum groups’ but which is not incompatible with another form coming out of cyclic cohomology.

The chapter also covers how this new science touches upon the biggest problems in theoretical physics at the moment, such as the origin of time, dark energy and the masses of elementary particles. One of its main physical predictions to date is of an

energy-dependence to the speed of light as a small correction to Einstein's theory. The origin of this is curvature of momentum space, a potentially new phenomenon in physics which we have previously termed 'cogravity'.

The chapter also leads into a wider philosophical debate about the nature of representation and reality in Physics. In this way we aim to make educators and working scientists in other fields aware of just how exciting and mysterious theoretical physics is at this time of revolution in our understanding of space and time.

1. Introduction

Space and time are things that we take for granted in almost all branches of Science. Events take place in space and time, wave-functions in quantum theory are spread over space and evolve in time, particles are lines in spacetime. But how much do we actually know about these things called 'space' and 'time' that we all take for granted? For the most part we still think of space as emptiness that we can move our hands around in, and time as another similar dimension generally by analogy with space even though we are not so free to move back and forth. Since Einstein we think of gravity as expressed in curved spacetime but still as a spacetime continuum. This will be covered in Sections 2 and 3.

In fact, physicists have known perfectly well for several decades that such a notion of a spacetime continuum has no scientific basis and our goal in Section 4 is to explain why not. It is so hard, however, to imagine an alternative that theoretical physicists have for the most part completely ignored this issue and continued to formulate fundamental physics as a quantum theory of particles or extended objects such as strings moving in a continuum, ignoring that it does not exist. This means that string theory in particular cannot live up to its claims as a theory of everything as it does not solve the fundamental problem.

For those who are serious about quantum gravity we need, then, to address the true or at least truer nature of space and time. We need first of all a new philosophy, and on the physics side the approach we will take is to treat quantum gravity as a black box and see what we can say effectively without knowing exactly what is inside. We can still learn lessons from specific ideas for quantum gravity but we do not need to and won't cover all the different approaches to quantum gravity itself in this chapter. Rather our theory provides a common framework that should feature as a correction to classical spacetime without knowing the whole theory. This might seem disappointing but I would argue that this is the correct, if old-fashioned, way to do all Science as a series of effective theories without worrying about 'ultimate theories' although providing insight into them. Our approach is similar to how a mathematical framework such as topology can be used to constrain complex systems that are not fully understood.

Meanwhile, the new philosophy on the mathematical side will be provided by the idea that geometry is algebra and algebra is geometry, and a coherent 'quantum geometry' that emerges from and expresses this. This is the topic of Section 5 and is the mathematical heart of the chapter. Going deeper into it is a kind of dualism between variables and functions, or between measured and measurer, which suggests in turn new

philosophical insights for the structure of quantum gravity taken from the author's own work, and which we overview in Section 8.

Science also requires facts and the new science keeps at its centre the possibility of experimental test even if the effects are tiny quantum gravity or Planck scale ones. We describe one such in Section 6. Our approach is also driven by long-standing and sometimes overlooked mysteries in theoretical physics and we will try to indicate how the new science of space and time and in particular the quantum spacetime hypothesis could impact upon some of these mysteries in Section 7.

Sections 2-4 are within established physics while Section 5 relates to what is now established mathematics, and between them they make up 2/3 of the chapter and most of what we wish to get across. The last 1/3 of the chapter comprising Sections 6-8 will be more rooted in the works of the author and others as an illustration of what is possible in the proposed new paradigm of quantum spacetime. The rest of the present Section 1 is an introduction to where we will be heading and again represents the author's personal perspective.

1.1 Theoretical Physics Needs Philosophy

If you look at the start of the 20th century you will find the emergence of ideas of relativism and logical positivism which were part of a creative intellectual climate that made possible respectively Einstein's formulation of gravity and quantum theory as an operational theory. Can we really expect the next revolution, even what some have claimed as the ultimate theory, without some similarly profound new input?

The problem here is that by the 1960s theoretical physics was already 'industrialized' with ample opportunity for variations and computations of different Lagrangians for researchers to make a career without the need for thinking about intractable or philosophical problems. Much progress was made early on, not least the discoveries in particle physics of quarks and of the electro-weak force, but since then there have been no fundamentally new discoveries in particle physics other than confirmation of the Higgs. In this vacuum many researchers in the 1980s turned to string theory but without actually addressing the most fundamental problems either. Indeed, one of its features was that it involved no new philosophy or really new methods. One took the standard methods and ideas of quantum field theory and adapted them to fields with values in a 'target space', a classical spacetime in which the string moved. 30 years later this approach has not provided any testable prediction for quantum gravity of reasonable credibility and in retrospect one could describe this lack of a new philosophy as a bug and not a feature. This is not to say that this or any other ideas for quantum gravity are necessarily wrong, only that we may need to take a look at what we are really trying to do.

It is also important in this discussion to realize the difference between philosophical thinking, which will concern us, and the rigors of professional History and Philosophy of Science, for which other chapters in this project would be more appropriate. The history and philosophy of science has an important retrospective role in the clarification of successful theories already found by physicists. By contrast, philosophical ideas and

their more precise mathematical implementations are now on the cutting edge of Science itself as a critical new ingredient and this seems often to be overlooked.

This touches upon another cultural difference that existed in the 1980s compared to the early 20th century, namely a disconnect between mainstream theoretical physics and pure mathematics. The author (Majid , 2008) has argued that Nature is ultimately unlikely to have used only the mathematics known in mathematics books written to date, and if so how can one do theoretical physics without first being attuned to the deepest principles of pure mathematics? It should therefore come as no surprise then that many of the ideas in later sections of this chapter originated in the work of interested mathematicians outside of the mainstream theoretical physics or strings communities.

1.2 Algebra with Everything

A standing joke about British cuisine is the notion that you have to have chips with everything. Here we will find the notion of algebra with everything. Why is this? It is because algebra is about proving things with symbols. As this is central to all of mathematics, algebra is central even if taken for granted. The symbols here are often place-holders for actual numbers and this is why algebras have the familiar properties of being able to add and multiply elements of the algebra. However, the algebra in some sense also represents the structure of the space of possible values and in this way algebra represents the geometry of that space. Thus you can do geometry – which is also the modern way to do gravity – as algebra.

On the other hand, if you study algebra for long enough you will eventually think of the algebra itself as the real thing and seek its representations. This is a point of view more natural to quantum theory, where typically we seek representations of matrix and Lie algebras, regarding the algebra as the collection of observables and its representations as defining possible states. These algebras are noncommutative in the sense that one can have $ab \neq ba$ for some elements a, b in the algebra. This is never possible for ordinary numbers but is possible for matrices or operators. This is why Heisenberg called his approach to quantum theory ‘matrix mechanics’.

Now if we want fundamentally to address quantum gravity then we first need to find a common language. One approach is to recast quantum mechanics as classical geometry but a new approach is to recast gravity as algebra. This can then be extended into the quantum domain by allowing the algebra of ‘coordinates’ of the geometry to be noncommutative. This is called noncommutative geometry and we will say more about it.

Another revolution beginning in the 1980s and involving algebra has been quantum groups. These arose out of mathematical physics both from the study of integrable systems and as toy models of quantum gravity (in the manner below in Section 1.3). By now they are of independent mathematical interest with many applications and properties including, for instance, being at the heart of why Chern-Simons quantum field theories lead to 3-manifold and knot invariants. These objects entail a new notion of symmetry that turns out to be more appropriate as, among other things, symmetries

of noncommutative geometry. This leads to what has been called the quantum groups approach to noncommutative geometry guided by principles of quantum symmetry.

Theoretical physics often has waves of interest with much activity that eventually fade but leave behind a cultural change. In the 1970s it was supersymmetry. Since the 1980s, we have mentioned quantum groups. Probably the greatest legacy of these two waves of interest on the part of physicists has been greater familiarity with and respect for algebra as a subject every bit as rich as geometry. Prior to this the main algebra that was really appreciated by physicists was the Heisenberg or ‘canonical commutation relations’ algebra, which is much like thinking that geometry is mainly about flat space rather than a much richer set of possibilities. This increased awareness has made possible the emergence of the new science of quantum spacetime.

Finally, in stressing algebra we will end up downplaying analysis. This is partly because analysis tends to be more technical and partly because it tends to be grounded in continuum assumptions that we will be challenging and this would confuse the picture at our present level of approximation. In fact many algebraic constructions have versions using operators and Hilbert spaces etc. and conversely many constructions that would seem to involve analysis have algebraic or category theoretic analogues.

1.3. Mach’s Principle and Born Reciprocity

Here we mention a couple of specific philosophical influences that will be relevant later. First of all the works of Ernst Mach have been presented by Einstein as inspiring his work. Underlying this is the deeper idea that a thing does not exist in isolation of the means by which to observe it and that the laws of physics somehow derive from this. Thus, if there was only one particle in the Universe then how would one know if it was accelerating? In that case the presence of other particles can be viewed as in some sense causing the acceleration of the first and this could be ‘why’ there is gravity. It is not a vision that is exactly realized in Einstein’s theory but such a Mach’s principle may still be relevant to quantum gravity (Majid, 1991).

Meanwhile, the philosopher and physicist Max Born proposed that a fundamental role in physics should be played by what is now called ‘Born reciprocity’ (Born, 1949), that both position space and momentum space should be equally treated in physics. This was evident in the formulation of classical mechanics at the end of the 19th century where position and momentum are part of a combined ‘phase space’. The momentum of a particle is normally proportional to its velocity and together the position and momentum make up the degrees of freedom of a classical particle. In the case of flat space or spacetime one can go from one to the other by Fourier transform – by the consideration of waves. These waves are labeled by momentum and the space of possible momenta is called momentum space. In quantum mechanics these plane waves can form wave packets centered around a particular momentum and these approximate in the appropriate limit to classical particles. This is the idea of ‘wave-particle duality’.

Fast forward now to the 1990s. Using quantum group methods it turned out that one can extend Fourier transform to nonAbelian groups and equate curvature on one side with noncommutative spaces on the other side. NonAbelian groups include such things as the

3-dimensional sphere and the related group of rotations $SO(3)$, and as spaces such groups tend to be curved. We summarize this in Figure 1:

	Position	Momentum
Gravity	Curved	Noncommutative
Cogravity	Noncommutative	Curved
Quantum Gravity	Both	Both

Figure 1. Table showing options related by quantum Fourier transform

The first line includes simple toy models of gravity in the sense of a constantly curved space as given by a nonAbelian Lie group. Gravity is an experimental fact so this line is a simplified model of something that we observe. Quantum Fourier transform swaps the columns and leads to the second line as a mathematical possibility or hypothesis inspired by Born reciprocity and which we accordingly have called ‘cogravity’ (Majid, 2000). This is a potentially new physical effect different from gravity because spacetime here is geometrically flat albeit quantum and now it is momentum space which is curved as expressed in a nonAbelian group structure. The first models like this appeared in the 1990s most notably the Majid-Ruegg model (Majid and Ruegg, 1994) and we will cover this in detail in Section~6. This second line is not yet experimentally observed but rather is an example of how philosophy can suggest a particular theoretical hypothesis. In physical terms it amounts to the hypothesis that quantum gravity effects can be modeled by quantum spacetime and/or curved momentum space.

If cogravity is a reasonable hypothesis then, since gravity *is* observed, we actually need to go further and have models with both gravity and cogravity as coexisting effects. This means that we should search for the simplest examples of objects to fill the third line of the table – something like nonAbelian groups but the spaces of which are quantum spaces, so both quantum and curved. This is the point of view that had led the author in the 1980s to one of the two main classes of such new objects called ‘quantum groups’, namely as toy models of quantum gravity (Majid, 1988). More generally one can consider the maintaining of Born reciprocity in a more general form as a foundation for quantum gravity and the deeper structure of space and time. In this scheme of things the quantum gravity version of Einstein’s equation for gravity should emerge as a natural self-duality condition in which the roles of position and momentum are parallel and reversible. So far such a vision has been only partly realized but remains a motivation.

2. Prehistory of Space and Time

This will not be any kind of scholarly account but rather we recall one or two matters to lead into later sections.

2.1. Euclidean Space and the Origins of Geometry

The ancient Greeks had a good grasp of geometry of the compass and ruler variety. Euclid’s formulation in *Elements* is famously in terms of the following self-evident postulates (Euclid, translation 1956):

1. "To draw a straight line from any point to any point."
2. "To produce [extend] a finite straight line continuously in a straight line."
3. "To describe a circle with any centre and distance [radius]."
4. "That all right angles are equal to one another."
5. "That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles."

From a modern point of view this is only appropriate to flat space. In nonEuclidean or curved space one still has a notion of geodesic analogous to a straight line but parallel geodesics can converge, a phenomenon visible on the surface of a sphere (all meridians or lines of constant longitude converge at the north and south poles, for example) or in spacetime as the phenomenon of gravitational lensing. The departure from Euclid's axioms is therefore a measure of gravity.

Plato in *Timeaus* around 350bc had taken a more symmetry-oriented view of geometry with a focus on properties of shapes of objects in spacetime – now called the Platonic solids. These are the five solid analogs of the regular polygon – made by repeating an identical face without gaps as in Figure 2:

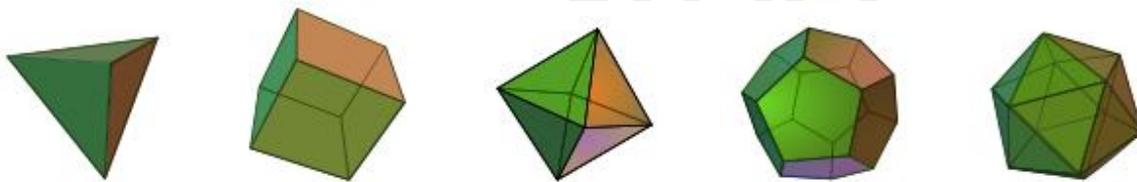


Figure 2. The five Platonic solids [creative commons license]

The proof that these are all is in Euclid's work while the shapes themselves are found independently in Neolithic sculptures a thousand years earlier. Plato identified them respectively with classical elements Fire, Earth, Air, Ether (the stuff that stars were made of) and Water. From a modern perspective the replication or 'transport' of equal lengths to different facets is a reflection of what we now would call flat space. It also illustrates the notion of discrete symmetry: the set of rotations and reflections of a shape that leave it unchanged. For example, the symmetries of the tetrahedron form the group A_4 of those permutations of the vertices that are expressible as an even number of transpositions.

Theoretical physicists are still looking to understand the different types of particles in Nature and as a pedagogical exercise suppose Plato was right and the different particles could be grouped into five types associated to the five solids. Thus an electron might be deemed to have 'internal structure' that of a tetrahedron, for example. The finite symmetry group A_4 would under this scheme be an analogue of the internal gauge symmetries in modern physics such as give rise to the electromagnetic and weak forces of Nature and matter would be viewed as finite geometry associated to each point of ordinary spacetime. We will see something like this in Connes' approach to matter in Section 7.4 but with the fundamental difference that one needs to use a finite but noncommutative geometry.

Another key ingredient of modern geometry underlies one of Zeno's nine paradox, namely the Achilles and the Tortoise as recounted in Aristotle (Aristotle, translation 1930):

“In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead.”

Thus if the tortoise starts 100 meters ahead, but the time Achilles has reached that point the tortoise will have run some other distance ahead of it, say 10 meters, and so on ad infinitum. The modern (and in fact Aristotle's) resolution of this is that the ticks of the clock in Zeno's analysis are not equally spaced. The relative distance between the runners shrinks to zero in a finite time because the time intervals get smaller and smaller in such a way that they do not add up to infinity. These ideas of convergent sums of infinite series eventually became, in the 19th century, the basis of modern analysis. The key intuitive assumption – which Zeno assumed – is that space and time are infinitely divisible; modern analysis is perfectly equipped to handle such notions as this and related notions such as ‘arbitrarily small’ and ‘limit’.

These notions underlie the real number system \mathbb{R} . Scientists almost universally assume the real number system and its enhancement the complex number system \mathbb{C} , but on what basis? No-one can ever definitively measure an irrational number as this would need infinite precision. And to a pure mathematician a real number is in fact nothing other than a certain Cauchy type of infinite sequence of rational numbers. In a complete metric space every Cauchy sequence converges. The rational numbers are not complete but by adding in all Cauchy sequences one makes it, by definition, complete. In short, the real numbers are merely a construct to simplify the otherwise more complicated properties of the rational numbers. One should regard the rational numbers (a pair of integers for the top and bottom of the fraction and an equivalence of different expressions of the same fraction) as the more primitive objects. You can count natural numbers on your fingers and with a little more care you can count fractions. In fact the real numbers are not the only completion, the others are the p -adic completion associated to each prime number p and these do not have infinite divisibility. They are said to be non-Archimedean, after the mathematician and scientist Archimedes a century or so after Aristotle, Euclid and Plato.

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Biographical Sketch

Shahn Majid is a pioneer of the modern theory of quantum groups and quantum spacetime, with three books and more than a hundred and fifty research articles. He received his B.A. and Part III diploma in Mathematics at the University of Cambridge and his PhD in 1988 at Harvard University. In 1993 he was awarded a one-time Konrad Bleuler Medal. He spent a decade at the Department of Applied Mathematics and Theoretical Physics, University of Cambridge as a fellow of Pembroke College, before moving to Queen Mary, University of London. He has been visiting professor at Harvard, the Perimeter Institute, Oxford University and Cambridge University as well as principal organiser along with Alain Connes and Albert Schwarz of an extended programme on Noncommutative Geometry at the Isaac Newton Institute. In 2009 he was a Leverhulme Trust Senior Research Fellow.