GRAVIMETRIC MEASURING TECHNIQUES

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Summary

The knowledge of gravity intensity is an essential tool for the geodynamical interpretations at local, regional and planetary level. Gravity measurements have important practical applications of economic interests for your society and future life conditions for the humanity.
Some examples are:

- Short distance gravimetric prospections are used to identify anomalies of density in the subsoil which are significant in oil researches (early discovery of salt domes in Texas) as well as for the detection of underground cavities (practical safety problems),

- Subterranean gravity measurements have been used to determine the extent of iron and chromites deposits,

- Measurements along the very long major faults, like the historic measurements along the Indonesian islands arc by Vening Meinesz (1930) made evident important mechanical stresses transmission in the Earth’s crust,

- Slow uplift of Scandinavia or subsidence of the Po Valley in Italy, the variations of thickness of the large glacial plateau in Greenland and Antarctica (owing to the large difference of densities between ice and rock) are also important examples of the usefulness of gravimetric measurements.

The subject was initiated by Galileo Galilei discoveries at the start of the 17th century and the first practical determinations were obtained with pendulums in 1668.

The progresses in instrumentation have been slow during three centuries. Since the International Geophysical Year (1957) spectacular progresses have been achieved thanks to the developments of new spring and cryoscopic differential gravimeters, free fall absolute gravimeters which benefit of the achieved high precision in time and length measurements, the flexibility of the electronics and the automation possibilities.

Measurements are now made not only on solid ground but also at sea and with airborne instruments on board of helicopters and small aircrafts.

Accuracies of the order of $10^{-9}$ of the gravity $g$ are now currently achieved in ground measurements.

Such accuracies necessitate careful corrections for tidal variations, atmospheric pressure variations, polar motion, vertical gradient of gravity, relativistic corrections, ship and aircraft accelerations.

(see: Geology, Tectonics, Applications of Gravimetry, Gravity Anomalies).

1. Introduction

Measuring the gravity acceleration became a scientific objective when Galileo Galilei performed his celebrated experiments with a tilted plane, his discovery of the isochronism of the pendulum oscillations (1583) and set forth the law of free fall, the proportionality of the travelled distance to the square of the time (letter to Paolo Sarpi, 16 October 1604). The christening of the gravity unit as “gal” (cm s$^{-2}$) commemorated this fundamental step in the modern development of Geodesy and Geophysics, a unit still used despite the recommendation to use the SI units (International system of units,
1954) i.e. the m s\(^{-2}\). This unit corresponds to a very high acceleration while the present gravity measurements are currently performed with accuracy not less than 10\(^{-8}\) m s\(^{-2}\) (ten nanometer s\(^{-2}\) or one microgal). Abundant literature still uses milligals (mgal) and microgals (µgal) because millions of existing measurements were expressed with such units.

The first measurements, about one century after Galileo were performed according his discoveries by counting the number of oscillations of a pendulum made by a very thin aloes fiber to which extremity a small metallic mass was suspended (Picard at Paris 1668, Richer at Cayenne 1672, Maupertuis and Clairaut in Laponia, Bouguer in Ecuador 1736). The length of the pendulum was adjusted at Paris so that the period oscillation was one second of time. Thus the measurements were “differential” as the observation at the different sites consisted in changing the length of the pendulum in order to keep a period of oscillation of one second of time.

This procedure emphasises the two essential difficulties of the process: one has to measure a change of length L with accuracy (about 3 millimetres between Paris and Cayenne) and one has to determine the period of oscillation T with a corresponding accuracy.

The difference of g is 2.9 gal (0.029 m s\(^{-2}\)) between Paris and Cayenne, 4.3 gal (0.043 m s\(^{-2}\)) between Tornea (Laponia) and Quito while the precision of measurements was better than 0.1 gal in the eighteenth century.

Such measurements made at different latitudes were used to determine the flattening of the Earth, i.e. the shape of the Earth.

In 1798 Laplace had at his disposal only 15 measurements when he wrote his “Mécanique Céleste”.

The accurate determination of L and T ensures the calibration of the instrument, which is the relentless condition for correct determinations of g. It is by no way an easy operation even if the principle is quite simple.
1.1. Absolute and differential measurements

While absolute measurements imply measurements of the two physical elements, length and time, the differential measurements which are made with respect to a site where the absolute value of $g$ is already known are much easier as they imply only one control measurement, either length as in the 1672 and 1736 experiments, either time.

Absolute measurements rest upon very careful manipulation and take at least one full day or more. They are thus expensive and are restricted to a limited number of sites for example 10 sites on an area of about 30,000 km$^2$ (Belgium) while differential measurements between two or more sites are easy and quick notwithstanding of course manipulations to be made with the utmost care so that ten or more sites can be visited within one day. Differential values of $g$ are interpolations between several absolute values and set up detailed networks, which are needed for geological researches on the earth’s crust and lithosphere structure.

(see: History of Geodesy, Gravimetry, Gravitational Field of the Earth, Physical Quantities and Units).

2. Absolute instruments

Owing to the difficulty of measuring time with a sufficient accuracy the elementary free fall method (4.905 meters during the first second) could not be proposed until the years 1960 when crystal and atomic clocks became currently available.
One had therefore to use vertical pendulums oscillating in vacuum by counting the number of oscillations during a sufficiently long duration of time. The difficulties in the length determination were solved in a more or less satisfactory way by using reversible pendulums (Kater, 1818). Nevertheless a serious systematic error was recognised in the famous basic determinations made at Potsdam (1898-1905) by Kühnen and Furtwangler with such a high care that they had been adopted as the unique world reference system until 1970. The accuracy of this determination (981 274 ± 3 mgal) was questioned by all the more modern measurements made with a precision slightly better than one mgal (10^-5 m s^-2) (1936-1970) which pointed out the necessity to decrease the Potsdam value by not less than 14 mgal (14 x 10^-5 m s^-2) that is about five times the precision of the measurements.

Since 1960 a number of free fall determinations were made in vacuum by interferometric measurements in different laboratories having at their disposal length scales defined by radiations wave length and standard frequency signals.

These extremely difficult experiments were confined to well equipped laboratories in few countries. The advent of transportable absolute gravimeters opens a new field in Geodesy and Geodynamics.

The first transportable instrument has been the Italian one, IMGC (1978) where an idea of Sakuma, a vertical launching of a corner cube observed during its flight up and down was applied. This instrument has been unique but allowed to perform a discrete number of absolute stations in Europe and Asia.

More recently (1983) a limited number of six JILAG (Joint Institute for Laboratory Geophysics, USA) instruments by Faller et al. have been developed. The principle is to observe the falling of a corner cube reflector contained in a co-falling servo-controlled motor-driven drag-free chamber, which falls over 20 centimetres in 0.17 second inside a vacuum chamber. One arm of a Michelson interferometer provides a length standard while the corner cube attached to the falling mass forms the movable arm of the interferometer. The time of occurrence of interferometer fringes is measured by a rubidium atomic clock. Drops can be produced every two seconds. The precision is 3 to 5 µgals.

Since 1993, the AXIS - FG5 (figures 2, 3) new instruments series benefit from important improvements resulting from the JILAG experience.
Figure 2: Principle of the FG5 absolute gravimeter

The reference corner cube is integrated into the seismometer proof-mass of an active inertial reference called the Superspring, which isolates the measurements from seismic noise, which could cause errors of the order of a few mgals. Hundred consecutive drops are combined to one “run” and an instrumental control of levelling is carried out after each run. Several runs are performed on each station distributed over one or two days. The length standard is provided by iodine-stabilised laser that is a practical realisation of the SI definition of the meter. Thus no calibration is required. It has a long-term sta-
bility of 2 x 10^{-11} per year. Time is measured by a rubidium atomic clock. In this way absolute gravity determinations are tied to national standard laboratories.

With this instrument the accuracy of a station is estimated to be about ± 2 μgal (± 0.02 μm s^{-2}) and the precision ± 1 μgal (0.01 μm s^{-2}). The vertical gravity gradient must be well known as it reaches 60 μgal (0.6 μm s^{-2}) for 20 cm difference of height (see § 4.6).

The GABL free-fall gravimeter constructed by the Institute of Automation and Electrometry at Novosibirsk (Russia) operational since 1976 is very similar in its principle and all its components. It ensured an accuracy better than 4 μgal (0.04 μm s^{-2}) in measurements performed in Eastern Europe, equatorial Asia and Australia.

The increase of the absolute measurements accuracy since the 17th century is illustrated by the figure 3.


3. Differential instruments

Relative gravimeters do not measure gravity but the difference of gravity between two points. The first differential measurements, as those between Paris and Cayenne, were also performed with dynamic instruments i.e. with pendulum devices as the von Sterneck four brass pendulums (1887) oscillating in to orthogonal planes and in opposition inside a vacuum chamber. Their precision could probably reach ± 1 mgal (10 μm s^{-2}). Later on quartz pendulums were used with better results until the 1950s. Such pendulums systems allow a total coverage of the earth’s surface from the poles to the equator with a difference of \( g \) amounting to 5 gals (5 x 10^{-2} m s^{-2}). The first intercontinental connections were realised with such equipments but due to a long tedious procedure only one site could be visited within about a week. The invention of static instruments in 1930, the so-called “gravimeters”, which use the torque exerted by an elastic spring as antagonistic to the gravity torque provided real interpolation instruments at the disposal of the geodesists and geophysicists (figure 4). All instruments used fine of special alloy metallic springs (ex. North American). Only the Worden gravimeter suspension and spring was made of fused quartz, which gave some advantages, notably with respect to temperature control. Gravity meters that have metallic springs may exhibit small errors due to their orientation in the earth’s magnetic field. This is avoided with fused quartz suspensions. The gravity torque exerted on the beam is compensated by the opposite torque exerted by one main zero length spring while an additional very fine spring, called the measuring spring, is to be used to reset the beam at its zero position. This is obtained by modifying the tension of this fine spring, turning its “measuring screw” which is provided with a micrometric dial scale.

The reading of this dial being \( d_A \) at station A and \( d_B \) at station B, the difference of gravity between A and B is given by the simple equation

\[
g_B - g_A = k (d_B - d_A )
\]  

(1)

The coefficient \( k \) is the calibration factor of the gravimeter, which must evidently be
determined with the utmost accuracy. The instrument’s maker gives its value in function of the screw dial but this value must be checked regularly.

Moreover the invention by Lucien LaCoste of the “zero length” spring permitted a perfect astatization of the balance. Such astatized relative instruments require a measure of only one parameter, the lengthening of the spring. Initially this restricted the use of the gravimeter into a limited zone of gravity variation, which could be changed, by the use of an additional spring called the reset spring. Now they are improved and permit worldwide changes of $g$ to be measured without the discontinuity resulting from a change of zone with an additional spring.

3.1. Properties of a “zero length” spring – Astatisation.

For a usual spring the moment exerted on the beam is not symmetrical with respect to the zero position and the free period of the beam is different on the zero position sides. To eliminate such a dissymetry one should have

$$\frac{dF}{dH} = \frac{F}{H}$$

which means that the tension $F$ is proportional to $H$ so that $H = 0$ when the spring is left free from any tension: all the coils press on each other to try to fulfil the condition $H = 0$. When stress is applied, the first result is merely to separate the coils. The importance of this property was discovered by Lucien LaCoste (1932). It enables the property of astatization to be established. To construct a zero-length spring it is necessary to obtain an helix with a pitch $k = 0$ or $1/T = 0$ (torsion) which means that the torsion produced in the wire by reeling must be cancelled by an equal but opposite torsion given to it just before reeling each coil. This combines a high accuracy with a wide range of $g$ variations.

![Figure 4. Principle of astatization](image)
3.2. Minimum point of sensitivity to tilt.

Great care must be taken for the levelling of such instruments as any change in the levelling has the same effect as a change of gravity and is amplified in the same way by the astatization property. Therefore, the levelling has to be controlled carefully during the measurements. To minimise possible disturbance one has to settle the instrument at the minimum point of sensitivity to tilt (which obviously corresponds to $\alpha + \beta = \pi/2$ in figure 4). This position is easily determined by slightly turning the levelling screws back and forth and comparing successively cross-level and longitudinal level bubbles readings with the position on the recorder.

The registered variations have a parabolic aspect, which shows that the minimum sensitivity to tilt corresponds to the top of a parabola (figure 5). Recent instruments are equipped with electronic levels and automatic correction.

3.3. The LaCoste Romberg astatised gravimeter.

This instrument is presently the most widely used. It has no auxiliary spring, the reset of the beam is obtained by moving the upper suspension point of the spring. The LaCoste Romberg gravimeters are equipped with capacitive plates and a capacitive position indicator (CPI card) of which the filtered output can be measured with a digital voltmeter giving a precision of $\pm 1 \mu$gal (10 $\mu$m s$^{-2}$). An electronic low pass filter smoothes the microseismic noise. In 1984 Harrison and Sato have proposed an electronic feedback circuit, which can be connected to the CPI and holds the beam of the gravimeter in a fixed null position. This so-called “zero method”, which eliminates elastic hysteresis, consists in measuring the change of the voltage applied on the capacitive plates to reset the beam at its zero position, a system which is free from periodic errors of the measuring screw. It evidently allows a higher precision (+/- 5 $\mu$gal in field measurements) than the optical reading of the dial (+/- 10 $\mu$gal).

However the electrostatic force being proportional to the square of the applied voltage, the feedback force to voltage relation is non-linear. The feedback system is linearized by applying two bias volta-ages. This system stabilises sensitivity against small tilts.
Figure 5. Sensitivity to tilt

Figure 6. Principle of LaCoste Romberg gravimeter
Figure 7. Photography of a LaCoste Romberg gravimeter

Bibliography


**Biographical Sketch**

**Paul Jacques Léon Camille** MELCHIOR is born at Mont-sur-Marchienne (Belgium) on september 30, 1925.

knighted Baron by royal favor of His Majesty King Baudouin (july 1993)

he married Anne-Marie Bary (september 16, 1950); they have three sons.

Doctor in mathematical sciences (University of Brussels, 1951).


Career at the Catholic University of Louvain: Lecturer (1964), Professor (1972), Professor emeritus (1990). Teaching Geodesy, Gravimetry, Geodynamics.

Main International responsibilities and Awards:

- Director of the International Centre of Earth Tides of the International Association of Geodesy (1958-1995).
- Secretary General of the International Union of Geodesy and Geophysics (1973-1991). Foreign Member of the Academies of Finland, Luxemburg, the Netherlands, Spain and Rumania.
- Past President of
  - ICSU Committee for Data in Science and Technology (CODATA) of the International Council of Scientific Unions
  - The Council of the Federation of the Services of Analysis of astronomical and geophysical data (FAGS)