ELECTRICAL RESISTANCE STRAIN GAGES

José L.F. Freire
Pontifical Catholic University of Rio de Janeiro – PUC-Rio

Keywords: strain, measurement, circuits, adhesives, Wheatstone bridge, potentiometer circuit, sensors, transducers, load cells, applications.

Contents

1. Introduction
2. Sensitivity of a thin metallic conductor to strain
3. Strain gage conductor materials
4. Characteristics of modern strain gages
5. Other topics concerning modern strain gages
   5.1. Conductor and Strain Gage Sensitivities
   5.2. Temperature Response of Strain Gage Installations
   5.3. Gage Response in Non-uniform Strain Fields
   5.4. Gage Response in Dynamic Strain Fields
6. Strain gage circuits
   6.1. Potentiometer Circuit
   6.2. Constant Voltage Wheatstone Bridge Circuit
   6.3. Constant Current Wheatstone Bridge Circuit
   6.4. Constant Voltage Wheatstone Bridge Circuits – Specific Cases
   6.5. Shunt Calibration
   6.6. Quarter Bridge: Three-wire Connection of a Strain Gage
   6.7. Null Balance Bridges
7. Strain gage measurement systems
8. Examples of strain gage applications
9. Conclusions
Glossary
Bibliography
Biographical Sketch

Summary

Electrical resistance strain gages are sensors fabricated from thin foil or wire-type conductors that respond to variations in their length with variations in their electrical resistance. Strain gages are used to measure linear strains that occur at surface points of an object when it responds to an actuating load. Strain gages are either bonded with adhesives to the surfaces of structures or are welded on. Strain gages are used to determine strains in localized areas of structural components in the laboratory, in the field or as sensors in transducers such as resistive accelerometers or load cells. They are accurate “point” measuring elements and can be used in cases of static or dynamic loading. The measurement systems employ conventional and in-shelf circuitry elements, and have the important advantage of leading to electrical and treatable output. Circuitries commonly used are the potentiometer and the Wheatstone bridge circuit.
1. Introduction

Electrical resistance strain gages are sensors made of thin foil or wire-type conductors that respond to variations in length with variations in electrical resistance.

Strain gages are used to measure linear strains that occur at surface points of an object when it responds to some actuating load, as shown in Figure 1. This figure shows a surface point on the object before and after a load was applied. The strain gage is bonded to the surface with an adhesive. Deformation of the surface element forces the strain gage to change its length. For the special conductor gage materials, the variation in length of the parallel segments of the wire-type conductor will be directly proportional to the variation in electrical resistance of the conductor. Eq. (1) shows the relationship between the variation in resistance of gage \( \Delta R \) and strain \( \varepsilon \) to be determined, where \( K \) is the gage factor. The static or dynamic variation in resistance is measured and registered by an auxiliary circuitry. Notation of variables used in Eq. (1) is presented in Figure 1.

\[
\frac{R_f - R_i}{R_i} = K \cdot \frac{L_f - L_i}{L_i} \quad \therefore \quad \frac{\Delta R}{R} = K \cdot \varepsilon \tag{1}
\]

Figure 1 and Eq. (1) show that strain gages measure the relative displacement between two points located at the extremities of their grids with changes in resistance. These displacement changes can be so small that other mechanical devices would not be able to measure them with the necessary resolution and accuracy.

An example of the necessary resolution is given in Figure 2. It shows a prismatic object loaded with an axial force \( P \), as occurs, for example, in a tensile test used to measure the elastic and plastic mechanical properties of a structural material. Inside the elastic range, the uniaxial stress-strain relationships given in Eqs. (2) hold, where \( E \) and \( \mu \) are...
are the Young modulus and Poisson coefficient, respectively. For low carbon steel – for example, ASTM A 36 – the yield strength is 250MPa or greater. Using $E = 200\text{GPa}$, $\mu = 0.3$, load $P = 12.5 \text{kN}$ and assuming that the cross section area of the prismatic object is square with sides equal to $w = 10\text{mm}$, it can be calculated that the tensile stress is $\sigma = 125\text{MPa}$ and the linear strain measured by the strain gage is $\varepsilon = 612.5 \times 10^{-6}$. The relative displacement occurring between two points, A and B, located at the surface of the prismatic object and far apart at $L = 10\text{mm}$ will be $\Delta L = 0.0006125\text{mm}$. These numbers show that working stresses as high as half the yield strength of the material of a structural member will cause very small displacements in a relatively large gage length of 10mm. Any device or method employed to measure this displacement will need a resolution of 1μm or at least 10μm to indicate strain steps of 1 to $10\times10^{-6}$. Electrical resistance strain gages can be used with this resolution and with an accuracy varying from numbers as low as 1% to as high as 0.1%, depending on the set ups and circuitry used in each measurement.

Figure 2. Elastic stress-strain uniaxial mechanical behavior relations

\[
\begin{align*}
\epsilon_x &= \frac{\sigma_x}{E} \\
\epsilon_y &= -\mu \epsilon_x = -\mu \frac{\sigma_x}{E} = \epsilon_z
\end{align*}
\]

2. Sensitivity of a Thin Metallic Conductor to Strain

Figure 3 shows a thin wire-type conductor subjected to a normal positive or negative change in length.
Resistance $R$ depends on resistivity $\rho$, length $L$ and area $A$, as shown in Eq. (3). The relative change in resistance is given by Eq. (4) in terms of relative changes of variables $\rho$, $L$ and $A$.

\begin{equation}
R = \rho \frac{L}{A} \tag{3}
\end{equation}

\begin{equation}
\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \tag{4}
\end{equation}

Term $\frac{dL}{L}$ is the instantaneous strain.

\begin{equation}
d\varepsilon = \frac{dL}{L} \tag{5}
\end{equation}

If the conductor has a circular cross section with diameter $D$, the infinitesimal variation of the area is:

\[ A = \frac{\pi D^2}{4} \rightarrow \frac{dA}{A} = 2 \cdot \frac{dD}{D} \]

The relative variation of the area can be written in terms of the infinitesimal strain and the Poisson coefficient:

\[ \frac{dD}{D} = d\varepsilon_x = -\mu \cdot d\varepsilon_y \rightarrow \frac{dA}{A} = -2 \cdot \mu \cdot d\varepsilon \]

The relative change in resistivity is assumed to be proportional to the relative change in volume, the proportionality coefficient designated as $c$, the Bridgeman constant.
\[
\frac{d \rho}{\rho} = c \frac{dV}{V}
\]  
(6)

The change in volume of the conductor is written as:

\[ V = L \cdot A \]
\[ \frac{dV}{V} = \frac{dL}{L} + \frac{dA}{A} = \varepsilon - 2\mu d\varepsilon \]

Therefore, it is possible to determine the dependence of the relative change in resistance \( R \) in terms of \( \varepsilon \) as:

\[
\frac{dR}{R} = [c(1 - 2\mu) + 2\mu + 1] d\varepsilon
\]
\[
\frac{dR}{R} = K d\varepsilon
\]

(7)

Integration of (7) gives (8) if \( K \) is considered constant:

\[
\ln \frac{R_f}{R_i} = K \ln \frac{L_f}{L_i}
\]

(8)

Eq. (9) will be valid if the total strain value is low and if coefficients \( c \) and \( \mu \) remain constant.

\[
\varepsilon = \int d\varepsilon = \int \frac{dL}{L} = \ln \frac{L_f}{L_i} \approx \frac{L_f - L_i}{L_i} \approx \frac{\Delta L}{L_i}
\]
\[
\frac{\Delta R}{R} = K \varepsilon
\]

(9)

In the case of \( c \) being equal to unity, which happens for a conductor made of the alloy Constantan (55%Cu + 45%Ni), proportionality constant \( K \) will be equal to 2 and will not depend on the change in value of \( \mu \) in the elastic-plastic transition.

### 3. Strain Gage Conductor Materials

Electrical conductor materials for strain gages must possess the capability of being formed as thin wires or very thin metal foils, and must have constant \( K \) values over a wide range of elastic (and plastic if possible) strains and temperature. The ones that are used the most are listed in Table 1. Constantan and Karma alloys are the ones used the
most in general applications within temperature ranges of \([-30^\circ C, 200^\circ C]\) and \([-50^\circ C, 250^\circ C]\), respectively.

<table>
<thead>
<tr>
<th>Material</th>
<th>Alloys</th>
<th>(K)</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advance or</td>
<td>45% Ni – 55% Cu</td>
<td>2.1</td>
<td>General use up to 8% strain. Widely used</td>
</tr>
<tr>
<td>Constantan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Karma</td>
<td>74% Ni - 20% Cr – 3% Al – 3% Fe</td>
<td>2.0</td>
<td>Wide range for temperature compensation, high fatigue strength</td>
</tr>
<tr>
<td>Isoelastic</td>
<td>36% Ni – 8% Cr – 0.5%Mo – 55.5% Fe</td>
<td>3.6</td>
<td>General use. Temperature sensitive.</td>
</tr>
<tr>
<td>Nichrome V</td>
<td>80% Ni – 20% Cr</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>Platinum-Tungsten</td>
<td>92% Pt – 8% W</td>
<td>4.0</td>
<td>High temperature applications (may be over 250°C)</td>
</tr>
<tr>
<td>Armour D</td>
<td>70% Fe – 20% Cr – 10%Al</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Strain gage conductor materials

4. Characteristics of Modern Strain Gages

The first strain gages were made from thin wire conductors that were bent to form several rows of parallel sensing legs to produce the sensing grids. These grids were cemented to thin special china paper backings to allow for electrical isolation and easy handling and mounting.

![Figure 4. Sketch of a strain gage layout](image)

Modern strain gages are made from thin conductor foils (e.g. Constantan foils) backed by flexible and non-conducting thin foils of polyimide, phenol or epoxy resins. The
strain sensitive grids are produced by a photo-resistive etching process. The backings give some stiffness to the foil grids to make their handling process relatively easy and to electrically isolate the grids from the prototype metallic materials.

The adhesives used to bond the strain gages to the prototype play an important role in the measurement system. They must be easy to apply, be compatible with both the prototype and backing materials, have a linear response to strain along the entire range of the measurement, and be time independent.

Figure 4 and Table 2 summarize most of the technological firsthand information needed to give sound and important information to a beginner. Circuitries for electrical measurements of resistance variation $\Delta R$ are commented on in Section 3 of this topic.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>History</td>
<td>Lord Kelvin worked on the basics in 1856. First wire grid gages were presented in mid-1930, by Ruge and Simmons independently of each other. Foil gages were first presented by Sanders and Roe after 1952.</td>
</tr>
<tr>
<td>Grid materials</td>
<td>Advance or Constantan, Karma alloy, Isoeelastic, Nichrome V, Armour D and Pt-W (see Table 1)</td>
</tr>
<tr>
<td>Backing or carrier materials</td>
<td>Thin paper, polyimide film, epoxy film, glass fiber epoxy-phenolic reinforced polymer film</td>
</tr>
<tr>
<td>Adhesive Materials</td>
<td>Epoxy (more stable, used in transducer applications), cyanoacrylate (very easy to use, general application up to 65°C), polyester, ceramic (high temperature applications)</td>
</tr>
<tr>
<td>Strain gage configurations</td>
<td>Uniaxial Biaxial rosette Triaxial stacked rosette Triaxial 45 degree rosette</td>
</tr>
<tr>
<td>Special types of strain gages</td>
<td>Weldable Linear array High temperature (weldable) Residual stress rosette</td>
</tr>
<tr>
<td>Stability</td>
<td>Combined effect of time of exposure to strain, temperature and humidity on the gage installation and measurement system can affect stability. Zero drift can be as low as a few micro-strains to several hundred, depending on foil and carrier material, adhesive, installation cleanliness, and stability of instrumentation.</td>
</tr>
<tr>
<td>Gage Factor</td>
<td>Constantan gages have gage calibration factor $K$ around 2.0.</td>
</tr>
<tr>
<td>Transverse sensitivity</td>
<td>$K_t = St/Sa = 0$ to $5%$, usually $&lt; 0.5%$, where $St$ and $Sa$ are transverse and axial sensitivities, respectively.</td>
</tr>
<tr>
<td>Temperature compensation</td>
<td>Available for prototype materials with expansion coefficients varying from zero to $26\times10^{-6}/^\circ\text{C}$</td>
</tr>
<tr>
<td>Grid dimensions</td>
<td>Grid dimensions vary in such a way that the most suitable geometry for a specific application will be found most of the time. There are different W/L ratios. Minimum and maximum L range from 1.0mm (stress concentration) to 150mm (concrete) applications.</td>
</tr>
</tbody>
</table>
Initial resistance

Common: 120 and 350Ω
Possible: 500, 1000, 2000, 5000Ω

Accuracy

Gage resistance = ±0.3%, Gage factor= ±1%. Transducers that employ strain gages have a much better accuracy due to calibration procedures and temperature corrections

Resolution

1με. Can be lower or higher, depending on the circuitry and measurement range and devices

Range

±1%, special gages with annealed foils may work up to 20%

Circuitry

Potentiometer and Wheatstone circuits. Four wire resistance direct reading circuit

Acquisition rate and dynamic range

Static to dynamic range. Dynamic measurements may require several MHz acquisition rate. Circuitry, amplifiers and reading-recording devices limit strain gage installation capabilities to measuring 100 ns pulses.

Influence of other parameters

Temperature
-40°C<θ<60°C general use- polyimide carriers and cyanoacrylate adhesive.
-195°C<θ<230°C Karma alloy fully encapsulated and epoxy adhesive.
Special high temperature gages - PtW alloy - weldable or bonded with ceramic adhesive (~600°C)

Humidity
Must be protected to avoid degradation of backing and adhesive and undesirable loss of isolation

Radiation
Large apparent strains are induced with time (zero drift)

Fatigue and strain cycling
Number of cycles to cause fatigue varies from ±1500x10^6 to ±2200x10^9, depending on the gage type. Fatigue strength depends on the quality of the installation

Magnetic fields
High field gradients influence the response of the gage and associated cabling

Hydrostatic pressure
Minimal influence (20με) up to about 20MPa pressure

Application

Widespread: experimental stress analysis, transducers (accelerometers with DC response and load cells), application to integrity monitoring, control of mixtures in chemical processes, weight control and measurement

Table 2. Characteristics of modern strain gages

The process of bonding a gage is described in Table 3. Of course, there are other processes and adhesives for more specific applications such as strain gage bonding for transducers, high temperature applications, or large strain measurements.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement system design</td>
<td>Selection of points where to measure strains, characteristics of measurement: static or dynamic, low or high temperature, low or high strain gradient, general stress analysis or transducer application, etc. Selection of gage, selection of adhesive and protection.</td>
</tr>
<tr>
<td>Locate point at surface of prototype.</td>
<td></td>
</tr>
<tr>
<td>Degreasing entire bonding and adjacent area. Check compatibility of prototype exposed surface with degreaser to avoid undesirable chemical attack.</td>
<td></td>
</tr>
<tr>
<td>Thorough cleaning of surface, taking off all paint and oxidation. Expose surface to bare metal.</td>
<td></td>
</tr>
<tr>
<td>Use hand sand paper up to number 220.</td>
<td></td>
</tr>
<tr>
<td>Clean bonding area completely with clean cotton and Freon, alcohol or acetone.</td>
<td></td>
</tr>
<tr>
<td>With hard lead pencil, mark position where to set gage.</td>
<td></td>
</tr>
<tr>
<td>Place gage with adhesive tape and check for position.</td>
<td></td>
</tr>
<tr>
<td>Pull back adhesive tape just enough to expose back side of gage that will receive adhesive.</td>
<td></td>
</tr>
<tr>
<td>Put small drop of adhesive on surface of gage and reposition adhesive tape to previous well-positioned place. Press gage against prototype surface using tip of finger on adhesive tape for about two minutes. A small silicone rubber pad may be used to spread pressure over gage and protect finger skin from excess adhesive.</td>
<td></td>
</tr>
</tbody>
</table>
Peel off adhesive tape. Clean surrounding gage area of excess adhesive.
Cement solder auxiliary tabs with cyanoacrylate.
Solder lead wires. Use 20 to 30W soldering iron.

<table>
<thead>
<tr>
<th>Check electrical contacts</th>
<th>Measure resistance of gage and circuit using ohmmeter.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measure resistance between gage and surface of metallic prototype. Resistance should be above 500 MΩ.</td>
</tr>
</tbody>
</table>

| Provide protection to installation | Using one or more coats, protect gage, electrical contacts and portion of lead wire that reaches installation. These should be selected to protect against electrical contact with environment, humidity and touch. |

Table 3. Steps for bonding, cabling installation, quality control and protection of strain gage installations

The high resistance of the gages is achieved with the use of relatively long and thin etched conductors. To make them relatively small in size, the gages are constructed with several parallel thin conductors that form a grid of dimensions $L$ and $W$ as showed in Figure 5. The initial resistance of normally marketed strain gages is 120 or 350Ω. Strain gages may also be found with resistances as high as 1,000 or 2,000Ω to be applied in special measurements; for example, when direct measurement of $\Delta R$ is required.

![Figure 5. Sketch of strain gage showing grid dimensions, axial (measurement direction) and transverse direction.](image)

The terminal tabs used for soldering the cables or for connecting the parallel conductors that form the grid are made much wider. This procedure helps to decrease their resistance and, therefore, their variation under strain. As a consequence, almost 100% of the resistance variation is caused by the straining of the thin parallel conductors, and the spurious response of the gages to transverse strains is usually very low.

Manufacturers of commercial strain gages state the strain gage calibration factor $K$ and the transverse sensitivity $K_t$ (see Section 5.1). Approximate values of $K$ are given in Table 1. The calibration of gage factor $K$ follows standardized procedures. The value of $K$ is determined with the help of an instrumented cantilever steel beam with a 0.285 Poisson coefficient.
Figure 6. Constantan foil strain gage with polyimide backing. Original size is $L = 75$ mm to be applied in concrete structures. Note the large, thick tabs for soldering the connecting cables and the $180^\circ$ turning corner connections of the wire-type parallel foil conductors.

Bibliography


Biographical Sketch

José L.F. Freire: B.S. (1972) and M.Sc. (1975) degrees in mechanical engineering from the Catholic University of Rio de Janeiro (PUC-Rio); Ph.D. (1979) in Engineering Mechanics from Iowa State University of Science and Technology (ISU); associate professor of Mechanical Engineering at PUC-Rio and chairman of the Structural Integrity Laboratory; member of the Society for Experimental Mechanics and is its past president; major areas of research: Experimental Stress Analysis, Pipeline Engineering and Structural Integrity of Equipment and Structures.