SYSTEM RELIABILITY ANALYSIS

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Contents

Summary

A survey of main approaches to the reliability evaluation is presented. Some simple analytical models for element reliability are discussed. Series and parallel configurations of compound systems as well as various types of redundancy are discussed and illustrated by examples. An introduction to the fault tree and event tree techniques is given. Some two-side estimates for the reliability measures of compound systems are presented.

1. Reliability Measures for Elements

Mathematical models of reliability theory may be divided into two groups. The first group consists of structural models based on the logical schemes. They describe, in the terms of mathematical logic (combinatorial logic), the interaction of elements and sub-systems entering the system in consideration. Only the statistical information on the reliability of elements and sub-systems is used here, without any information on physical properties, mechanical interactions, etc. The structural models are presented by block diagrams and graphs such as trees of events. The information on the reliability of elements is assumed known in the form of assessed reliability measures (see *Reliability Evaluation, Non-destructive Testing and Fault Correction*).

The second group of mathematical models takes into account mechanical, physical, chemical, etc. processes affecting the change of the properties of a system and its elements. For example, the models of mechanics of structures are widely used in the reliability analysis in civil and mechanical engineering. The components of machines and structures are in a complex interaction of dynamical character. The behavior of these items essentially depends on their interaction with environment, on the type and intensity of their functioning. To predict this behavior, the processes of deformation, damage accumulation, fracture, and wear under unsteady loading and temperature have to be considered. The principal approach to predicting the reliability measures for mechanical systems is based on mathematical models and the statistical information on material properties, component parameters, loads, and actions.

Many engineering systems, especially in communication and information industry, consist of a large number of mass produced elements. These systems are operating in conditions that are relatively stationary and homogeneous. The reliability tests for such elements are comparatively simple, and the operating conditions can be easily simulated in a laboratory. Statistical treatment of test results allows finding the appropriate analytical relationships and the estimates of reliability measures. If an element is non-restorable, the reliability function, i.e. the probability of the non-failure operation, or the failure rate is used as the reliability measures. The simplest models suitable for elements are considered hereafter.

The so-called *exponential law of failures* is most frequently used. The reliability function for this model

$$R(t) = \exp(-\lambda t), \quad (1)$$

corresponds to the constant failure rate λ and the mean operating time to failure

$$T = 1/\lambda.$$

Following the model based on the *Weibull distribution,* the reliability function is defined as follows:

$$R(t) = \exp\left[-\left(\frac{t}{t_c}\right)^{\alpha}\right]. \quad (2)$$

Here t_c and α are positive parameters. Equation (2) describes a rather wide family of distributions including the exponential law (1) at $\alpha = 1$. At $\alpha > 1$ this equation suits for the aging elements with the failure rate increasing in time. The mean operating time to failure in this case is

$$T = t_c \Gamma\left(1 + \frac{1}{\alpha}\right), \quad (3)$$

and the variance of the time to failure is

$$w_T^2 = \left[\frac{\Gamma(1+2/\alpha)}{\Gamma^2(1+1/\alpha)} - 1 \right]. \tag{4}$$

Here $\Gamma(\cdot)$ is the gamma-function. An asymptotic approximation for the right-hand side of equation (4) gives the coefficient of variation:

$$w_T \approx \frac{\pi}{\alpha\sqrt{6}} + O\left(\frac{1}{\alpha^2}\right). \tag{5}$$

Equation (5) shows that the distribution of times to failure becomes more compact when the power exponent α grows. Hence, this exponent characterizes the stability of properties within a large sample of similar elements. This stability depends on the stability of manufacturing, homogeneity of raw materials, etc. It characterizes the level of quality in general. The equation

$$R(t) = R(0)\exp\left[-\int_0^t \lambda(\tau)d\tau \right] \tag{6}$$

is a generalization of equations (1) and (2) with the time-dependent failure intensity $\lambda(t)$. Equation (6) takes into account that the probability of failure at t = 0 may be not equal to zero.

In many applications the failure intensity varies in time non-monotonically (Figure 1). At the initial stage 0 < t < T_b the failure rate is relatively high that corresponds to early failures. After the burn-in stage, the weakest elements are removed from the sample. Then a sufficiently long stage takes place during which the failure rate is approximately constant (line 1). From some moment t = T_a the rate begins to grow due to aging, damage accumulation, wear, etc.

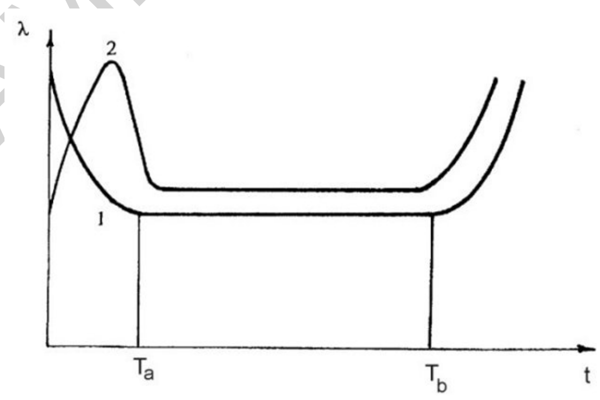

Figure1. Failure rate versus operating time: (1) including the burn-in stage (2) In the presence of rapidly aging elements.

Sometimes the increase of the failure rate is observed on the initial stage, as it is shown by line 2 in Figure 1. For such cases the reliability function may be presented in the form

$$R(t) = \sum_k p_k R_k(t), \quad \sum_k p_k = 1. \tag{7}$$

This model describes the mixture of elements with reliability functions $R_k(t)$ and their relative portions p_k. The situation shown by line 2 takes place if the mixture contains a portion of weak elements with the low time to failure.

Gamma-distribution is also used in the reliability analysis. The corresponding probability density of the time to failure is

$$f(t) = \frac{t^{\alpha-1}}{t_c^{\alpha}\Gamma(\alpha)} \exp\left(-\frac{t}{t_c}\right), \tag{8}$$

with $\alpha > 1$ for aging elements. Equation (8) contains the characteristic time t_c. The probability $R(t)$ of non-failure operation is equal to the integral of $f(t)$ on the segment [0, t]. When $\alpha = 1$, equation (8) results into the exponential law (1).

2. Statistical Estimation of Reliability Measures

To model the reliability of repairable elements (repairable items in the general case), the theory of random processes is used. As an example, consider the model of the homogeneous Poisson stream of events with the parameter λ equal to the mean number of failures in the time unit. The probability that k failures occur on the time segment [0, t] is given by the *Poisson law*:

$$Q(t) = \frac{(\lambda t)^k}{k!} \exp(-\lambda t), \quad r = 0, 1, 2, \ldots. \tag{9}$$

This simple model may be interpreted according to the following scheme. An item is used or subjected to testing until the failure of a certain element. Then this element is replaced with a new one from the same set of elements. Usage or testing proceeds until the next failure and the next replacement. Let the time for the replacements to be small compared with the duration of the operating times between neighboring failures. Then the life of the system, from the viewpoint of reliability, is characterized by the sequence t_1, t_2, … of moments corresponding to failures. The operating time between failures is a random variable, and the sequence of failures form a *stream of random events*.

It is to be noted that the reliability measures discussed above relate to probabilistic models, i.e. they have to be interpreted as the predicted measures. Treating experimental observation data, we deal with corresponding statistical estimates of the probabilistic measures. For example, the estimate for the probability of non-failure for the time period t is given as

$$\breve{R}(t) = 1 - \frac{r(t)}{N}. \tag{10}$$

Here N is the number of items operating at initial time moment $t = 0$, and $r(t)$ is the number of items failed in the segment $[0, t]$. The estimate of the mean operating time to failure is

$$\breve{T} = \frac{t_1 + ... + t_N}{N}, \tag{11}$$

where t_k is the time to failure for the k-th item. Similarly, the estimate of the failure rate is

$$\breve{\lambda}(t) = \frac{r(t + \Delta t/2) - r(t - \Delta t/2)}{[N - r(t)]\Delta t}. \tag{12}$$

To receive reliable statistical estimates following equations (10) - (12), the sampling volume N is to be sufficiently large. The adequate statistical analysis includes the assessment of confidence intervals, checking good fitness of distributions, etc. These problems have to be solved using the techniques of applied mathematical statistics (see *Bibliography*).

3. Series configuration systems

To present the logical interaction of elements composing a system, various diagrams and graphs are used. Among them are *block diagrams* where the elements are presented as blocks connected in a system according to their interaction with respect to reliability. It is assumed that in all block diagrams discussed later the failures of elements are independent.

Let the elements interact in such a way that the failure of any element results into the failure of the system as a whole. In this case one says of *series* or *sequential configuration* (Figure 2a). The up state of the system is a random event that is equivalent to the intersection of independent events, which are the up states of each separate element. The probability of non-failure operation of the system R is equal to the product of corresponding probabilities R_1, …, R_m for elements. Thus,

$$R = \prod_{k=1}^{m} R_k, \tag{13}$$

where the time dependence symbol is omitted. At $R_1 = … = R_m = R_0$ equation (13) gives

$$R = R_0^m. \tag{14}$$

As an example, let consider the case when all the elements are identical (in the probabilistic sense) and follow the exponential law. Substituting equation

$$R(t) = \exp(-\lambda t) \tag{15}$$

gives the probability of non-failure operation of the system in the segment [0, t]:

$$R(t) = \exp(-m\lambda t)\,. \tag{16}$$

The mean time to failure is determined by equation

$$T = \int_0^\infty R(t)\,dt\,. \tag{17}$$

Substituting equation (15) results in

$$T = t_c / m\,. \tag{18}$$

Here the notation t_c is used for the mean time to failure for elements.

Equations (13) – (18) illustrate the well-known fact that a system formed of the series of elements is more vulnerable with respect to failures than each element of the system. The decrease of reliability is the higher the more is the number of elements in the series. If the number m is large, it is practically impossible to form a system of high reliability. For example, at m = 10^3, R_0 = 0.99, equation (2) gives the reliability measure R = 0.9999. The mean time to failure of the system is 10^3 times less than that of each element.

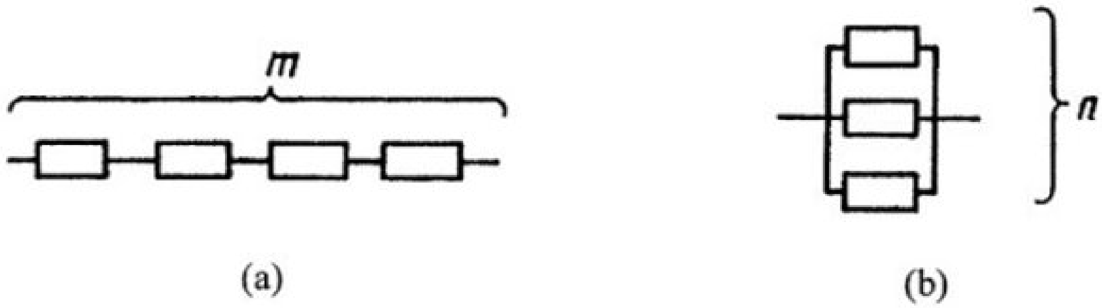

Figure 2. Block diagrams for series (a) and parallel (b) configurations.

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Bibliography

Ang H.S., Tang W.H. (1984). *Probability Concepts in Engineering Planning and Design*. New York: John Wiley and Sons. [This textbook contains a detailed discussion of the fault tree and event tree techniques].

Barlow R. and Proschan F. (1975). *Statistical Theory of Reliability and Life Testing Probability Models*. New York: Holt, Rinehard and Winston. [This is a comprehensive textbook with the detailed presentation of the system reliability theory].

Dhillan B. and Singh Ch. (1981). *Engineering Reliability*. *New Techniques and Applications*. New York: John Wiley. [This is a reference book on different techniques based on the system reliability theory with numerous practical applications].

Kapur P. and Lamberson L. (1977). *Reliability in Engineering Design*. New York: John Wiley. [This textbook contains the presentation both of system and physical versions of the reliability theory].

Probabilistic Safety Assessment (1993). New York: American Nuclear Society. [This is the volume of the conference proceedings dedicated to safety and reliability of nuclear power plants. There are a number of practical examples in this volume as well as the references to commercial programs for the system reliability analysis].

Ushakov I.A., ed. (1985). *Reliability of Engineering Systems. Handbook*. Moscow: Radio and Svyaz' (in Russian). [This is a handbook on reliability theory with the impact in the system reliability analysis].

Biographical Sketch

Bolotin Vladimir V., was born on March 29, 1926, Tambov, Russia. He graduated from the Moscow Institute of Railway Engineers as the Civil Engineer (Bridges and Tunnels) in 1948. He received from the same Institute the Degree of Candidate of Sciences in 1950 and the Degree of Doctor of Sciences in 1952.

Professional employment: 1950 – 1951, Assistant, 1951 – 1953, Docent of the Moscow Institute of Railway Engineers. 1953 – present, Professor of the Moscow Power Engineering Institute/Technical University; 1958 – 1997, Head of the Department of Dynamics at this University. 1980 – present, Head of the Laboratory of Reliability at the Mechanical Engineering Research Institute of the Russian Academy of Sciences; 1997 – present, Chief Scientist at this Institute.

He has authored/coauthored about 300 research papers and 20 monographs, textbooks and handbooks. Among them are : Dynamic Stability of Elastic Systems (in Russian 1956, translated into English, German, Japanese and Chinese); Nonconservative Problems of the Theory of Elastic Stability (in Russian 1961, translated into English and Japanese). Statistical Methods in Structural Mechanics (in Russian 1961, 1965, translated into English, German, Japanese, Polish, Czech, Hungarian and Portuguese). The latest books are published in English: Stability Problems in Fracture Mechanics, John Wiley, 1996; Mechanics of Fatigue, CRC Press, 1999.

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