CONTROL POINT SURVEYING AND TOPOGRAPHIC MAPPING

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Summary

Surveying is based on geodesy. With precise survey instrument, the Earth must be treated as an ellipsoid instead of a sphere. The ellipsoid that is chosen for geodetic survey is called reference ellipsoid. The shape of the Earth, so called geoid, is expressed as an equipotential surface of gravity. Usually, position on the ground is measured in longitude, latitude on the reference ellipsoid and height on the geoid. For convenience to measure the three dimensional values, a lot of monuments whose longitude, latitude or height is known are settled as reference point all over the countries. Control point surveying is a surveying to establish the reference point network, and is also to

determine the coordinates of a new point based on the reference point network. Some precise measuring instruments such as total station and level are used for the surveying. In addition, rapid development of the space geodesy enables us to establish an international reference frame covering the whole globe. Using the frame, Geographical Positioning System (GPS) survey is rapidly being introduced into common control point surveying. The trend also requires us to reconstruct the individual national reference frame to harmonize with the international reference frame. Based on these control points, precise mapping of the topography is enabled. The map, so-called topographic map, is an exact mathematical projection of the undulations of land and manmade features at reduced scales on to a paper. Most of the mapping is done with aerial photographs. The field of photogrammetry is one of the survey technologies to make a topographic map from the aerial photographs. The control point surveying or table surveying are also quite effective in making a topographic map within a local survey. These days, most of the surveying instruments include a small computer inside, and data output from the computer can easily construct the geographic information system instead of paper topographic map.

1. Introduction

The technique of control point surveying is based on geodesy. Geodesy has a long history extending back to a few thousand years. The objective of geodesy is to measure the Earth with instruments which can precisely measure angles, distances and heights etc. The history of geodesy is deeply linked with the history of physics and mathematics. The shape of the Earth is ellipsoid and is evidenced by universal gravitation theory. Only precise geodetic survey can detect the small difference between the major and minor axes of the Earth. The progress of mathematics has also been required to treat an elliptical equation and least squares method.

Almost all of the countries establish their geodetic networks with geodetic control points. The networks give us a precise reference for positioning. Along the traditional manner of geodetic control, they settled an origin of the datum. From the datum origin, control point survey has started to cover the territory of the country with a lot of control points. Usually the accuracy of the control points reaches the order of centimeters as reference. The reference of the position is one of the most basic infrastructures for human activities such as public works, mapping, construction, navigation, cadastre etc.

Using the geodetic control points as anchor points, precise topographic maps can be produced. The topographic maps are drawings that show the elevations and undulations of land, the locations of buildings, roads, and railroads, etc., the utilization of land, geographical names and other information for characteristic features of local areas, at reduced scales of about 1:50,000 or lager. To make the topographic maps, photogrammetric method is used. High level mathematical knowledge is required to treat map projection and adjustment of the error.

As any other science, geodesy and mapping technology are making a marked advance. Space geodetic technology has been able to measure distances of 10,000 km in order of centimeters. The network of the space geodesy has covered the Earth. Geodetic control is no longer the task of one country. Strong international cooperation is required in the field. Satellite imaging and GPS measuring are rapidly introduced in topographic mapping.

2. Geometric Background

The first step of the geodetic control work is usually to choose a reference ellipsoid because the first approximation to the Earth's surface is an ellipsoid. Precise measurements with geodetic instruments can detect small deviations of the Earth's shape from sphere and ellipsoid as Earth's reference, while it seems perfect sphere from thge photo taken from the moon. The undulations such as mountains and hills are measured from the reference ellipsoid.

2.1. Ellipsoid

An ellipsoid is a solid shape which is produced by rotating an ellipse around its minor axis. An ellipse is defined as a curved line which has two points geometrically termed as foci and in which the sum of distances from the two focal points to any point on the ellipse is constant as illustrated in Figure.1.

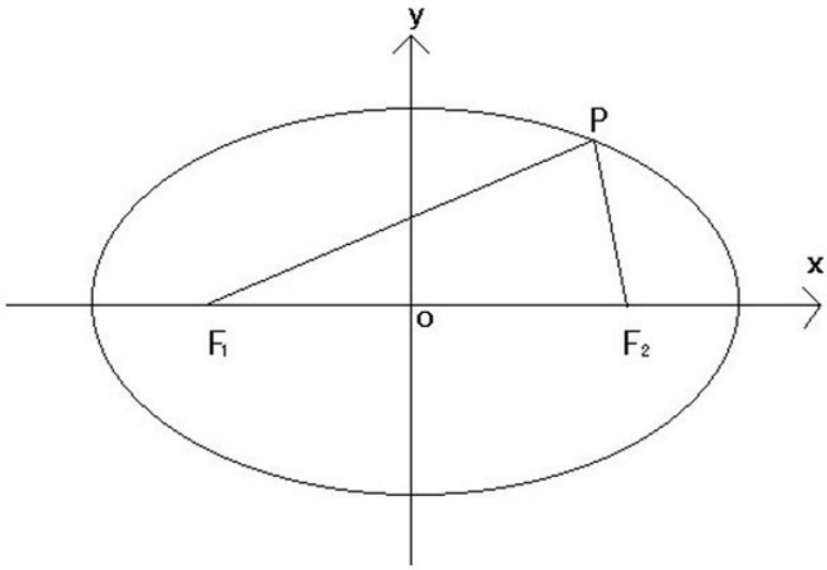

Figure.1 An ellipse

If the coordinates of the foci are $F_1(\text{-}X,0)$ and $F_2(\text{-}X,0)$, and the sum of the distances is $2a$, and the point on the ellipse is $P(x, y)$, the ellipse is described by the equation:

$$((x + X)^2 + y^2)^{1/2} + ((x - X)^2 + y^2)^{1/2} = 2a$$

It is rearranged as:

$$x^2/a^2 + y^2/b^2 = 1$$

where a and b respectively are termed semi-major axis and semi-minor axis of the ellipse where $b = (a^2 - X^2)^{1/2}$. To express the character of an ellipse, flattening f as $(a - b)/a$ and eccentricity e as $e^2 = (a^2 - b^2)/a^2$ are used in geometric geodesy.

The geodetic latitude φ at a random point P on the ellipse is defined as an angle to be formed between a normal at the point P and the major axis. By differentiating Eq.(1),

$$dy/dx \;=\; -(b^2/a^2)\;(x/y)$$

According to the definition of the geodetic latitude,

$$dy/dx \;=\; \tan(\varphi + 90)$$

Hence,

$$b^2 x \sin\varphi \;=\; a^2 y \cos\varphi$$

By substituting this into the formula of an ellipse,

$$x \;=\; a\cos\varphi/(1 - e^2 \sin 2\varphi)^{1/2}$$

$$y \;=\; a(1 - e^2)\sin\varphi$$

An ellipsoid is a figure to be produced by rotating an ellipse. The expression of an ellipsoid using rectangular coordinates is:

$$x^2/a^2 \;+\; y^2/a^2 \;+\; z/b^2 \;=1$$

The definition of the geodetic latitude of point P in the surface of an ellipsoid is represented by angle between the normal line of the ellipse at point P and the equatorial plane. The definition of the geodetic longitude is represented by angle from the zero meridian to a meridian passing point P, as measured counterclockwise.

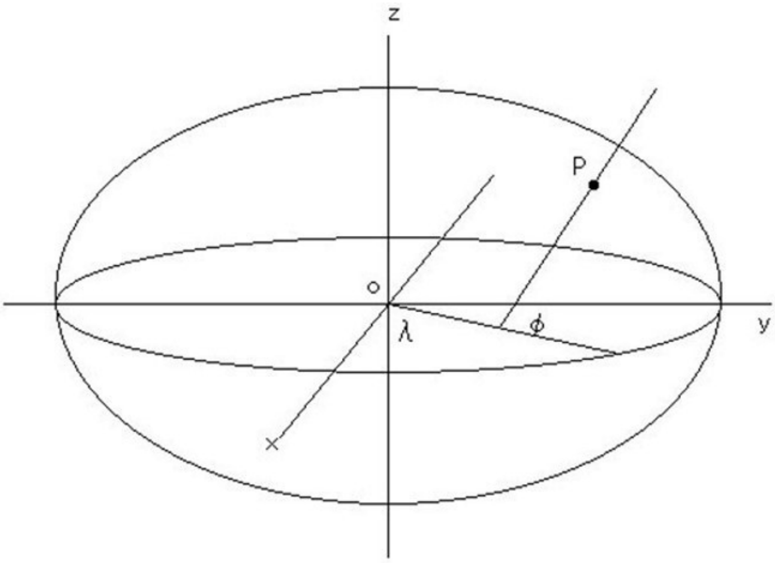

Figure.2 Ellipsoid

2.2. Transformation between Geodetic Systems

Usually geodetic systems of all countries are not same. To compare coordinates between different geodetic systems, relationship between the coordinate systems must be known. Relationship between two geodetic systems is expressed by using components of revolution, $(\varepsilon_x, \varepsilon_y, \varepsilon_z)$ and origin shifting components $(\Delta X, \Delta Y, \Delta Z)$ and taking difference of scale factors (*S*). The relationship among the rectangular coordinates is respected as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^{\text{System1}} = (1+S) R_3 \varepsilon_z R_2 \varepsilon_y R_1 \varepsilon_x \quad y \begin{bmatrix} x \\ z \end{bmatrix}^{\text{System 2}} + \Delta Y \begin{bmatrix} \Delta x \\ \Delta z \end{bmatrix}$$

where (R_1, R_2, R_3) represent matrices of revolution for axes $x, y,$ and z:

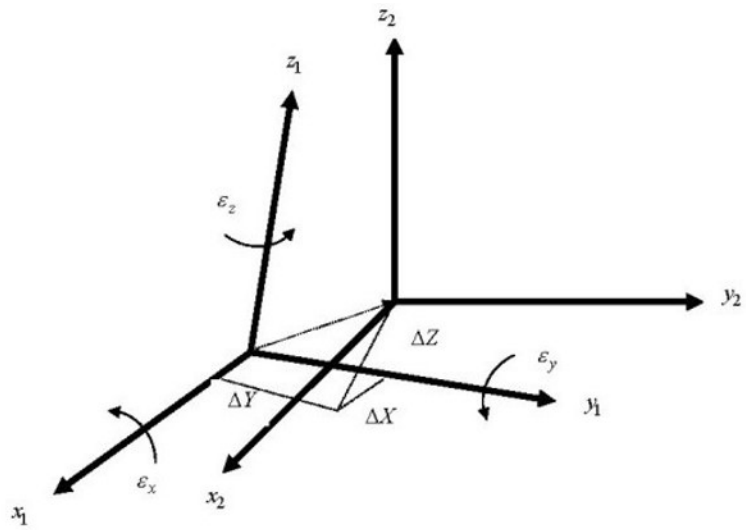

Figure 3. Transformation from Geodetic System 1 to Geodetic System 2

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Biographical Sketch

Shoichi Oki holds a B.Sc in Science from Tohoku University and an M.Sc in Science from Tohoku University. His master's research was analysis of plasma waves in space by Very Long Baseline Interferometry(VLBI) survey and computer simulation.

He was hired after graduation from the university by Geographical Survey Institute (GSI). He conducted geodetic survey, topographic mapping and digital mapping. During 1991 to 1993 he was transferred temporarily to the National Land Agency as a chief of National Land Information Office. During 1996 to 1997 he stayed in Germany as a guest scientist of Technical University Munich to research digital photogrammetry. During 1997 to 1998 he worked to establish International VLBI Service as the VLBI group leader of GSI. He participated in basic survey planning and system reforming of survey as a member of strategic planning team of GSI. In 2000.04 he was prompted to the Deputy Director for cadastral survey of the National Land Agency.