NETWORK ANALYSIS

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Contents

1. Introduction
2. Passive Components
2.1. Resistors
2.2. Capacitors
2.3. Inductors
3. Voltage and Current Sources
4. Circuits
4.1. Single-phase Circuits
4.2. Three-phase Circuits
5. Ohm's Law, Kirchhoff's Voltage Law and Kirchhoff's Current Law
5.1. Ohm's Law
5.2. Kirchhoff's Voltage Law
5.3. Kirchhoff's Current Law
6. Node and Mesh Analysis
6.1. Node Analysis
6.2. Mesh Analysis
7. Network Theorems
7.1. Linearity
7.2. Superposition Theorem
7.3. Thevenin's Theorem
7.4. Norton's Theorem
7.5. Tellegen's Theorem
7.6. Reciprocity Theorem
7.7. Substitution Theorem
7.8. Compensation Theorem
7.9. Maximum Power Transfer Theorem
8. Two-Port Networks
8.1. Introduction
8.2. Open-circuit Impedance Parameters
8.3. Short-circuit Admittance Parameters
8.4. Equivalent Relationship between T and "Π" Networks
9. Advanced Circuit Analysis Techniques
9.1. Fourier Analysis
9.2. Wavelet Analysis
Acknowledgements
Glossary

Bibliography
Biographical Sketches

Summary

Electricity is very important nowadays and power system is essential for electricity to exist in our daily life. A good and careful analysis of power networks is necessary to maintain the efficiency and safety. This chapter mainly describes the basic components of a power system including passive components like resistors and capacitors. Subsequently, basic laws of current and voltage will be briefly described. Then, a more advanced theorem about networks will be discussed. Finally, there will be discussions about two port network analysis, Fourier analysis and Wavelet analysis.

1. Introduction

Efficiently operating and generating electrical power has always been the main goal for power engineers to achieve. Carefully planned and effectively operated networks through analyses are essential to reach the goal. Power systems have been operating for a very long time in modern society and the methods of network analysis also has evolved over a very long time.

This chapter is aimed mainly at introducing the basic power system components such as resistors, capacitors and inductors. These three components which are characterized by their properties respectively as resistance, capacitance, and inductance, contribute the most important part of a power system, impedance which is a combination of resistance, capacitive and/or inductive reactance.

Impedance is the ratio of voltage drop to current flow in an electrical circuit. To implement or calculate the impedance is crucial for network analysis. The basic analysis of a power network is applying Ohm’s Law and Kirchhoff’s Voltage Law (is also known as law of compatibility) and Kirchhoff’s Current Law (also known as the law of continuity). Ohm’s law describes the constitutive relation between the voltage and current in a circuit element. On the other hand, Kirchhoff’s laws are dealing with interconnectivity constraints in a network configuration.

The most basic network is a linear network in which the parameters of resistance, inductance, and capacitance are constant with respect to voltage or current or the rate of change of voltage or current and in which the voltage or current of a source is either independent of or proportional to other voltages or currents, or their derivatives. Unknown parameters in the network are therefore easy to be estimated and calculations are straight forward. Theorems such as Superposition Theorem, Thevenin’s Theorem or Norton’s Theorem can be applied here.

In a more complex non-linear network, impedances depend on the values of voltage and current we can simulate them as variable inductances or capacitances. Then, with a simple single-frequency voltage source, current components of different frequencies, called harmonics, will be produced. To analyze such a network, more advanced methods will be required such as Fourier analysis and wavelets.

2. Passive Components

A component is said to be passive if the total energy delivered to it from the rest of the circuit is always non-negative. There are generally three kinds of passive components

2.1. Resistors

A resistor is an electrical device whose primary function is to introduce resistance to the flow of electric current. The magnitude of opposition to the flow of current is called the resistance of the resistor. The resistance R of a resistor is given by

$$R = \frac{\rho\, l}{A} \tag{1}$$

where ρ is the resistivity of the material; l is the length of the resistor and A is the cross-sectional area perpendicular to current flow.

2.2. Capacitors

A passive electrical device that is intentionally designed to store energy in an electric field is called a capacitor. The ability of a capacitor to store energy in an electric field is called the capacitance. A capacitor is physically made of two conducting plates separated by a dielectric medium and its capacitance C is given by

$$C = \frac{\varepsilon\, A}{d} \tag{2}$$

where A is the area of the plate; ε is the dielectric constant of an insulating material and d is the separation between plates. C is measured in Farads (F).

In an alternating current (ac) circuit, the opposition offered to the flow of current by capacitance is called capacitive reactance X_C which is defined as

$$X_C = -j\frac{1}{2\pi fC} = -j\frac{1}{\omega C} \tag{3}$$

where f is the frequency of the ac circuit and the angular frequency, $\omega = 2\pi f$.

2.3. Inductors

An inductor is an electrical device designed to store energy in a magnetic field. Inductance is a measure of the ability of an inductor to store energy in its magnetic field. An inductor can be made by winding a coil of wire around a toroidal core and its inductance L, measured in Henrys (H), is given by

$$L = \frac{N^2 A\mu}{2\pi r} \tag{4}$$

where N is the number of turns of wires; A is the cross-sectional area of the torus; μ is the permeability of the material of torus and r is the radius of the torus. Inductive reactance X_L of an inductor is defined as

$$X_L = j2\pi fL = j\omega L \quad (5)$$

3. Voltage and Current Sources

An electric device is said to be active if it is capable of delivering energy. These devices are intended to supply energy to an electric circuit and they are called sources. Sources are categorized into one of two types: voltage sources and current sources. For a voltage source, the voltage is specified, but the current is determined by the rest of the circuit. For a current source, the current is specified with the voltage determined by the rest of the circuit.

Besides these two sources, some devices act like controlled sources such as transistors and amplifiers and they are called dependent sources. They consist of two elements: the controlling element and the controlled element. The controlling element is either an open circuit or a short circuit. The controlled element is either a voltage source or a current source. There are four types of dependent sources, namely voltage-controlled voltage source (VCVS), current-controlled voltage source (CCVS), voltage-controlled current source (VCCS) and current-controlled current source (CCCS). For VCVS, the controlling element is an open circuit and the open circuit voltage, v_c , is the controlling signal of this dependent source. The controlled element is a voltage source, v_d and is controlled by v_c such that $v_d = gv_c$ where g is the gain of the dependent source. For CCVS, the controlling element is a short circuit and the short circuit current, i_c , is the controlling signal. The controlled element is a voltage source, v_d , and is controlled by i_c such that $v_d = gi_c$ where g is the gain. For VCCS, the controlling element is an open circuit and the open circuit voltage, v_c is the controlling signal. The controlled element is a current source, i_d , and is controlled by v_c such that $i_d = gv_c$ where g is the gain. For CCCS, the controlling element is a short circuit and the short circuit current, i_c , is the controlling signal. The controlled element is a current source, i_d , and is controlled by i_c such that $i_d = gi_c$ where g is the gain.

4. Circuits

An electrical circuit is an interconnection of electrical components such as passive components and power sources linked together in a closed path so that an electric current may flow continuously. In a direct current (DC) circuit, both current and voltage have their values unchanged. In an alternating current (AC) circuit, their values change periodically. In general, there are two types of AC circuits, namely single-phase circuits and three-phase circuits.

4.1. Single-phase Circuits

Figure 1 shows a single-phase circuit in which there are one voltage source and a pure resistor. The source is sinusoidal in nature. By plotting the instantaneous value of the applied voltage against the time, a sine curve is formed. The applied voltage is thus called as a sinusoidal voltage. Figure 2 shows the waveform of the current and the applied voltage in this circuit.

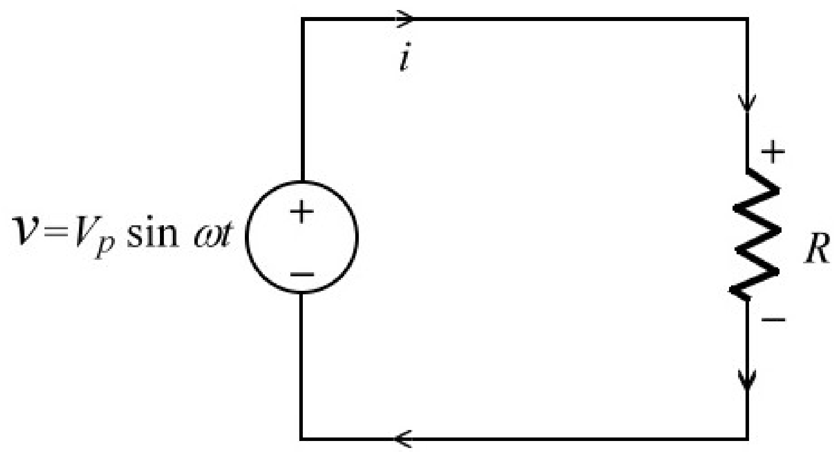

Figure 1: A single-phase circuit with one voltage source and one pure resistor

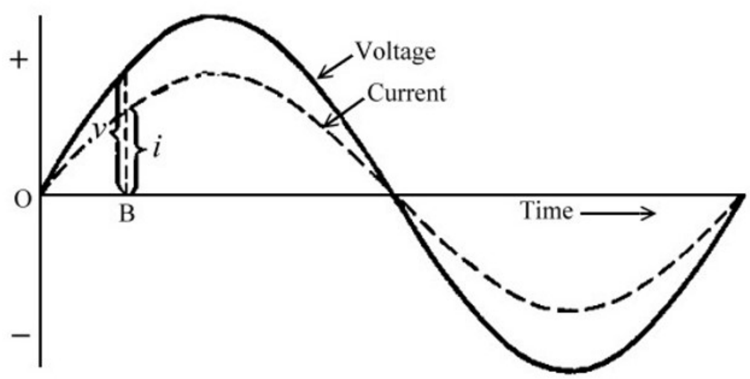

Figure 2: The waveform of current and voltage in a purely resistive circuit

It is observed that the current is also sinusoidal if the applied voltage is sinusoidal. The current and voltage are denoted by $i(t)$ and $v(t)$ respectively which are expressed by Eqs. (6) and (7).

$$i(t) = I_p \sin(\omega t) \tag{6}$$

$$v(t) = V_p \sin(\omega t) \tag{7}$$

where I_p and V_p are the peak value of the current waveform and the voltage waveform respectively; ω is the angular frequency.

In Figure 3, the instantaneous value of a sinusoidal waveform can be represented by the vertical projection of a phasor rotating in a counterclockwise direction at an angular speed of ω. A phasor is like a vector with the difference that the angles in a phasor diagram denote phase angles. To facilitate the power calculation in an ac circuit, the length of a phasor is set to the value equal to the peak value divided by $\sqrt{2}$. This magnitude is called a root-mean-square (rms) value of a concerned quantity which is often either an ac current or an ac voltage. Also, the phasor has direction as mentioned later. For circuit analysis in ac circuit, a phasor is used instead of using a sinusoidal waveform to simplify the calculation. In a purely resistive circuit, both voltage phasor and current phasor are in the same direction meaning that they are in-phase. They can be directly represented by their rms values which are real numbers. Therefore, the voltage phasor is set to V_R and the current phasor is equal to I_R where $V_R = V_p / \sqrt{2}$ and $I_R = I_p / \sqrt{2}$.

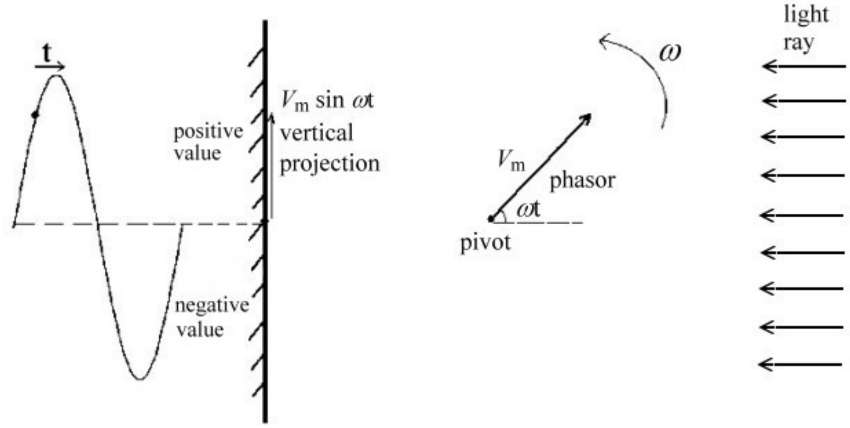

Figure 3: The phasor representation of a sinusoidal waveform

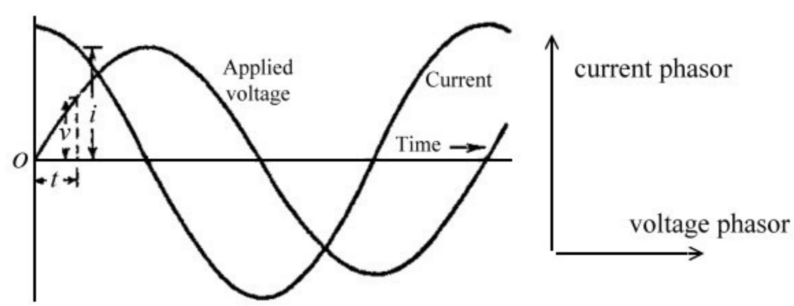

Figure 4: The current and voltage waveforms and their phasors in a purely capacitive circuit

In a purely capacitive circuit, the current and voltage waveforms are not in-phase. Figure 4 shows their waveforms in this circuit and their corresponding phasors. It is observed that the current phasor leads the voltage phasor by 90^o. To distinguish the direction of two phasors, the system of a complex number is employed. In this case, the voltage phasor is a real number V_C and the current phasor is an imaginary number jI_C, where $j = \sqrt{-1}$. V_C and I_C are the rms values of the voltage and current respectively.

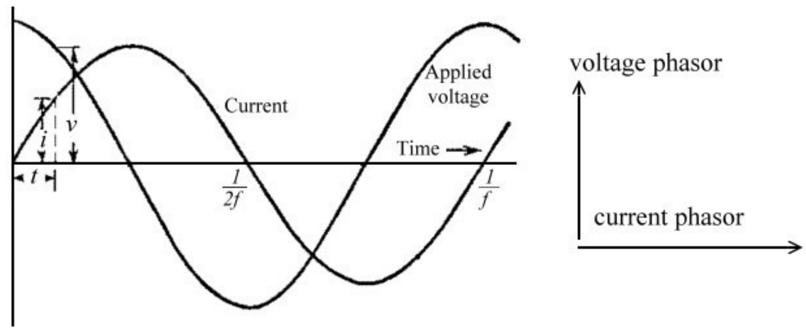

Figure 5: The current and voltage waveforms and their phasors in a purely inductive circuit

On the other hand, in a purely inductive circuit, the voltage current situation is reversed.; the current phasor lags the voltage phasor by 90^o as shown in Figure 5. The voltage phasor becomes an imaginary number jV_L and the current phasor is now a real number I_L. Similarly, V_L and I_L are their rms values. In practice, an ac circuit consists of resistors, capacitors and inductors. The current phasor and the voltage phasor are not in-phase or with a phase angle difference of exactly 90^o. In general, they are represented by complex numbers which have both non-zero real and imaginary parts.

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Bibliography

Deller, J.R., Jr., (1994) "Tom, Dick, and Mary discover the DFT", IEEE Signal Processing Mag., 11 (2), 36-50 pp. [In-depth analysis on discrete Fourier transform]

Drof, R.C., (1997). The Electrical Engineering Handbook, 63 pp., CRC Press LLC, New York, USA. [A comprehensive reference book on electrical engineering].

Drof, R.C., Svoboda, J.A. (1996). Introduction to Electric Circuits, 3rd Ed. 192 pp., John Wiley & Sons, Inc., New York, USA. [This provides background theory of electric circuits]

Jenkins, W.K., Desai, M.D., (1986) "The discrete-frequency Fourier transform", IEEE Trans. Circuits Syst., (CAS-33) 7, 732-734 pp. [In-depth analysis on discrete-frequency Fourier transform]

Meyer, Y., (1993). Wavelets Algorithms & Applications, 4 pp., Society for Industrial and Applied Mathematics, Philadephia, USA. [A comprehensive reference book on the algorithms of Wavelets analysis]

Schumaker, L.L., Webb G., (1994) Recent Advances in Wavelets Analysis, 259-309 pp., Academic Press, Inc., New York, USA. [A reference book for Boxcar / wavelet Transform]

Scott, D.E. (1987). An Introduction to Circuit Analysis, 12 pp., McGraw-Hill, Network, USA. [An introductory text on circuit analysis]

Stuller, C. (1995). Circuit Analysis, 2^{nd} Ed. 57 pp., Houghton Mifflin Company, Boston, USA. [This provides the basics of various circuit theorems]

Temes, G.C., LaPatra J.W. (1977). Circuit Synthesis and Design, 55 pp., McGraw-Hill Kogakusha Ltd., London, UK. [An introductory text on circuit synthesis].

Tung, J.L., Kwan, B.W. (2001). Circuit Analysis, 208 pp., World Scientific, New York, USA. [A comprehensive reference book on Fourier analysis]

Young, R.K., (1993). Wavelets Theory and It Applications, 1-44 pp., Kluwer Academic Publishers, Boston, USA. [A comprehensive reference book on Wavelets Theorems]

Zheng, T., Makram, E.B., Girgis, A.A. (1999) “Power System Transient and Harmonic Studies using Wavelet Transform”, IEEE Trans. Power Delivery, 14, pp. 1461-1468. [In-depth analysis on Wavelet analysis]

Biographical Sketches

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