

# MULTI-CRITERION ANALYSIS IN WATER RESOURCES MANAGEMENT

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## Summary

Some of the basic features and concepts of multi-criterion analysis (MCA) are presented. Thus, terminology and notation that we have found to be practical and may

be considered as fairly standard throughout the field of MCA are reviewed. In addition, distinctions are made between different types of solutions that can be derived using the different types of MCA techniques available for consideration. A typology of such techniques consisting of five groups is also presented. At the end of this article, a paradigm of the multi-criterion process is provided to unify the elements of this brief presentation of the conceptual framework of MCA applied to water resources systems.

## **1. Concepts and Terminology in Multi-criterion Decision Making**

Terms often used in an MCA problem include attributes, criteria, objectives, goals and constraints. Universally accepted definitions of such terms do not seem to exist in the MCA literature. Many authors have used a number of these terms, such as goals, purposes, criteria and objectives interchangeably while others make clear distinctions, at least partially, in their usage. The purpose of this section is to give distinct definitions of the essential MCA terms used in water resources systems analysis.

### **1.1 Objectives**

Objectives indicate the directions of state change of a system desired by the decision maker(s). They reflect the aspirations of whoever is providing the value structure and as such indicate the directions sought. There are three possible ways to reach an objective: maximizing it, minimizing it or maintaining it at an existing position. The first two are self-evident. An example of the third situation would be a reservoir manager wishing to maintain a constant supply of water to a downstream river reach where both an excess and a deficiency of water would adversely affect hydroecological sustainability. Another viewpoint is to consider five types of “aspirations,” which are objectives over a range: near a target, greater than a threshold, less than a threshold, inside of an interval, outside of an interval. Extended definitions of objectives with respect to these concepts are available in references listed in the bibliography.

Another aspect of objectives that needs to be raised at this point has to do with their generation. There is substantial information on this process in the literature, for example, approaches include: (a) examination of relevant literature to see how others have been modeling the same kind of problem; (b) analytical study of the problem, and (c) casual empiricism. The analytical approach suggests that by building a model of the system under consideration and identifying the relevant input and output variables, the appropriate objectives for the problem will crystallize. The casual empiricism approach, on the other hand, suggests observing people to see how, in fact, they are presently making decisions relevant to the given problem. Any one of these approaches can help in generating objectives and a combination may help even further.

The process of modeling and solving a problem with two or more non-commensurable and conflicting objectives is known in the literature as multi-objective decision making (MCA). Objectives are non commensurable if their level of attainment, with respect to given attributes cannot be measured in common units. Objectives are conflicting if an increase in the level of one objective can only be achieved by decreasing the attainment

level of another objective. Usually, a conflict arises when the attainment of each objective in a problem requires the shared use of limited available resources. Examples of objectives are optimization of economic payoff, environmental quality, water supply, water quality and mitigation of natural and man-made hazards, ecological preservation and sustainable development.

## 1.2 Attributes

These refer to the characteristics, factors, qualities, performance indices or parameters of alternative management schemes or other decision processes. An attribute should provide a means for evaluating the levels of attainment of an objective; as such, it is defined here as a measurable aspect of judgment by which a dimension of the various decision variables or alternative management schemes under consideration can be characterized. This characterization, in turn, is made possible through determination of at least one empirical indicator, such as dissolved oxygen for each attribute (water quality). Then, to make the measurement complete, scales are constructed one for each attribute in the form of a set of estimates with an order relation.

The choice of scale type depends on the technique of measurement to be used and on the magnitude of the properties being measured. Then, depending on the desired accuracy of measurement, values are determined for the magnitude characterized by the empirical indicator and these values need to be in the region of feasible estimates. In model-based mathematical system theory, a distinction is made between performance indices (or attributes) such as reliability, and resource indices, such as money or land.

A decision analysis problem consisting of more than two attributes is known as a multi-attribute decision problem and may be solved using an MCA procedure. The procedure involves the selection of the “best” alternative course of action from a given number of alternatives described in terms of their attributes. Examples of attributes are flood damages, sediment yield, nitrate concentration.

## 1.3 Criteria

The dictionary meaning of criteria is standards, rules or tests on which judgments or decisions can be based. In decision-making theory, however, a criterion may represent either an attribute or an objective. In this sense, a multi-criterion decision problem means either a multi-attribute or a multi-objective decision problem or both. MCA is, therefore, used to indicate the general field of study that includes decision making in the presence of two or more conflicting objectives and/or decision analysis processes involving two or more attributes.

## 1.4 Decision Variables, Alternative Schemes and Parameters

Decision variables are the vehicles used to specify decisions made by a decision maker. In mathematical programming, they represent the numerical variables whose values are to be determined and are denoted by  $x_j$ ,  $j = 1, \dots, J$ . The symbol  $x_j$ , a decision variable, represents the quantity within a set of  $J$  quantities, for example, water release from a reservoir at a given time.

In most mathematical programming problems, decision variables are continuous and also assumed to have implicit upper boundaries. In problems involving mixed numerical and non-numerical data, the different objectives can only be approached using a set of discrete alternative actions. The members of this set are carefully selected by considering all important, relevant information on the problem and its objectives and the alternative actions themselves, as stated by Gershon et al. Different ways of selecting alternatives are available. If too many alternatives are made available in the process, then some sort of filtering or screening mechanisms such as ELECTRE 1 and exclusionary screening, a method described by Goicoechea et al., can be used to eliminate the dominated alternatives. Once the selection process for the set of alternatives to be considered is complete, a relationship between the alternatives and the criteria of the problem under consideration is developed using some measurement scales. The information consisting of criteria, alternatives and measurement scales is then used to construct an evaluation matrix of criteria versus alternatives, upon which MCA solution techniques are applied in order to select the “best” alternative possible plan(s). This concept has been extensively developed in many case studies. Note that alternatives should be evaluated over a common set of criteria.

During the problem formulation stage of the mathematical decision process, one should decide which quantities are to be treated as decision variables and which ones are to be taken as fixed. The quantities whose values are fixed are called parameters. These quantities remain relatively fixed because the values are either objectively assigned and we are not at liberty to change them, or we have learned from experience that particular values of the respective quantities always give good results, leaving us with no reason to treat such quantities as decision variables. In any case, mathematical relationships between the decision variables and the parameters constitute a major part of the problem formulation stage of the decision analysis process.

### 1.5 Constraints

Constraints are restrictions on attributes and decision variables that may or may not be expressed mathematically. They are usually dictated by such factors as environment, physical processes, economics, cultural, legal and/or resources aspects, which must be satisfied in order to produce an acceptable solution. In mathematical form, constraints describe dependencies between decision variables and parameters, and may be stated in the form of equalities (mass balance), inequalities (resources constraints) or probabilistic/fuzzy statements (reliability constraints).

### 1.6 Decision Space and Objective Space

A multi-criterion programming problem can be represented in a vector notation as:

$$\text{“Satisfy”} \quad f(x) = (f_1(x), f_2(x), \dots, f_I(x)) \quad (1)$$

$$\text{Subject to} \quad g_k(x) \leq 0, \quad k = 1, 2, \dots, K \quad (2)$$

$$x_j \geq 0, \quad j = 1, 2, \dots, J \quad (3)$$

Here there are  $I$  objective functions each of which is to be “satisfied” subject to the constraint sets (2) and (3). The region defined by this constraint set is referred to as the feasible region in the  $J$ -dimensional decision space. In this expression, the set of all  $J$ -tuples of the decision variable  $x$ , forms a subset of a finite  $J$ -dimensional Euclidean space; in many other applications,  $x$  is defined to be discrete. In the further special case when  $X$  is finite, then the most satisfying alternative plan has to be selected from that finite set  $X$ . It is important to note at this point that the word “optimum” which includes both the maximization of desired outcomes and minimization of adverse criteria is replaced by the word “satisfactum” and “optimize” is replaced by “satisfy” in this discussion. The reason is that when dealing with two or more conflicting objectives one cannot, in general, optimize all the objectives simultaneously, as an increase in one objective usually results in a deterioration of some other(s). In such circumstances, tradeoffs between the objectives are made in order to reach solutions that are not simultaneously optimum but still acceptable to the decision maker with respect to each objective, as described in standard texts.

In a mathematical programming problem such as the one defined by Eqs. (1), (2) and (3), the vector of decision variables and the vector of the objective functions  $f(x)$  define two different Euclidean spaces. These are (1) the  $J$ -dimensional space of the decision variables in which each coordinate axis corresponds to a component of vector  $x$ , and (2) the  $I$ -dimensional space  $F$  of the objective functions in which each coordinate axis corresponds to a component of vector  $f(x)$ . Every point in the first space represents a solution and gives a certain point in the second space that determines the quality of that solution in terms of the values of the objective functions. This is made possible through a mapping of the feasible region in the decision space  $x$  into the feasible region in the objective space  $f$ , using the  $I$ -dimensional objective function.

The definition and concepts given above can be easily understood with the help of a simple continuous example. For this purpose, consider the following bicriterion and bivariate linear problem.

$$\begin{aligned} \max. f_1(x) &= 2x_1 - x_2 \\ \max. f_2(x) &= -x_1 + 3x_2 \end{aligned}$$

subject to:

$$\begin{aligned} g_1(x) : x_1 + x_2 &\leq 10 \\ g_2(x) : x_1 &\leq 7 \\ g_3(x) : x_2 &\leq 6 \\ g_4(x) : -x_1 + x_2 &\leq 4 \\ g_5(x) : -x_1 &\leq 0 \\ g_6(x) : -x_2 &\leq 0 \end{aligned}$$

The feasible region in the decision space  $x$  of this problem is shown in Figure 1.

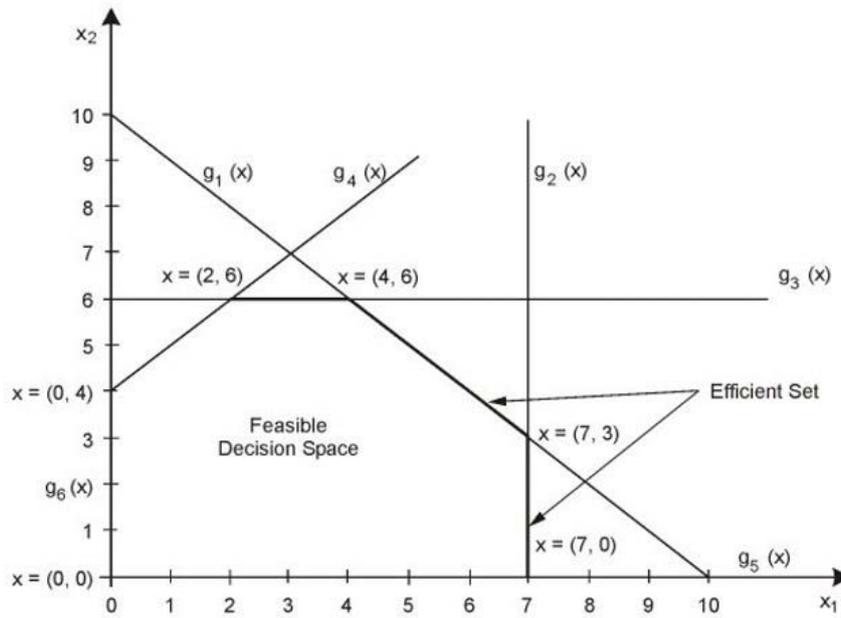


Figure 1. Feasible region of decision variables.

It is the convex space bounded by all the relevant constraints, that is, any point  $x$  in the feasible region satisfies these constraints. On the other hand, any constraint whose boundary does not intersect the feasible space is redundant.

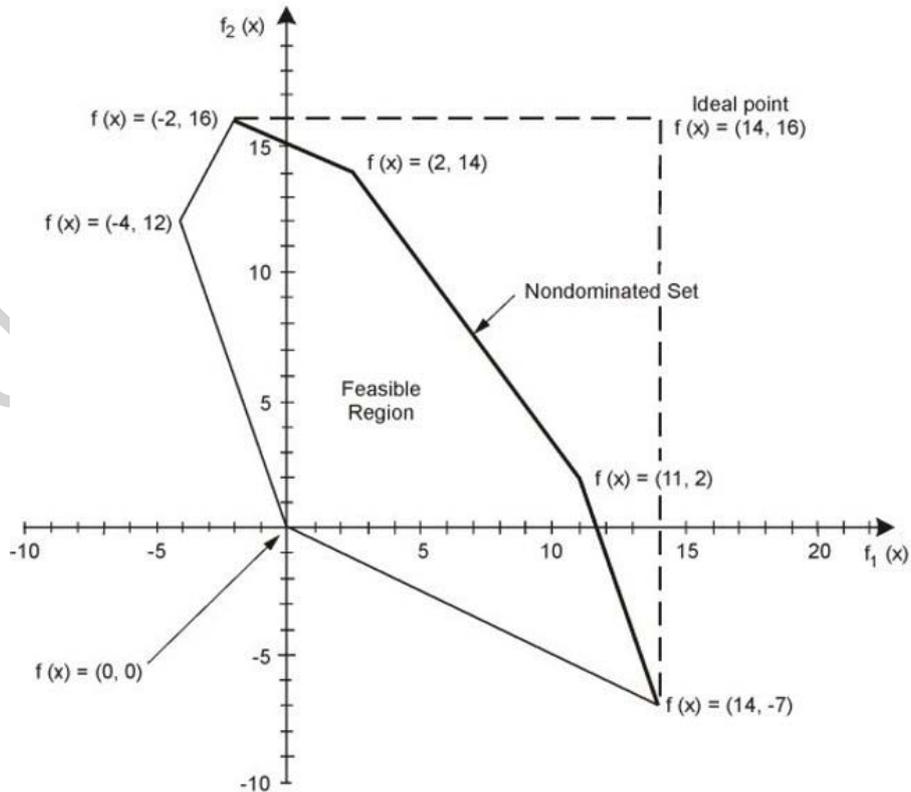


Figure 2. Feasible region of objective functions.

The feasible region in the objective or payoff space  $f(x)$  is a transformation of the feasible decision space and is determined by enumeration of all the extreme points and subsequent computation of the value of each objective function at each of the corner solutions as shown in Figure 2. This figure illustrates how the non-dominated set can be identified in this feasible region. To find a final satisfying solution, an interaction between the analyst and decision-maker is required. Possible mechanisms for this interaction are provided next.

## 2. The Roles of the Decision Maker and Analyst

The key element in any decision-making process is the presence of the decision maker. The decision maker is the individual or group of individuals whose *desirata* are supposed to be satisfied by the outcome of the multi-criterion decision process. It is the responsibility of the decision maker to identify both the decision problem and specify the objectives of that problem. It is also the decision maker who directly or indirectly furnishes the final value judgment that may be used to rank available alternatives, so that a *satisfactum* can be determined. The analyst, on the other hand, is responsible for defining the decision model, conducting the multi-criterion decision process and presenting the results to the decision maker. This requires that a wide range of activities be carried out by the analyst in the form of appropriate problem formulation, and quantitative and qualitative analyses of that problem. In addition, some interaction between the decision maker and the analyst are indicated in these works.

The interaction between the analyst and the decision maker is an inherent characteristic of the decision process. The minimum interaction requirements are that the decision-maker be able to specify his/her preference structure with respect to the objectives of the problem under consideration, and then decide the acceptability of the solutions to the problem when presented to him/her by the analyst. The interaction becomes more elaborate and quite complex if interactive decision-making aids are utilized as these involve a constant elicitation of preferences of the decision-maker.

### 2.1 The Decision Maker's Preference Structure

A very important component of an MCA process concerns the priorities often attached to each one of the various criteria under consideration. These priorities may be represented as quantitative numbers referred to as weights or coefficients of importance by Roy, or by means of ordinal expressions which are denoted as priorities.

The weights and priorities in the decision makers' view represent the relative importance of the objectives or utilities of a problem to one another and thus constitute a major part of the decision-maker's preference structure in a particular MCA problem. From applications of multi-criterion techniques it appears that such preference structures of the decision maker can have a major influence on the final evaluation results. In case a particular set of weights does not result in a satisfactory solution, the weights can be changed in order to reach a more acceptable solution. The process of changing progressively or iteratively the weights until an acceptable solution is reached can help the decision maker to arrive at his/her "true" preference structure.

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**Lucien Duckstein** was a professor of Systems and Industrial Engineering and also of Hydrology and Water Resources at the University of Arizona Tucson, USA, from 1962 to 1997.

He has then become a professor emeritus at the same institution and has since returned to his native city, Paris, France, as a professor at ENGREF (French Institute of Agronomy, Water Resources and Forestry). His research areas cover multiobjective analysis, decision theory, statistical and Bayesian decision theory, fuzzy logic with applications to hydrology and water resources.

**Aregai Teclé** is a professor of Hydrology and Decision Systems Analysis in the School of Forestry, Northern Arizona University in Flagstaff, Arizona, USA since 1988. He teaches Environmental and Surface Hydrology, Watershed Restoration as well as Multiobjective Decision-Making and Conflict Resolution in Natural Resources Management. His research areas cover modeling surface hydrology and watershed management, water quality and pollutant movement, riparian restoration and analysis of the impacts of fire and human activities on stream basins, as well as multi-objective decision-making and conflict resolution in watershed management.