MATHEMATICS, CIVILIZATION, AND PROGRESS

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Contents

Summary

This chapter provides an overview of the comparative history of mathematics in the context of the ideas of civilization and progress. The narrative is guided by the idea that mathematics is a social practice and not a set of ideas revealed through some sort of heavenly or Platonic discourse. This implies that mathematics has a normative dimension in terms of its cultural and professional settings. The chapter begins with a critical review of the terms of the discourse, namely mathematics, civilization, and progress. Following some general remarks on mathematics and civilization, the reader is guided through case studies of mathematics in its civilizational and cultural contexts including mathematics in China, India, and Greece, as well as modern Europe. A more analytical section follows, summarizing the idea of the social roots of mathematics, the development and functions of puzzle solving and proofs, and generalization as a way to think about "abstraction" in concrete terms. The narrative follows an historical social science perspective that draws on the ideas of classical and contemporary social theorists from Durkheim and Spengler to Randall Collins and Sal Restivo.

1. Prologue: Interrogating the Terms of Our Discourse

Let us begin by briefly interrogating the three terms that make up the title of this essay. First, we should ask, borrowing the title of the book by Reuben Hersh (1999): *What is mathematics, really*? Mathematics has been shrouded in mystery and halos for most of its history. The reason for this is that it has seemed impossible to account for the nature and successes of mathematics without granting it some sort of transcendental status. Classically, this is most dramatically expressed in the Platonic notion of mathematics.

Briefly, what we call Platonism in mathematics refers to Plato’s theory of Forms. Skirting the complexities of scholarly discourse, Plato is associated with the idea that there are “Forms” or “ideals” that are transcendent and pure. These immaterial Forms exist in a realm outside of our everyday space and time. They are the pure types of the ideas and concepts we manage in our everyday world.

This over-simplifies Plato but is consistent with a long tradition in the history and philosophy of mathematics. Consider, for example, the way some scholars have viewed the development of non-Euclidean geometries (NEGs). The mathematician Dirk Struik (1967: 167), for example, described that development as “remarkable” in two respects. First, he claimed, the ideas emerged independently in Gòttingen, Budapest, and Kazan; second, they emerged on the periphery of the world mathematical community (most notably in the case of Kazan and to a lesser extent Budapest). And the distinguished historian of mathematics, Carl Boyer (1968: 585) characterized the case as one of “startling…simultaneity.”

These reflect classical Platonic, transcendental views of mathematics. One even finds such views in the forms of the sociology of knowledge and science developed from the 1920s on in the works of Karl Mannheim and Robert K. Merton and their followers. Mannheim, for example, wrote in 1936 that 2+2 = 4 exists outside of history; and Merton championed a sociology of science that focused on the social system of science and not on scientific knowledge which he claimed lay outside of the influences of society and culture.

His ambivalence about this is reflected in his critical reply to G.N. Clark’s criticism of Boris Hessen’s historical materialism of Newton’s *Principia* (Merton, 1967: 661-663). Clark opposed Hessen’s (Marxist) political economy of the *Principia* with a defense of Newton’s “purely” scientific motives. Merton argued that individual motivations do not change the structural facts of the matter and in this case they support Hessen’s argument. This doesn’t reach to the core of the social construction of scientific knowledge but it does demonstrate at least an appreciation for the contextual foundations of that knowledge.

There are a couple of curiosities in the case of non-Euclidean geometry (NEGs). Even a cursory review of the facts reveals that NEGs have a history that begins already with Euclid’s earliest commentators, runs over the centuries through names like Saccheri, Lambert, Klügel, and Legendre, and culminates in the works of Lobachevsky (1793-1856), Reimann (1826-1866), and J. Bolyai (1802-1860). The concerns over Euclid’s parallels postulate moved geometers eventually to the systematic development of NEGs. The issue was that the parallels postulate, the fifth postulate in Euclid’s system, did not possess the axiomatic self-evidence of the first four postulates, and it could not be derived from the first four. The three creators of NEGs were by no means isolated and working independently. All were connected to Gauss (1777-1855) who had been working on NEGs since the late 1700s.

J. Bolyai was the son of one of Gauss' friends, W. Bolyai. Gauss and his friend Bolyai were at the University of Göttingen where the parallels postulate was the subject of lectures by Kastner and a number of dissertations. Reimann was Gauss' dissertation student. And as for Lobachevsky, he did indeed work at a university on the periphery of the European mathematical community, the University of Kazan. However, the university was staffed by distinguished German professors, including Gauss' teacher, J.M. Bartels. J. Bolyai developed ideas on non-Euclidean geometries (NEGs) as early as 1823. His "The Science of Absolute Space" was published ten years later in a book written by his father. Lobachevsky published on the foundations of geometry from 1825 on. Reimann's *Habilitationschrift* was on the foundations of geometry. Gauss, we know, wrote about NEGs in letters to W. Bolyai (December 17, 1799), Taurinus (November 8, 1824), and to Besel (January 27, 1829). He also wrote about NEGs in published notes from 1831 on. There are two short reviews on NEGs in *Göttingische Anziegen* in 1816 and 1822. One has to wonder why in the face of the facts of the case Struik and Boyer chose to view things as "remarkable" and "startling."

Classically, the story of the development of NEGs was told in the context of "pure" mathematics. Thus, to take the case of Riemann as an example, the story was that he constructed the generalization of elliptic geometry as a purely mathematical exercise. The idea that there was a concrete possibility of practical applications for this exercise was not a consideration. In the light of a more realistic sociological and network analysis, Riemann's work along with that of Gauss, Lobachevsky, Bolyai, Helmoltz, and Clifford, the story of NEGs takes on a different shape.

To some extent, they all agreed that Euclidean geometry was an unimpeachable system of ideal space and logic. It could be read as a game played in accordance with a set of formal rules. In fact, however, they interrogated Euclidean geometry in terms of whether it was a valid representation of "actual space." This should be tested not by mathematics, not what is within the confines of the social world of mathematics per se, but should be tested scientifically – by observation and some mode of experimentation.

The sociological generalization this leads to is that if you are given a "genius" or a startling event, search for a social network –cherchez le réseau. No one has made the case for social networks as the roots of ideas more powerfully than Randall Collins (1998). The rationale here should become clearer over the course of this chapter.

Even more curious in the case of the sociology of knowledge is the fact that already in his *The Elementary Forms of Religious Life* published in French in 1912, Emile Durkheim had linked the social construction of religion and the gods to the social construction of logical concepts. Durkheim's program in the rejection of transcendence languished until the emergence of the science studies movement in the late 1960s and the works of David Bloor, Donald MacKenzie, and Sal Restivo in the sociology of mathematics.

It is interesting that a focus on practice as opposed to cognition was already adumbrated in Courant's and Robbins' classic "*What is Mathematics*?" (1906/1995). We must turn to active experience, not philosophy, they wrote, to answer the question "What is mathematics"? They challenged the idea of mathematics as nothing more than a set of consistent conclusions and postulates produced by the “free will” of mathematicians. Forty years later, Davis and Hersh (1981) wrote an introduction to “the mathematical experience” for a general readership that already reflected the influence of the emergent sociology of mathematics. They eschewed Platonism in favor of grounding the meaning of mathematics in “the shared understanding of human beings…” Their ideas reflect a kind of weak sociology of mathematics that still privileges the mind and the individual as the creative founts of a real objective mathematics.

Almost twenty years later, Hersh, now clearly well-read in the sociology of mathematics, wrote “*What is Mathematics, Really*?” (1997). The allusion to Courant and Robbins is not an accident. Hersh does not find their definition of mathematics satisfactory. In spite of his emphasis on the social nature of mathematics, Hersh views this anti-Platonic anti-foundationalist perspective as a philosophical humanism. While he makes some significant progress by comparison to his work with Davis, by conflating and confusing philosophical and sociological discourses, he ends up once again defending a weak sociology of mathematics. The modern sociology of mathematics associated with the science and technology studies movement that emerged in the late 1960s has established mathematics as a human construction, a social construction. Mathematics is manufactured by humans on the earth out of the material and symbolic resources at their disposal in their local environments. While traditional philosophical and sociological discourses have become estranged, especially in the arena of science studies, there are efforts abroad to reconcile the two disciplines consistent with the interdisciplinary turn in contemporary research and theory. In this sense, one can consider Hersh’s philosophical humanism a step in that direction.

The second term in my title, “civilization” is not without its controversial features. In the ancient world, “civilized” peoples contrasted themselves with “barbarians;” in modern times, “civilized” has been opposed to “primitive” or “savage.” The relevance of the concept of civilization to the topic of mathematics lies in its association with the idea of progress. More to the point is the fact that different civilizations (variously “nations,” “societies,” and “cultures”) are associated with different mathematical traditions (v. Restivo, 1992, 23-88). To the extent that humans have developed in ways that can be captured in the ideas of “evolution” and “progress,” mathematics, in conjunction with science and technology more generally, is assumed to have contributed positively to and benefited from those developments. The Scottish philosopher Adam Ferguson (1723-1816) is credited by Benveniste (1954) with introducing the term “civilization” in its modern sense into the English language in his *Essay on the History of Civil Society* (1767), and perhaps as early as 1759. The term also appears in the works of Boswell (1772), Adam Smith (1776), and John Millar (1771). Mirabeau (1757) introduces the term in French in his *L'Ami des hommes ou traité de la population*. Just as the individual grows from infancy to adulthood, Ferguson wrote, the species advances from “rudeness to civilization.” Set at the pinnacle of forms of society, civilizations are characterized by complexity, hierarchies, dynamism, advanced levels of rationality, social progress, and a statist form of government. Rousseau, by contrast, viewed civilization as opposed to human nature.

If we adopt Ferguson’s view of civilization, then clearly mathematics has been both a result of the emergence and development of civilization and a contributor to that development. If on the other hand we adopt Rousseau's viewpoint, the virtues of science and mathematics and the very idea of "civilization" are made severely problematic.

What about progress, the very idea? Arguably, the idea comes into Western and world culture in the Old Testament with its conception of linear time and a God that moves through time with humans (e.g., Sedlacek, 2011: 47). The idea of scientific and technological progress was fueled by the seventeenth century advances in science and literature by such cultural giants as Galileo, Newton, Descartes, Molière, and Racine. The idea of social progress was added later. Early in the eighteenth century, the Abbé de Saint Pierre advocated establishing political and ethical academies to promote social progress. Saint Pierre and Turgot influenced the Encyclopedists. The great *Encyclopédie* was produced by a group of eighteenth century philosophers under the direction of Denis Diderot.

It defines the Enlightenment program of promoting reason and unified knowledge. It was at this point that social progress became mated to the values of industrialization and incorporated into the ideology of the bourgeoisie. Scientific, technological, and social progress were all aspects of the ideology of industrial civilization. Veblen, for example, argued that the various sciences could be distinguished in terms of their proximity to the domain of technology. Thus, the physical sciences were closest to that domain, even integral with it, whereas such areas as political theory and economics were farther afield. We have entered an era of machine discipline unlike any in human history. And now we stand on the threshold of machines that will discipline us with conscious awareness and values, including social and sociable robots (the so-called robosapiens), and cyborgs.

There have been attempts to identify a type of progress that is independent of material or technological criteria (see, for example, the discussion in Almond, Chodorow, and Pearce, 1985, and the classic criticisms in Roszak, 1969/1995). For many ancient as well as modern thinkers, the idea of progress has always been problematic. We are right to be concerned about the actual and potential impacts of our new bio- and nano-technologies. But one finds similar concerns in Plato's *Phaedrus.* There, in the dialogue between Theuth and king Thamus concerning the new technology of writing, Theuth makes promising predictions about the impact of writing.

The king claims to be in a better position to do what in effect is a "technology assessment," and concludes that writing will have the opposite of the effects predicted by Theuth. The cultural meaning of science has fared no better. Where the Rousseaus and the Roszaks saw danger and alienation in science, the Francis Bacons and Bronowskis saw civilization and progress. When the biochemist J.B.S. Haldane wrote about a future of human happiness built on the application of science, Bertrand Russell replied with a vision of science used to promote power and privilege rather than to improve the human condition. St. Augustine worried about the invention of machines of destruction; Spengler predicted that humans would be annihilated by Faustian man. Fontenelle, in the first modern secular treatise on progress published in 1688 argued that science was the clearest and most reliable path to progress. Rousseau, by contrast, argued that science and the arts have corrupted our minds. The author will draw attention to some additional examples in his conclusion in this chapter.

By its intimate association with the very foundations of science, mathematics does not escape this ambivalence. But it stands apart from science in terms of its stronger association with human progress. In the seventeenth and eighteenth centuries, a wave of positivism fueled by Newton's achievements evoked nothing but the promise of progress among mathematicians of that period. The historian Florian Cajori (1894: 4) had no question about the connection between mathematics and human progress. For Alex Bellos (2010: ix), mathematics is ("arguably") the foundation of all human progress.

Progress, then, can be viewed in terms of "amelioration" or "improvement" in a social or ethical sense. Are we more advanced than cultures that are less dominated by machines and machine ideology? How do we measure the primacy of humans and ecologies and how do we sustain them in any given culture? Can we bring them to fruition and nourish them in any culture, or are some more friendly to the primacy of humans and ecologies than others? These issues are really matters of degree associated with the degree to which individuation of the self (and then the myth of individualism, selfishness, and greed) has progressed in any given society. Furthermore, the degree of awareness of and attention to ethics, values, and social justice has to come into consideration here. It is impossible to even discuss the idea of progress without engaging ideas about and the value of the person, freedom, and democracy.

It may be possible to define progress in a way that takes it out of the realm of hopes, wishes, and dreams and plants it more firmly on a meaningful (and even perhaps measurable) foundation. Following Gerhard Lenski (1974: 59), progress can be defined as the process by which human beings raise the upper limit of their capacity for perceiving, conceptualizing, accumulating, processing, mobilizing, distributing, and utilizing information, resources, and energy in the adaptive-evolutionary process. The relationship between adaptation and evolution is a paradoxical one. On the one hand, survival depends on the capacity to adapt to surroundings; on the other hand, adaptation involves increasing specialization and decreasing evolutionary potential. Adaptation is a dead end. As a given entity adapts to a given set of conditions, it specializes to the point that it begins to lose any capacity for adapting to significant changes in those conditions. The anthropologists Sahlins and Service (1960: 95-97) summarize these ideas as follows:

Principle of Stabilization: specific evolution (the increase in adaptive specialization by a given system) is ultimately self-limiting.

General evolution (progressive advance measured in absolute terms rather than in terms of degrees of adaptation in particular environments) occurs because of the emergence of new, relatively unspecialized forms.

Law of Evolutionary Potential: increasing specialization narrows adaptive potential. The more specialized and adaptive a mechanism or form is at any given point in evolutionary history, the smaller is its potential for adapting to new situations and passing on to a new stage of development.

We can add here the Law of Adaptive Levels: adaptation occurs at different levels across various life orders and systems and occurs at different speeds in different spatial arenas. This law draws attention to the complexity of adaptation and the general processes of variation and selection. Adaptation suggests an active agent in a stable environment. But active agents can and do change their environments in ways that make different demands on the adapting agents. Looked at another way, environments have agential like dynamics. Law of Agent-Environment Entanglement.

Perhaps the most important aspect of the ideology of science is that it is (in its mythical pure form) completely independent of technology. This serves among other things to deflect social criticism from science onto technology and to justify the separation of science from concerns about ethics and values. Interestingly, this idea seems to be more readily appreciated in general by third world intellectuals than by the Brahmin scholars of the West and their emulators. Careful study of the history of contemporary Western science has demonstrated the intimate connection between what we often distinguish as science and technology. It has also revealed the intimate connection between technoscience research and development and the production, maintenance, and use of the means (and the most advanced means) of violence in society. Not only that, but this is true in general for the most advanced systems of knowledge in at least every society that has reached a level of complexity that gives rise to a system of social stratification.

Contradictions and ambivalence about science, technology, and progress may be built into the very core of our cultural machinery. Agricultural activities in the ancient Near East reduced vast forests to open plains, and wind erosion and over-grazing turned those areas into deserts. Deforestation in ancient China led to the development of the loess plateau. Loess sediment gives the Yellow River (nicknamed “China’s Sorrow”) its signature color and flooding pattern. Was deforestation necessary for building China into the greatest civilizational area on earth between the first and sixteenth centuries of the common era? Or were there conservation principles that the ancient Chinese could have relied on without detracting from their cultural development? There is some evidence that at least some of the deforestation they caused could have been avoided. The deforestation experiences of China, Rome, and other civilizational areas of the ancient world are being repeated today and offer cautionary tales for an era characterized by many hard to monitor emerging and converging technologies, that is, technocultural systems.

At the end of the day, it should be clear that progress is not easy to define, and that it is even harder to point to examples of progress that resist critical interrogation. How can we sustain the idea of progress in the face of the widespread ecological, environmental, and human destruction that has characterized the industrial age? The fact is that the destruction and danger we see all around us is integrally connected to the very things we use to mark the progress of humanity. For these reasons, we must be cautious when considering whether any of the sciences, engineering disciplines, or mathematics have contributed to or served as signposts of progress. Mathematics, like all systems of knowledge, does not exist in a vacuum. It is always connected to social institutions and under the control of the most powerful institutions in any given society. All of this may put too much of the onus on the sciences and technology when what we are dealing with is culture in general. Is it possible that cultures by their very natures inevitably destroy planets?

It should be clear from this brief introduction that the terms of our title, “mathematics,” “civilization,” and “progress” are all imbued with some level of ambivalence and uncertainty. It remains to be seen whether in the rest of this chapter we can find our way to greater certainty about the meaning and implications of these terms.

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Hersh, R. (1999), *What is Mathematics, Really?* New York: Oxford University Press. [Mathematics for the modern reader by a mathematician who appreciates the new sociology of mathematics developed by Bloor on the one hand and Restivo on the other.]

Horton, R. (1997), *Patterns of Thought in Africa and the West: Essays on Magic, Science and Religion,* Cambridge; Cambridge University Press. [The distinguished anthropologist and philosopher was at the center of debates that crystallized in the 1950s and 1960s concerning the nature of and relationship between “primitive” (read, primarily, African) and “advanced” (read “the West”) systems of knowledge and belief. In this collection of his essays, Horton addresses the debates by exploring African beliefs, rituals, and cosmologies in the context of open and closed systems thinking. His objective is to demonstrate the kinship between primitive and modern thought and between science and religion more generally. Among the most important provocations for this and related debates are studies of Zande (n. Azande) logic. Were they “pre-logical,” as some claimed (e.g., in my view arguably Levy-Bruhl) or perfectly coherent in context (as Evans-Pritchard argued)? In the early years of the science studies movement David Bloor took up this question and argued for a cultural relativism that provoked new interest in this topic. This literature is relevant to my topic but requires more direct attention than I can give it here.]

Kavolis, V. (1985), “Civilizational Analysis as a Sociology of Culture,” *Sociological Theory 85,* 31, Spring 1985, 29-38. [The comparative civilizational analysis of any phenomenon, Kavolis argues, requires that we spell out the structure of each level of sociocultural organization and the connections and resources linking all the levels that bear on the particular phenomenon in a particular civilization. We don’t require an elaborate analytical apparatus but it helps to carry an awareness of such an apparatus in the study of a particular problem in terms of the relations between general civilizational structures, processes, and issues. Civilizational analysis is about the distinctive varieties of and changes in human experience, and the universe of symbolic designs in social behavior.]

Kline, M. (1962), *Mathematics: A Cultural Approach,* Reading, Massachusetts: Addison-Wesley. [A good introduction, even where dated factually and ideologically, to mathematics in relation to the humanities and the search for truth.]

Kramer, E. E. (1970), *The Nature and Growth of Modern Mathematics.* New York: Hawthorne Books. [Kramer discusses the lives and contributions of prominent mathematicians from Pythagoras and Newton to the modern period. Mathematical concepts such as binary operations, point-set topology, post-relativity geometries, optimization and decision processes ergodic theorems epsilon-delta arithmetization, and integral equations are discussed with admirable clarity.]

Lenski, G. (1974), *Human Societies.* New York: McGraw-Hill. [A cultural ecology of the evolution of societies. This is a major textbook contribution to macrosociology, now in its 11th edition from Paradigm Publishers in Boulder, Colorado, 2008 co-authored by P. Nolan.]

Mannheim, K. (1936), *Ideology and Utopia.* Eugene, Oregon: Harvest Publishers. [This is a complex treatise in the context of Mannheim’s legacy in relationship to European and America sociology. It is a founding document in the emergence of sociology but especially of the sociology of knowledge. In terms of its relationship to the substance of this essay, the significance of this book is that while situating knowledge in its social, cultural, and historical contexts, Mannheim exempts the formal science from his analysis. There cannot be, he claims in these pages, a sociology of 2+2=4. This idea carried into the emergence of the sociology of science in the 1930s and was not seriously challenged until the birth of the science studies movement in the late 1960s.]

McClain, E.G. (1976), *The Myth of Invariance: The Origins of the Gods, Mathematics and Music from the Rg Veda to Plato*. York Beach, ME: Nicolas-Hays, Inc. [McClain’s thesis is that in the ancient civilizations music was a science that bridged the gap between the everyday world and the divine. The invariance of music contrasted sharply with the variability of the everyday world. Music expresses and motivates mathematics. Music as a science is revealed by studying the mathematical relationships between musical notes. In this context music is properly understood as an expression of and the motive for mathematical study. The "key" to unlocking this science comes from a study of the mathematical relationships between various musical notes. The Pythagorean notion of number was in fact more general and more widespread and an essential feature of the very idea of culture and civilization. The presentation is controversial in some details but we are learning more and more about the centrality of musicality in humans and this book speaks to that idea.]

Merton, R.K. (1968), *Social Theory and Social Structure*, enlarged ed., New York: The Free Press. [Merton founded and dominated the sociology of science with his students from the late 1930s to the late 1960s. The Mertonian paradigm, consonant with Mannheim's sociology of knowledge, focused on the social system of science – for example, norms, values, the reward system, stratification in science, age-grading – but exempted scientific knowledge per se from sociological scrutiny.]

Merton, R. K. (1961), , "Singletons and Multiples in Scientific Discovery: a Chapter in the Sociology of Science," *Proceedings of the American Philosophical Society,* 105: 470–86. Reprinted in R. K. Merton, *The Sociology of Science: Theoretical and Empirical Investigations*, Chicago, University of Chicago Press, 1973, pp. 343–70.

Merton, R. K. (1973), "Resistance to the Systematic Study of Multiple Discoveries in Science," *European Journal of Sociology*, 4:237–82, 1963. Reprinted in R.K. Merton, *The Sociology of Science: Theoretical and Empirical Investigations*, Chicago, University of Chicago Press , pp. 371–82. [This and the following paper introduce Merton's idea that all discoveries are in principle multiples. These papers are milestones in the Merton corpus.]

Merton, R.K. (1958), "The Matthew Effect in Science," *Science*, 159(3810): 56-63, January 5, 1968. [Widely cited in the sociology of science, the Matthew effect, or the principle of accumulated advantage, refers to situations in which the rich get richer and the poor get poor. Merton named the effect after Matthew 25: 29 (NRSV Bible). In science as in other activities, power and economic or social capital can be leveraged to gain additional power and capital. This is one the causes of mis-eponymy along with historical amnesia and random and deliberate acts of misattribution.]

Mesquita,M., S. Restivo and U. D'Ambrosio (2011), *Asphalt Children and City Streets: A Life, a City, and a Case Study of History, Culture, and Ethnomathematics in São Paulo*, Rotterdam: Sense Publishers. [An innovative plural voiced ethnography and history of street children that attends to how mathematics plays into their survival strategies.]

Morris-Suzuki, T. (1994), *The Technological Transformation of Japan: From the Seventeenth to the Twenty-first Century,* Cambridge: Cambridge University Press. [Japan did not miraculously leap into the technological forefront of twentieth century societies. Its rise to superpower status, as this book makes clear, is rooted in its history. This is the first general English language history of technology in modern Japan. One of the significant features of this book is its consideration of the social costs of rapid changes in technology.]

Nasr, S. H. (2007), *Science and Civilization in Islam*. Chicago: Kazi Publications. [The first one volume English language book on Islamic science from the Muslim perspective. Hossein explains the place of science in Muslim culture as he unfolds its content and spirit.]

Needham, J. (1959), *Science and Civilization in China. Volume III: Mathematics and the Sciences of the Heavens and the Earth.* Cambridge: Cambridge University Press. [One of several volumes in the monumental study that uncovered the hidden history of science and technology in China. Needham documented that not only did China have a history in science and technology, it was the leading civilizational center of science and technology in the world between the early Christian period in Palestine and 1500 CE. The explanatory framework is Marxist cultural ecology.]

Neugebauer, O. (1952), *The Exact Sciences in Antiquity.* Princeton: Princeton University Press. [It took a long time for historians of science to overcome the ideology of the once and always Greek miracle. Neugebauer contributes to the demise of this myth in this non-technical discussion of the influence of Egyptian and Babylonian mathematics and astronomy on the Hellenistic world. An early look into the sophistication of ancient Babylonian mathematics.]

Noë, A. (2010), *Out of Our Heads: Why You Are Not Your Brain, and Other Lessons from the Biology of Consciousness*, New York: Hill and Wang. [Makes an important contribution to getting away from classical ideas about the primacy of the brain in consciousness. His approach is radically social but in a strange way that makes biology, rather than sociology, the science of the social. But just because of this twisted logic, he furthers the interdisciplinary agenda of figuring out a non-reductionist way to link biology and society.]

Polster, B. and M. Ross, (2011), "Pythagoras's Theorem ain't Pythagoras's," *h t t p : / / education . theage . co m . au / cmspage . php ? I ntid = 147 & intversion = 79*, March 7, 2011.

Resnikoff, H.L. and R.O. Wells, Jr. (2011), *Mathematics and Civilization.* New York: Dover (paperback). [This book was originally published by Holt, Rinehart, and Winston in 1973 and later published by Dover with supplemental materials. Still a good general introduction to the reciprocal relations between mathematics and human culture with an emphasis on the technical mathematics. No great demands are made on the mathematical aptitudes of readers, and the more sophisticated reader will find some of the treatment, especially on the calculus, technically deficient.]

Restivo, S. (1979), "Joseph Needham and the Comparative Sociology of Chinese and Modern Science: A Critical Perspective," pp. 25-51 in R.A. Jones and H. Kuklick (eds.), Research in the Sociology *of Knowledge, Sciences, and Art*, Vol. II. JAI Press, Greenwich. [The most extensive journal length critical survey of Needham's monumental history. Supports and extends his sociological materialism based on a human ecological perspective.]

Restivo, S. (1981), "Mathematics and the Limits of Sociology of Knowledge," *Social Science Information*, V. 20, 4/5: 679-701. [The new sociology of science associated with the science studies movement that emerged in the late 1960s challenged the status of mathematics as the arbiter of the limits of the sociology of science and knowledge. This is one of the early examples of the challenge by one of the founders of the modern sociology of mathematics.]

Restivo, S. (2001/1992), *Mathematics in Society and History*. New York. [This is the first book devoted completely to the sociology of mathematics as a subfield of sociology. The author reviews precursors, Spengler's thesis on numbers and culture, mathematical traditions in different civilizations, and develops an original conception of the sociology of pure mathematics.]

Restivo, S. (1993), "The Social Life of Mathematics," pp. 247-278 in S. Restivo, J.P. van Bendegem, and R. Fischer (eds.), *Math Worlds: Philosophical and Social Studies of Mathematics and Mathematics Education*, Albany, NY: SUNY Press. [A collection of essays, including a set of papers by German scholars previously unavailable in English, focused on recent developments in the study and teaching of mathematics. The authors are all guided by the idea that mathematical knowledge must be grounded in and reflect the realities of mathematical practice.]

Restivo, S. (1994), *Science, Society, and Values: Toward a Sociology of Objectivity*. Bethlehem PA: Lehigh University Press. [This book introduces Restivo's main contributions to the sociology of science between 1966 and the early 1990s. Based on his work in the ethnography of science, the history of science in China and the West, his social problems approach to understanding modern science, and other contributions, he develops a sociological perspective on objectivity.]

Restivo, S. (2007), "Mathematics," Monza: Polimetrica. June, 2007 The Language of Science (ISSN 1971-1352). [An overview of the sociology of mathematics written for an online encyclopedia.]

Restivo, S. (2011), *Red, Black, and Objective: Science, Sociology, and Anarchism,* Surrey: Ashgate Publishers. [This book explores the implications of the science studies movement for science and society in the context of an anarchist tradition. The particular tradition the author has in mind here makes anarchism one of the sociological sciences. Here he follows Peter Kropotkin. The book is grounded in the empirical studies carried out over the last forty years by researchers in science studies (and more broadly science and technology studies). The author's perspective is at once empirical, normative, and policy-oriented.]

Restivo, S. and H. Karp (1974), "Ecological Factors in the Emergence of Modern Science," pp. 123-142 in S. Restivo and C.K. Vanderpool (eds.), *Comparative Studies in Science and Society.* Columbus, OH: C. Merrill. [The authors theoretically link organizational and institutional structures to their ecological contexts and apply this theory to the problem of why modern science emerged in the West and not in China.]

Restivo, S. and R. Collins (1982), "Mathematics and Civilization," *The Centennial Review* Vol. XXVI, No. 3 (Summer 1982), pp. 277-301. [An introduction to the comparative historical sociology of mathematics that modifies Spengler's ideas on numbers and culture in the context of sociological theory.]

Restivo, S. and J. Croissant (2008), "Social Constructionism in Science and Technology Studies," pp. 213-229 in J.A. Holstein & J.F. Gubrium, eds., *Handbook of Constructionist Research*, New York: Guilford. [The authors aim is to clarify the widespread misconceptions, misapplications, and misconstruals of this term which they identify as the fundamental theorem of sociology. They are at pains to argue that the term does not imply or entail any form of relativism; it is compatible with a realistic sociology that recognizes objectivity and truth as real. They are real however in an institutional sense. This view, as Durkheim pointed out more than one hundred years ago, is consistent with the idea that there is a reality "outside of us," but we do not have access to a *ding an sich*.]

Reyna, S.P. (2007), *Connections: Brain, Mind and Culture in Social Anthropology,* New York: Taylor and Francis. [Reyna returns to the Boasian (Franz Boas) roots of anthropology to creatively construct a new paradigm for connecting the biological and cultural domains without submitting to the Sirens of reductionism.]

Rosental, C. (2008), *Weaving Self-Evidence: A Sociology of Logic,* Princeton: Princeton University Press. [Rosental traces the history of a theorem in the foundations of fuzzy logic to demonstrate the inherently social nature of logic. He describes the process by which logical propositions are produced, disseminated, and established as truths.]

Roszak, T. (1995), *The Making of a Counter-Culture: Reflections on the Technocratic Society and its Youthful Opposition, with a new introduction,* Berkeley; University of California Press (orig. publ. 1969). [Published in the middle of the 1960s sociocultural revolution, this book spoke directly to those who would become "the children of the 60s" while simultaneously bewildering their parents. Roszak coined the term "counter-culture" and damned the technocracy that was at the heart of the problems the protesters were angry about. A literate effort to explain the disaffection of young people and the young at heart during this tumultuous period.]

Sahlins, M. and E. Service, eds. (1960), *Evolution and Culture,* Ann Arbor: University of Michigan Press (co-authored by T.G. Harding, D. Kaplan, M.D. Sahlins, and E.R. Service). [In the hands of these authors, evolutionary anthropology becomes a predictive tool that can be applied to theorizing the future of human societies. Current events on the world scene, including the political and economic rise of China and the troubled status of the United States would not have surprised these anthropologists. This is a classic and still relevant contribution to our understanding of culture and cultural change writ large.]

Schechter, Eric (2005), *Classical and Nonclassical Logics: An Introduction to the Mathematics of Propositions*, Princeton: Princeton University Press. [Classical logic--the logic crystallized in the early twentieth century by Frege, Russell, and others--is computationally the simplest of the major logics, and adequate for the needs of most mathematicians. But it is just one of the many kinds of reasoning in everyday thought. Schechter introduces classical logic alongside constructive, relevant, comparative, and other nonclassical logics. This is the first textbook to make this subject accessible to beginners.]

Sedlacek, T. (2011), *Economics of Good and Evil,* New York, Oxford University Press. [Sedlacek understands economics as a social, cultural, and historical phenomenon. It is a product of our civilization not a pure, value free science. In this sense, Sedlacek lines up with contemporary students of the sociology of science and mathematics. Economics, at the end of the day, is about "good and evil." In viewing economics as a moral enterprise he reminds us that the author of The Wealth of Nations, Adam Smith, is also the author of The Theory of Moral Sentiments.]

Selin, H. (ed.) (2001), *Mathematics Across Cultures,* New York: Springer. [A survey of Islamic, Chinese, Native American, Aboriginal Australian, Inca, Egyptian, and African mathematics, with essays on rationality, logic and mathematics. Shows how science and math practice is situated in its cultural context.]

Smith, D. E. (1958), *History of Mathematics.* New York: Dover. [A classical two volume introduction to the history of mathematics from ancient Egypt to modern times, very lucid survey including biographical notes and chronology.]

Sohn-Rethel, A. (1975), "Science as Alienated Consciousness." *Radical Science Journal,* Nos. 2/3: 65-101. [Working from inside a sophisticated Marxist paradigm, Sohn-Rethel discusses science as an alienated and alienating form of knowledge and knowledge building. He is known for linking Kant and Marx in his work on epistemology.]

Spengler, O. (1926), *The Decline of the West.* New York: A. Knopf. [Spengler's readers append adjectives like "audacious," "profound," "magnificent," "exciting," and "dazzlingly" to this book which flaws and all is one of the most amazing efforts in human history to capture humanity's march through space and time. It's relevance for the topic at hand is that Spengler's theses are grounded in a radically cultural understanding of the relationship between culture and mathematics.]

Stigler, Stephen S. (1980), ‘Stigler’s Law of Eponymy’, *Trans. N. Y. Acad. Sci.* (2) 39, 147–157.

Leon Stover (1974), *The Cultural Ecology of Chinese Civilization.* New York: Signet. [An innovative interpretation of peasants and elites in what Stover refers to as a “once and always Bronze Age culture.” A paradigm for understanding the nature of Chinese culture and the continuities between the age of the dynasties and the modern era from Sun Yat Sen and Chiang Kai Shek to Maoism and modernism.]

Struik, D. (1967), *A Concise History of Mathematics*,. New York: Dover Publications. [The fourth revised edition of this classic was published by Dover in 1987. Struik, a Dutch mathematician and Marxist theory, was a professor at MIT for most of his professional career and to my knowledge the first person to identify the sociology of mathematics as a field of study. This book is a very readable introduction to the history of mathematics, concise as advertised but with a lot of substance poured into the books roughly 230 pages. Struik covers the period from the ancient world to the early twentieth century.]

Sugimoto, Masayoshi and D.L. Swain, (1978), *Science & Culture in Traditional Japan.* Cambridge MA: MIT Press. [Between 600 and 1854 CE, Japan was impacted by a first and second Chinese cultural wave and the first Western Cultural Wave in the nineteenth century. The authors focus on how these cultural waves set the stage for the development of an indigenous science and technology.]

Verran, Helen. (1992), *Science and an African Logic.* Chicago: The University of Chicago Press. [An empirical study that supports the idea of mathematics and logics as culturally situated. Quantity is not always absolute (as in 2=2=4) but sometimes relational, as in Yoruba. Verran’s experience and research as a teacher in Nigeria is the basis for this important contribution to the sociology of mathematics and ethnomathematics.]

Wigner, E. (1960), “On the Unreasonable Effectiveness of Mathematics in the Natural Sciences,” *Communications on Pure and Applied Mathematics* , 13: 1–14. [A classic paper in defense of the idea that the mathematical structure of a physics theory often points the way to further advances in that theory and even to empirical predictions. It is basically an argument in support of “pure” mathematics.]

Wright, Ronald (2004), *A Short History of Progress*, Philadelphia: Da Capo Press. [Looking over the long history of humanity, Wright sees not the unfolding a linear evolution of progress but rather a series of “progress traps.” He reveals a history of “progress and disasters” that should serve as a warning to humanity and especially to those people who assume that progress is an inevitable and positive manifestation of human exceptionalism.]

Zaslavsky, C. (1999), *Africa Counts: Number and Pattern in African Cultures*, 3rd ed. Chicago: Lawrence Hill Books (orig. publ. 1973 by Prindle, Weber, and Schmidt). [This is one of the earliest efforts to document the experience of mathematics in a non-Western culture and to view it in a positive civilizational perspective.]

Zeleza, Paul Tiyambe and Ibulaimu Kakoma (2005), *Science and Technology in A*frica. Trenton NJ: Africa World Press. [The authors deal with scientific and technology literacy, production, and consumption in modern Africa. The focus is on developments in information technology and biotechnology in the context of The Knowledge Society in a globalizing context.]

Biographical Sketch

Dr. Sal Restivo is widely recognized as one of the founders of the field of Science and Technology Studies (STS), a pioneer in ethnographic studies of science, a founder of the modern sociology of mathematics, a contributor to public sociology and a prominent figure in the radical science movement of the 1960s. Dr. Restivo was Professor of Sociology, Science Studies, and Information Technology in the Department of Science and Technology Studies, at Rensselaer Polytechnic Institute in Troy, New York until his retirement in June 2012. He is Special Lecture Professor in STS at Northeastern University in Shenyang, China; a former Special Professor of Mathematics, Education, and Society at Nottingham University in Great Britain; and a former Hixon/Riggs Professor of Science, Technology, and Society at Harvey Mudd College. In 2012, he was a Senior Postdoctoral Fellow at the University of Ghent in Belgium. He is a founding member (1975) of and a former president (1994/95) of the Society for Social Studies of Science.