RIVER NETWORKS

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Contents

Summary

In this chapter we review a particular case of graphs composed by river networks. These geophysical graphs are a particular kind of network. Given its physical properties, they do not have "loops" (technically "cycles") and are therefore better designed by the term "trees". Knowledge of these structures is nowadays extremely important especially for the increasing value that freshwater resources are assuming in our society. As graphs they are particularly important as a paradigmatic example of transportation graphs and for the possibility to define energy functionals to describe their evolution.

1. Introduction

What does an oak leaf have in common with a river?

The answer is rather obvious: they both hinge upon a tree-like structure a situation common to the skeleton of a variety of complex networks. But why is this so?

A glance to Figure.1 provides an intuitive answer. Suppose we are faced with the problem of sending supply of water from a set of points (the black dots in the picture) lying within an extended area to a well localized final destination (the red dot in the lower left corner). Different strategies can be devised to this aim. In the first (see (A)) each point sends its share to a selected nearest-neighbor according to a well defined pattern (known as Hamiltonian walk in Statistical Physics). This is a globally efficient system, as the shipment is carried out according to an ordered path which does not allow for superposition that would lower the efficiency of the system. On the other hand, it is also locally inefficient as, for instance, water from point immediately north of the final destination could flow along a direct much shorter southward path without making a long detour.

Consider now the opposite strategy (as sketched in (B)) where each point acts individually by choosing the shortest route. This makes the system locally efficient but globally highly costly as the total distance traveled by the entire system is much larger than in the previous case, due to the many superpositions (dashed as well as solid lines have been used in the figure (B) to enhance clarity). A reasonable trade-off between the two opposite strategies is provided by the tree-like structure shown in Figure (C) where each point attempts to find the shortest path but repetition is avoided through a hierarchical construction, so that a global efficiency can be achieved. A quantitative mathematical proof of the above statement can be obtained by showing that a tree-like structure is the one minimizing both the average individual path and the total traveled distance. Later on within this chapter we shall briefly touch upon these methods.

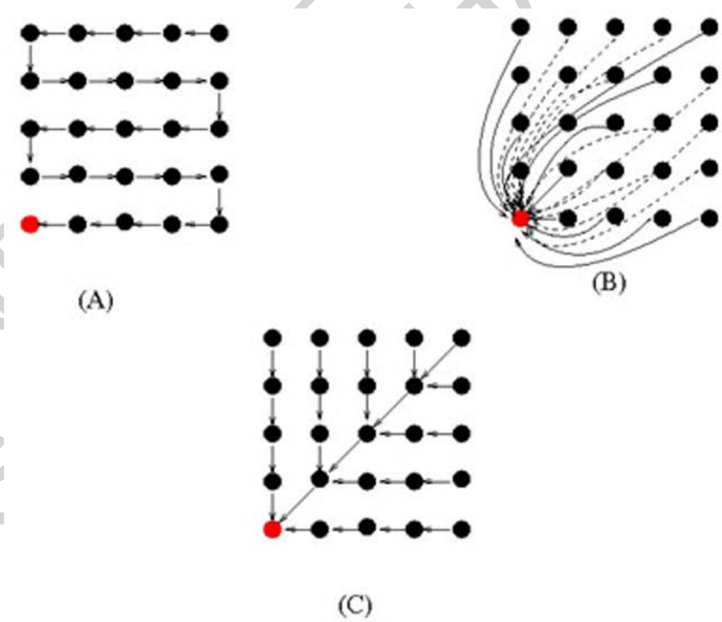

Figure 1. Three examples of structures with a different efficiency in the transfer of water...

A tree-like structure is then an efficient (the most efficient in fact!) structure to convey matter (or energy) from an extended source (the black points) to a single outlet (the red point).

This simple mechanism is particularly striking in the case of river networks in a drainage basin due to the following physical mechanism originatine the final structure: Gravity drives water downhill along the steepest path; as water flows, landscape is eroded and its morphology then evolves until a very unstable situation characterized by large local slope is reached; a sedimentation process then occurs reporting the local slope to a smaller value (in general different from the original one) so that water can again flow along a new path and process iterates until a steady state is reached.

In this respect, a river network is a two-dimensional projection of the three-dimensional tree-like structure given by computing the steepest descent on a rugged surface. The above mechanism has far reaching consequences which are of paramount importance for geophysical science as will be further discussed below. The breadth of knowledge involved in the study of river networks, however, far exceeds hydrology touching upon mathematical aspect of graph theory (mentioned elsewhere in this book) and theoretical aspects characteristic of statistical physics such as scaling theories and critical exponents. While the former has been more or less always been present in the subject, the latter has crept into the field only very recently when multidisciplinary teams formed by hydrogeologists and statistical physicists have teamed up to tackle this very fascinating topic [see e.g. Rodriguez-Iturbe&Rinaldo 1997].

This chapter was written with the explicit objective of avoiding technicalities and to provide an intuitive picture of this topic which could be of some interest to non-experts. As such, it hinges mostly on personal work carried out over the past and it does not have the pretency of being an exhaustive review of of the field.

2. Drainage Basins, River Networks and Digital Elevation Map (DEM)

River basins are the fundamental natural system hydrological phenomena and a major topic in the more general field of geomorphology. Transport through river channels has also major consequences for public safety, management of water resources, and environmental sustainability.

We begin this Section by remarking that a river system may be divided in three distinct regions called the production zone, the transportation (or transfer) zone and the delivery (or deposition) zone. Properties of these three regions are different and require different approaches. What we will be discussing in this chapter is the *production zone*, and this will be identified hereafter as a river basin.

A drainage basin of a river is defined as the part of the territory where all the rainfall is collected by the same river and transferred to one or more outlets. A river network is composed of the collection of all the paths formed by every tributary of the main river in its drainage basin. Clearly, the formed paths of the network strongly depend upon the morphology of the landscape on which the river flows; this in turn is influenced by various external agents (rain, sedimentation, erosional process etc etc) and by the flow of the river itself, thus creating a feedback mechanism in which the direction of the flow is dictated by the landscape morphology but the former also affects the latter in a nontrivial way.

Hydrogeologists have devised a very clever technique from satellite images to represent river networks, called digital elevation map (DEM) that allows us to determine the average height of areas (pixels) of the order 10^{-2} km^2. The method is based on the phenomenological evidence that water falling from one site follows the steepest descent between those available. From one satellite photograph the zone is divided into portions whose linear size is about 20 - 30 m and each of these regions is treated as a single point. Then an average height h is assigned to these coarse-grained sites (from a satellite it is easy to compute the elevation field of a region). The network is built using the elevation field and starting from the highest mountain according to the steepest descent rule as shown in Figure 2. Essentially then, a river network is a two-dimensional representation of the three-dimensional landscape morphology, not necessarily (although frequently) related with the actual river flowing on that landscape in reality.

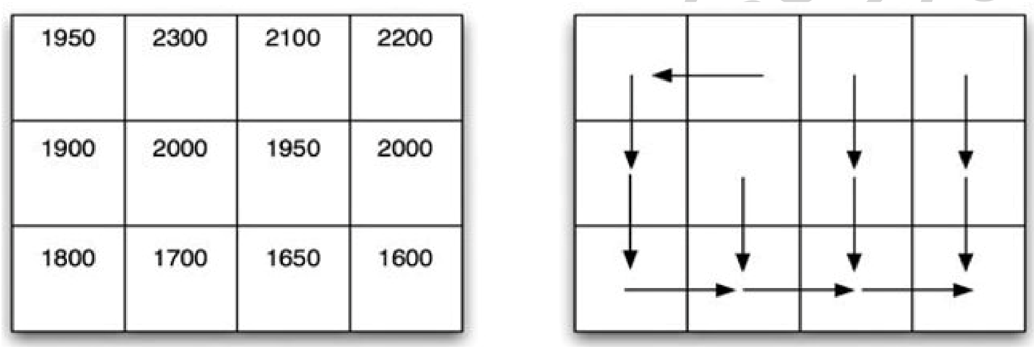

Figure 2. A cartoon of the Digital Elevation Map procedure and the resulting river flow.

By DEM techniques, systems with sizes varying from hundreds of meters to thousands of kilometers can be observed, thereby spanning more than four orders of magnitude. Below few hundreds of meters local morphology obviously starts to play a very relevant role whereas above some thousands of kilometers this description is inadequate as phenomena characteristic of continental length scales set in. Yet, within this very large window and irrespective of the geographical locations, distinct river basins present a common tree-like structure with many common properties that we are going to describe next.

3. Scale Invariance and Fractals

Consider the river network shown in Figure 3 as obtained by a real river One immediately notices the main (i.e the longest) stream spanning the basin from one side to the other, all smaller streams being tributary to this main one.

As the thickness of each line in the Figure represents the total amount of water flowing along that stream, the main stream is obviously also the one transporting the largest amount of water. A closer look, on the other hand, identifies a set of smaller streams more or less of the same order of magnitude which can be reckoned as the main tributaries.

Each of them has its own drainage basins and could be regarded as the main stream of smaller basins, at a smaller length scale. This observation could be iterated many times until one reaches the smallest stream. The system is then self-similar in the sense that all parts statistically resembles identical one another within their own length scale: this is the main property of a fractal, so a river network does have fractal structure.

One possible way for making this feature more quantitatively explicit is the so-called Horton-Strahler ordering. The main idea behind this ordering scheme is very intuitive. Think about two small creeks roughly of equal size which at some point merge together. It is natural to rename the larger creek originating from there with a new name.

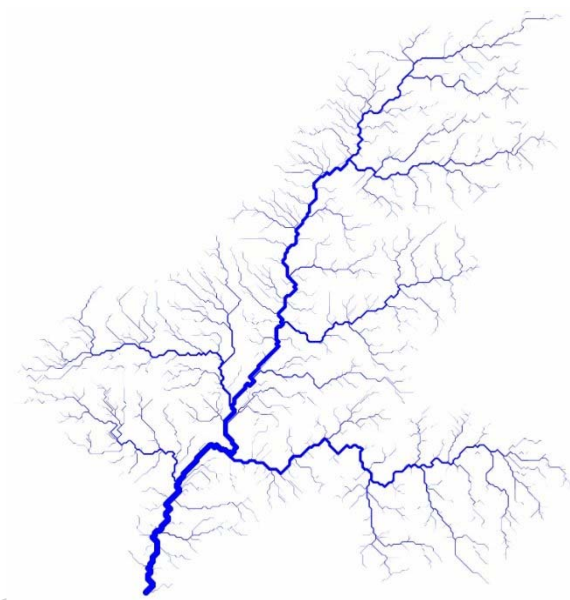

Figure 3. A real river network obtained from the DEM technique (courtesy of Riccardo Rigon).

On the other hand, after the confluence of the Nile with a small tributary, the downstream resulting river is effectively of the same Nile size, as the contribution of the small tributary can be considered as negligible, and it is again natural to maintain the Nile denomination.This scheme is patterned after what geographists have done for decades to denote the real rivers.

With reference to Figure 4 we define the following rules:

- Channels with no tributaries (that originate at a source), are defined to be first-order streams;
- When two streams of order i join, a stream of order $j+1$ is created;
- When two streams of different order join, the channel immediately downstream has the higher order of the two combining streams.

An example of this ordering scheme, called Horton-Strahler ordering in river basins geomorphology, is depicted in Figure 4 where the thickness of the line is proportional to the order of the stream.

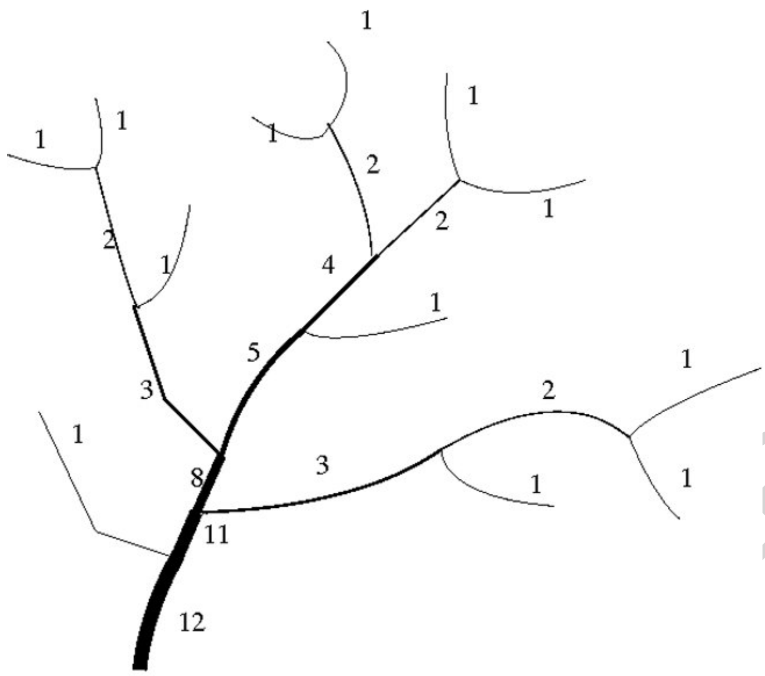

Figure 4. The structure of rivers according to the Horton-Strahler ordering

Define $N(i)$ the number of streams of order i and $L(i)$ the average length of a stream of order i, one introduce the branching and length ratios

$$\begin{cases} R_{\mathrm{B}} = \dfrac{N(i)}{N(i+1)} \\ R_{\mathrm{L}} = \dfrac{L(i+1)}{L(i)} \end{cases}$$

It has been observed (by Horton) that for a very large class of river basins, $3 \le R_{\mathrm{B}} \le 5$ and $1.5 \le R_{\mathrm{L}} \le 3.5$.

The self-similarity of basins and rivers can then be translated into a hierarchical structure of relative sizes among different tributaries as described by the above scheme, where different numerical indices (branching ratio R_{B}, length ratio R_{L} and other quantities) provide a quantitative measure of its self-similarity character. Remarkably, it turns out that for a very large range of river sizes, from few meters to hundred of kilometers, those indices are more or less identical one another, a clear indication of the self-similarity of the river basins.

They can also be computed exactly in a deterministic structure denoted as Peano river network (see Figure 5). With much more mathematical effort, they can also be estimated in a random tree-like structure known as spanning tree (for a construction of the Peano river network see Figure 6)

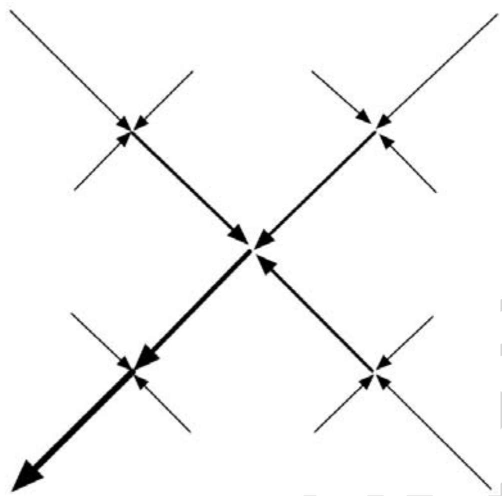

Figure 5. The Peano river network.

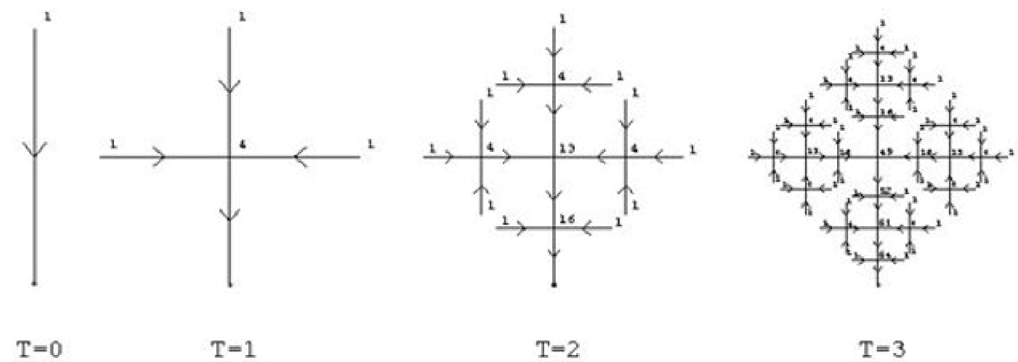

Figure 6. Different steps of construction of a Peano Basin

We can translate this bit of information still in another way. As usual in science, it proves convenient to identify the relevant length scales of the problem. It is apparent from the figure that the drainage basin has a very high aspect ratio, that is rather elongated. This means that there are two different length scales $L_{\|}$ and $L_{\perp}$ along the parallel and perpendicular direction, respectively, their product being approximately equal to the area *A* of the drainage basin, but with $L_{\|} > L_{\perp}$.

One of the fundamental laws known to the hydrogeologists is called Hack's law (see Fig 6) and it states that,

$$L_{\|} \sim A^{h},$$

where $h \approx 0.6$ for all river basins in the wide window range mentioned earlier. This "universality'" in the value of h is still another manifestation of the self-similarity of the system. Note that a value $h \approx 0.5$ would correspond to a symmetrical basin with the two length scales approximately equal, so $h > 0.5$ is a representation of the fact that $L_{\|} > L_{\perp}$ (as shown in Figure7). Very large basins ($L_{\|} > 10^4 \text{km}$) are observed to be rather more symmetric so that a value of h much closer to 0.5 is actually observed.

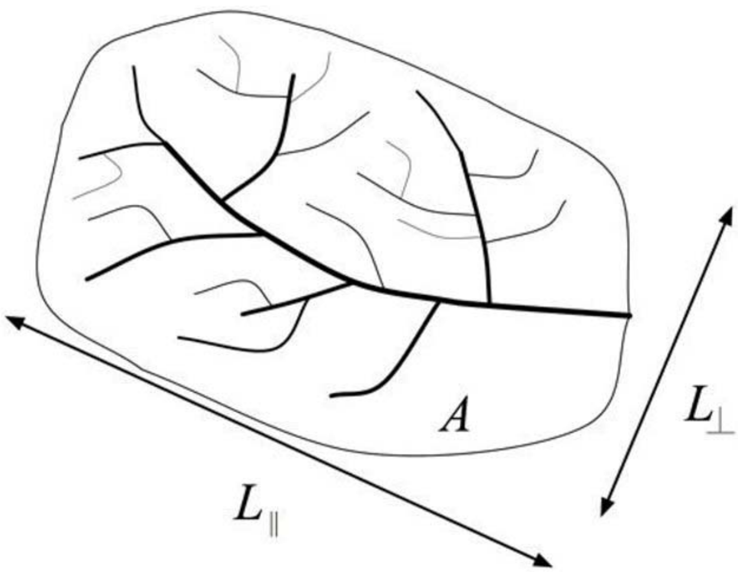

Figure 7. The various lengths one can define in a river basin.

Hack's law is an empirical law derived from field observations. Other similar laws have been devised by hydrologists over the years: they have proven to be extremely useful to rationalize theories aiming to explain the observed behavior on the basis of fundamental principles of engineering fluid and soil mechanics. However the universal behavior of the topological indices and laws, such as Hack's law, strongly suggests that the actual details of the local morphology and fluid properties should not play a fundamental role, the main observed features being independent of them.

This situation here is similar to what is found in statistical physics of non-equilibrium critical phenomena.

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Biographical Sketch

Achille Giacometti is currently Associate Professor of Condensed Matter Physics at the University Ca' Foscari of Venice.,He studied in Padua with Amos Maritan and Attilio Stella and then moved to Purdue University (USA) for his Ph.D. He then held post-doctoral positions at IFF-FZ in Julich (Germany) and at SISSA/ISAS in Trieste. He has worked on several topics in Statistical Physics including growth processes, Self-Organized Criticality and River Networks.