DESIGN AND TUNING OF FUZZY SYSTEMS

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Summary

Fuzzy Systems have attracted the interest in numerous real world scenarios because these approaches assume that we do not deal with exact measurements or 'pure' random values. Fuzzy Logic deals with partial membership to the set and the key concept is the degree of set membership. These mathematical models (fuzzy sets) are the cornerstone of the Fuzzy Systems. A Fuzzy System is a system of variables which are associated using fuzzy logic, and are described in an interpretable way using linguistic expressions. This chapter will describe different existing methods for designing and tuning of fuzzy rule-based (FRB) systems. The different methods will be analyzed taking into account the historical developments, the needs of the contemporary computing, communication, robotics and other applications that require such systems to be implemented.

In general, the methods and approaches can be divided into the FRB which designed by experts or autonomously by learning. However, the tuning is usually associated with the parameter learning and adaptation and is often considered as an optimization task aiming to produce an end result which deviates as less as possible from the desired or actual behavior which a FRB system aims to predict, control, classify or, simply, model.

“So far as the laws of mathematics refer to reality, they are not certain. And so far they are certain, they do not refer to reality.”  
Albert Einstein

“Everything is a matter of a degree”  
Bart Kosko
1. Introduction

In 1965, Lofti Zadeh’s presented his seminal work in which he used the term “fuzzy sets” for the first time. As he described in the paper “Fuzzy Sets”: more often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership… Yet, the fact remains that such imprecisely defined “classes” play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction”. (Zadeh, 1965)

With this idea in the background, he proposed the “fuzzy sets” as mathematical models of linguistic expressions which represent a “class” with a continuum of grades of membership.

Fuzzy Logic deals with partial membership to the set; it means that an entity could be in two or more sets at the same time but to different degrees. Thus, the key concept is the degree of set membership. These mathematical models (fuzzy sets) are the cornerstone of the Fuzzy Systems. A Fuzzy System is a system of variables which are associated using fuzzy logic. These systems are described in an interpretable way using linguistic expressions.

Fuzzy Systems have attracted the interest in numerous real world scenarios because most traditional approaches from classical statistics assume that we deal with exact measurements or ‘pure’ random values. The reality, however, is neither of the two extremes (neither ‘purely’ deterministic, nor ‘purely’ random).

In normal set theory (where \( S_i \) represents a certain set), an object (represented as \( x \) ) can either:

a) belong to the set (\( x \in S_i \))

b) not belong to the set (\( x \notin S_i \))

However, a real-world scenario does not usually have a precise measurement or clear cut boundaries. For this reason, in a fuzzy system, an object can partially belong to a certain fuzzy set. Thus, the belonging (membership) to a set needs to be described by a value. This value of membership to the set \( i \) is represented as \( \mu_i \), and is normalized such that \( \mu_i \) is in \([0,1]\). Also, it is required that the total membership to all sets of an object adds up to 1:

\[
\sum_{i=1}^{k} \mu_i = 1 \text{ where } k \text{ is the total number of sets.}
\]

Taking into account the previous aspects, Fuzzy logic is a powerful methodology of how to describe rules which have high generalization and summarization ability.

In order to clarify the proposed ideas, let us consider an example: Consumer buying behavior is influenced by several factors such as age and income. If we want to determine the influence of these factors on their buying behavior, we can create
different fuzzy rules. These rules link age and income with the possibility of a user of buying a specific product. The following rule is an example of this:

IF (Age is Young) AND (Income is High)

THEN (Purchase is High)

Figure 1 represents the membership function distribution of the different fuzzy sets that represent the age in a fuzzy system.

![Fig 1](image)

**Figure 1. Example of the fuzzy sets that representing the age.**

This representation is called complete, if for any value of the variable (e.g. age) there exists at least one fuzzy set that describes it. Thus, a fuzzy set is described by its membership function.

### 1.1. Types of Membership Functions

Although there are various types of membership functions, we could consider the following types as the most commonly used ones:

a) **Triangular:** The mathematical expression for the triangular membership function can be defined as:

\[
\text{triangle}(x : L, C, R) = \begin{cases} 
0 & x < L \\
\frac{x-L}{C-L} & L \leq x \leq C \\
\frac{R-x}{R-C} & C \leq x \leq R \\
0 & x > R 
\end{cases}
\]

where \( L \) denotes ‘left’; \( R \) denotes right and \( C \) denotes centre.

In this case, the precise appearance of this function depends on the values of 3 parameters: \( L \), \( C \) and \( R \). This type of membership function is shown in Figure 2.
b) **Trapezoidal:** The trapezoidal membership function is:

\[
\text{trapezoid}(x : L, \overline{L}, R, \overline{R}) = \begin{cases} 
0 & x < L \\
\frac{x - L}{\overline{L} - L} & L \leq x \leq \overline{L} \\
1 & \overline{L} \leq x \leq R \\
\frac{R - x}{R - \overline{R}} & \overline{R} \leq x \leq R \\
0 & x > R
\end{cases}
\]

where \( L \) denotes left lower boundary; \( \overline{L} \) denotes left upper boundary, \( R \) denotes right lower boundary, and \( \overline{R} \) denotes right upper boundary.

These values are graphically represented in the following figure.

Figure 3. Example of Trapezoidal Membership Function.

c) **Gaussian:** The Gaussian membership function is one of the more widely used functions and it can be defined using exponential:
Gaussian \( (x : x^*, \sigma) = \exp \left( -\frac{(x - x^*)^2}{\sigma^2} \right) \)

where the focal point \( (x^*) \) represents the centre, and spread of the Gaussian is defined by \( \sigma \). As it can be seen in Figure 4, the values of this type of membership function are smooth and non-zero at all points.

**Figure 4. Example of Gaussian Membership Function.**

d) **Bell Shaped:** The Bell Shaped membership function has symmetrical shape and it is specified by three parameters using the following expression:

\[
\text{Bell Shaped}(x : A, B, C) = \frac{1}{1 + \left| \frac{x - C}{A} \right|^{2B}}
\]

where the parameter \( C \) represents the center of the curve, and \( A \) shows the width of the curve. This function is represented in Figure 5.

**Figure 5. Example of Bell Shaped Membership Function.**

e) **Sigmoidal.** This type of membership function can be open to the right or to the left and it is given by the following expression:
Sigmoidal \( x : A, C = \frac{1}{1 + e^{-A(x-C)}} \)

where \( C \) represents the distance from the origin and \( A \) determines steepness of the function. Depending on the sign of \( A \) the function is inherently open to the right (if \( A \) is positive) or to the left (if \( A \) is negative).

Figure 6. Example of Sigmoidal Membership Function.

1.2. Fuzzy Rule Based Systems

A Fuzzy Rule Based (FRB) System involves some degree of ‘computational intelligence’ since they are able to learn, approximate reasoning and support decisions. The interest in these systems has increased during the last decades since they provide a flexible and robust methodology to deal with noisy and incomplete data. Besides, the transparent and human interpretable rule-based structure of a FRB system is expressive enough to represent imprecise qualitative knowledge.

The most important phase in the design of a FRB system is the creation of its rules. In the early approaches (during 1970-1980s), these rules were created using the knowledge and experience of a human expert. These experts were able to create a system which consists of several rules (IF-THEN rules). In this sense, they also use a method called Group Decision Making (GDM) which consists of multiple experts interacting to reach a (common) decision. Thus, different experts process the same input and then a group compromises the decision (the best alternative) is formulated by considering the different preferences of the experts.

One of the pioneering FRB systems which used expert knowledge was DENDRAL (Heller, et al., 1974) which was developed to deduce the molecular structure of organic compounds from knowledge about fragments into which the compound had been broken. This system is still one of the most promising successes in Artificial Intelligence. Other pioneers in this field are the DARC system (Heller, et al., 1974) and the CHEMICS systems (Abe, et al., 1981). However, the creation of a FRB system by an expert is a difficult task since the number of parameters that define the rules of a system is usually very high, the relation between these parameters are not usually intuitive, the consequence of the different parameters and its values is usually difficult to detect. For this reason, the expert knowledge has been usually used in conjunction with the...
information that can be extracted from the input-output (I/O) data. Specially, fuzzy clustering has been used widely for this task since obtaining different clusters, different rules can be represented. This trend started in 1980-1990s. However, the structure of the FRB systems created in this way was still usually fixed.

In order to solve this problem and create structures that are able to adapt to the changes of the data, a fast, recursive, incremental, memory efficient and adaptive algorithm has been proposed by using Evolving Fuzzy Systems (EFS) which will be detailed later on. This trend started around year 2000 and is still under intensive development by the researchers worldwide.

2. Fuzzy Systems

The main components of a FRB system are: the rule base and an associated inference process. The structure of a fuzzy system consists of a set of fuzzy rules (linguistically expressed) which are composed of antecedent (IF) and consequent (THEN) parts. The antecedent part consists of a number of fuzzy sets that are linked with fuzzy logic operators such as ‘AND’ (conjunction), ‘OR’ (disjunction), ‘NOT’ (negation) and several families of operators that have been introduced in the fuzzy set theory for these logical connectives (such as min and max operators) (Klir & Folger, 1988). The number of fuzzy rules and inputs is part of the structure of the system.

During the last decade of the previous century there was an increase of applications of fuzzy logic-based systems mainly due to the introduction of fuzzy logic controllers (FLC) by Ebrahim Mandani in 1975 (Mandani & Assilian, 1975), the introduction of the fuzzily blended linear systems construct called Takagi-Sugeno (TS) fuzzy systems in 1985 (Tomohiro & Michio, 1985), the theoretical proof that FRB systems are universal approximators(Wang & Mendel, 1992) and recently, the AnYa type FRB (Angelov & Yager, 2011).

2.1. FRB Systems Types

Depending on the structure of the IF-THEN rules, FRB systems can be classified into the following main types:

1) Mandani-type (Mamdani, 1977),
2) Takagi-Sugeno-type (Tomohiro & Michio, 1985) and

2.1.1. Mamdani Type

In this type of systems (also called linguistic systems), the antecedent (IF-part of the rule) and the consequent (THEN-part of the rule) are fuzzy propositions. Their rules are of the following form:

\[ Rule_i : \text{IF } (x_1 \text{ is } A_1') \text{ AND } (x_2 \text{ is } A_2') \text{ AND } ... \text{ AND } (x_n \text{ is } A_n') \text{ THEN } (y \text{ is } B), \quad k = 1, 2, \ldots, R. \]
where \( x_i, i = 1, 2, \ldots, n \) is the input variable, \( A_j^i, j = 1, 2, \ldots, n \) and \( B \) are linguistic terms (such as Small, Large, High, Low, etc.) represented by fuzzy sets, \( y \) is the output associated with the given rule, and \( R \) is the number of rules in the model.

This kind of linguistic fuzzy models are useful for representing qualitative knowledge.

For example, the following fuzzy rule could be part of the rules of a Mamdani type fuzzy model:

\[
\text{Rule}_i : \text{IF (Car Weight is High) AND (Volume of Cylinders is High)} \\
\text{THEN (Miles Per Gallon is Low)}
\]

where High, Low, etc. are fuzzy sets defined by their membership functions.

2.1.2. Takagi-Sugeno Type

In this type of systems (also called TS systems), the antecedent is defined in the same way as in the Mandani type, while the consequent is defined as a function of the input variables:

\[
\text{Rule}_i : \text{IF (} x_1 \text{ is } A_1^i \text{) AND (} x_2 \text{ is } A_2^i \text{) AND … AND (} x_n \text{ is } A_n^i \text{)} \\
\text{THEN (} y = a_0 + a_1 x_1 + \ldots + a_n x_n \text{), } i = 1, 2, \ldots R.
\]

where \( x_i \), \( A_j^i \), and \( y \) are input variables, linguistic terms, and output variable associated with the rule respectively, and \( a_0, a_1, \ldots, a_n \) are consequence parameters. This model combines the linguistic description with standard functional regression: the antecedents describe fuzzy regions in the input space in which the consequent functions are valid.

The following fuzzy rule is an example of a rule that could be part of a Takagi Sugeno (TS) type fuzzy model:

\[
\text{Rule}_i : \text{IF (Car Weight is High) AND (Volume of Cylinders is High)} \\
\text{THEN (MPG = a + (b * Car Weight) + (c * Volume of Cylinders))}
\]

where MPG denotes miles per gallon.

2.1.3. AnYa Type

This type of systems (called Granular Decomposition with Input Vector Membership but it will be referred by AnYa) was recently introduced in (Angelov and Yager, 2010).

In this novel method, the system design process is significantly simplified and it will be very detailed in the next section.
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performance evaluation of the developed Sequential Adaptive Fuzzy Inference Systems (SAIFS) algorithm for classification problems.


**Biographical Sketches**

**Dr Plamen Angelov** is a Reader in Computational Intelligence and coordinator of the Intelligent Systems Research at Infolab21, Lancaster University, UK. He is a Senior Member of the IEEE and Chair of the Technical Committee on Evolving Intelligent Systems, Systems, Man and Cybernetics Society, IEEE. He is also a member of the UK Autonomous Systems National TC, of the Autonomous Systems Study Group, NorthWest Science Council, UK and of the Autonomous Systems Network of the Society of British Aerospace Companies. He is also a founding member of the Centre of Excellence in CyberSecurity officially recognized by UK GCHQ for the period 2012-2017. He authored or co-authored over 160 peer reviewed publications in leading journals (50+) peer-reviewed conference proceedings, a patent, a research monograph, a number of edited books, and has an active
research portfolio in the area of computational intelligence and autonomous system modelling, identification, and machine learning. He has internationally recognised pioneering results into on-line and evolving methodologies and algorithms for knowledge extraction in the form of human-intelligible fuzzy rule-based systems and autonomous machine learning.

Dr. Angelov is also a very active researcher leading numerous projects (over a dozen for the last five-six years) funded by UK and EU research councils, industry, HM Government, including UK Ministry of Defence (total funding in order of tens of millions pounds with well over £1M for his group alone). His research contributes to the competitiveness of the industry, defence and quality of life and was recognised by ‘The Engineer Innovation and Technology 2008 Award in two categories: i) Aerospace and Defence and ii) The Special Award.

Dr. Angelov is also the founding Editor-in-Chief of the Springer’s journal on Evolving Systems and serves as an Associate Editor of several other international journals, including IEEE Transactions on Systems, Man and Cybernetics, on Fuzzy Systems, Elsevier’s Fuzzy Sets and Systems, Journal on Automation, Mobile Robotics and Intelligent Systems etc. He also Chairs annual conferences organised by IEEE acts as Visiting Professor (in Brazil, Germany, Spain) regularly gives invited and plenary talks at leading companies and universities More information can be found at his web site www.lancs.ac.uk/staff/angelov.

José Antonio Iglesias is a teaching assistant and researcher at Carlos III University of Madrid, Spain. He has published more than 35 journals, conference papers and book chapters and he is committee member of several international conferences. He takes part in several national and European research projects. He is member of the Fuzzy Systems Technical Committee (IEEE/CIS) and Chair of the Education Activities Task Force. He is also member of the Standards Committee of the computational Intelligence Society (IEEE) and the University Curricula. He is Editorial Board of the Evolving Systems Journal and publicity Chair of Evolving and Adaptive Systems Conferences. He is also member of RoboCup Spanish National Committee. His research interests include agent modeling, plan recognition, sequence learning, machine learning, and evolving fuzzy systems. More information can be found at his web site: www.caos.inf.uc3m.es/~jiglesia/