

DESIGN IN THE FREQUENCY DOMAIN

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Summary

In the material to follow, two design measures are chosen: degree of stability (as measured by phase margin) and steady state accuracy. It is observed that the sign of the phase margin is an unambiguous indicator of stability (assuming proper care is taken in calculating the sign of the phase margin) while the sign of gain margin is ambiguous for that purpose. Six compensators are analyzed, design rules are derived and design is

applied to a common problem. PI and lag compensators improve steady state accuracy (with a modest loss of stability). PD and lead compensators improved stability (but steady state accuracy is not as good as with PI and lag compensators). PID and lag-lead compensators improved both stability and steady state accuracy, as compared to the system having only the plant and unity feedback.

1. Introduction

Control system science is indebted to the work of pioneers such as Nyquist and Bode who were initially interested in creating regenerative feedback amplifiers in the 1930s and 1940s, largely for application in the communications industry. Nyquist's elegant use of complex variables and Bode's intuitive use of frequency response graphics formed the basis for design methods that remain in common use today.

For the purposes of this section, a stable system is defined as one in which all closed-loop poles (roots of the controlled characteristic polynomial) are in the open left-half plane. The natural response of such a system will tend toward zero as time increases and a bounded input will produce a bounded output. Conversely, an unstable system is defined as having at least one closed-loop pole in the open right half plane and/or repeated closed-loop poles along the imaginary (vertical) axis. Such a system possesses an unbounded natural response. A bounded non-zero input produces an unbounded output, unless all closed-loop right half plane poles are cancelled by equal closed-loop right half plane zeros. For purposes of this section, in that latter case, the system shall still be considered to be unstable. The term marginally stable is used here to describe the remaining type of system having simple imaginary (vertical) axis closed-loop poles with all other closed-loop poles being in the open left half plane. Such a system has a bounded natural response, which does not converge to zero as does a stable system. Some bounded inputs produce bounded outputs, while other bounded inputs (having a natural frequency located at a closed-loop imaginary axis pole) produce an unbounded output.

If the system is stable, two commonly used measures of the degree of stability are gain margin and phase margin, which indicate the amount of magnitude and phase change, respectively, required to move the system to marginal stability from stability. These measures indicate the margin by which gain may be changed if the system dynamics are known or, alternatively, the amount by which the assumed system dynamics may be in error, if gain is not changed (or some combination of changes). While the sign of the dB value of gain margin may be positive or negative for stable or unstable minimum-phase or non-minimum phase systems, positive phase margin (if properly calculated as mentioned later) implies that a system is stable. Similarly, negative phase margin implies instability while zero-valued phase margin implies marginal stability, using the stability definitions just presented. We shall therefore use phase margin as a stability measure. Steady state accuracy will also be considered.

Six compensators are considered. PI and lag compensators improve steady state accuracy (possibly to the detriment of stability). PD and lead compensators improve stability (possibly to the detriment of steady state accuracy). PID and lag-lead compensators improve both stability and steady state accuracy. Design rules are

suggested and a common plant is used to compare and contrast the use of the six compensators.

2. Gain and Phase Margins

Figure 1a shows a typical control system whose degree of stability is to be evaluated. In Figure 1b, a gain K is added to the forward path, in an effort to measure the degree stability of the system. For convenience, when $K=1$ in Figure 1b, the system will be referred to as being nominal in that the original system of Figure 1a emerges. The transfer function including K is

$$T(s) = \frac{KG(s)}{1 + KG(s)H(s)} \tag{1}$$

If s is a closed-loop pole, then

$$1 + KG(s)H(s) = 0$$

$$KG(s)H(s) = -1 = 1 \angle -180^\circ$$

There is thus a magnitude condition

$$|KG(s)H(s)| = 1 \tag{2}$$

and an angle condition

$$\angle KG(s)H(s) = -180^\circ \tag{3}$$

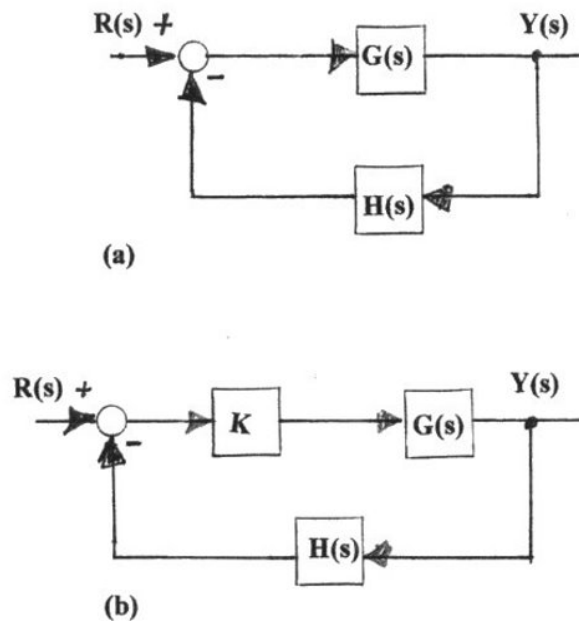


Figure 1: Control System Nomenclature (a) Nominal System (b)Gain Included to Obtain GM and PM

It is of interest to know how much adjustment of K from the nominal value of 1 is required to make the system marginally stable. There are at least two possibilities: the magnitude of K can change to a value called the gain margin (GM) while the phase does not change or the phase of K can change by the negative of an angle called the phase margin (PM), while the magnitude does not change. To find GM and PM,

1. Adjust the system magnitude until the system becomes marginally stable by selecting K to be a positive real number with zero phase angle, that is, $K=GM\angle 0^\circ$.
2. Adjust the system phase until the system becomes marginally stable by selecting $K=1\angle -PM$.

2.1. Gain Margin

Following 1, above, K is to be a positive real number called the gain margin (GM). In general, if K is to be adjusted until the system becomes marginally stable, then the denominator of (1) is zero for some value of s that is purely imaginary. Let that s be called $j\omega_{PC}$. Then (3) can be solved to obtain ω_{PC}

$$\angle G(j\omega_{PC})H(j\omega_{PC}) = -180^\circ \quad (4)$$

The frequency ω_{PC} is called the phase crossover frequency. The GM follows from (2) with $s=j\omega_{PC}$

$$K = GM = \frac{1}{|G(j\omega_{PC})H(j\omega_{PC})|} \quad (5)$$

With $K = GM$, the system is marginally stable. Since the dB of some magnitude m is

$$dB(m) = 20 \log_{10}(m)$$

it follows that

$$dB(GM) = -dB(|G(j\omega_{PC})H(j\omega_{PC})|)$$

2.2. Phase Margin

Following 2, above, K is to have a unity magnitude and a phase equal to the negative of the phase margin (PM). Again, the denominator of (1) is zero for some value of s that is purely imaginary. Now, let that s be called $j\omega_{GC}$. Then (2) can be solved to obtain ω_{GC} , assuming the magnitude of K to be 1.

$$|G(j\omega_{GC})H(j\omega_{GC})| = 1 \quad (6)$$

The frequency ω_{GC} is called the gain crossover frequency. For that frequency, (3) can be solved to obtain PM, knowing that the phase of K is $-PM$

$$-PM + \angle G(j\omega_{GC})H(j\omega_{GC}) = -180^\circ$$

$$PM = 180^\circ + \angle G(j\omega_{GC})H(j\omega_{GC}) \quad (7)$$

Ideally, a stable system should be designed with sufficient gain margin and phase margin, that any expected combination of gain change and uncertainty as to plant dynamics will not generate instability.

Suppose the type of system stability (stable, marginally stable or unstable) is unknown. Can gain margin or phase margin be used to determine stability type (as well as degree of stability)? The next two examples illustrate the fact that the sign of dB(GM) is ambiguous in determining stability. What about using the sign of the phase margin? It is imperative that PM be calculated correctly. At low frequency, the phase of $G(s)H(s)$ can be expressed as a negative angle. As frequency increases, that phase becomes progressively more negative or less negative, but is always expressed as a negative angle, until the frequency is reached where the magnitude $|G(j\omega_{GC})H(j\omega_{GC})|$ equals one thus defining ω_{GC} . Then, $PM = 180^\circ + \angle G(j\omega_{GC})H(j\omega_{GC})$. It has often been observed, using a complete Nyquist plot, that if $G(s)H(s)$ is a rational polynomial ratio in s with no poles or zeros in the open right half plane, and there is only one value of ω_{GC} then a positive value of PM occurs for a stable system, a negative value occurs for an unstable system, and a zero value occurs for a marginally stable system.

It is my experience, based on many examples, that if phase is calculated as mentioned above, the sign of PM is also an unambiguous indicator of stability type for non-minimum phase systems, when there is only one value of ω_{GC} . While that is not a proof, the lack of a counter example is encouraging. Of course, whenever type of stability is in doubt, as when there are multiple values of ω_{GC} with PM of differing sign, one can be more certain by examining the complete Nyquist plot or the root locus. In the following examples, the sign of the PM is an unambiguous indicator of stability type.

2.3. Examples

Consider two nominal systems. The first is

$$G_1(s)H_1(s) = \frac{10(s+10)^2}{s^2(s+2)}$$

From the Bode plot of Figure 2, $\omega_{PC} = 7.75$ rad/sec, $GM = 0.3$ (-10.5 dB), $\omega_{GC} = 14.6$ rad/sec, $PM = 28.9^\circ$. Notice from the root locus of Figure 3 (with $K=1$) that the nominal system is stable having closed-loop poles at $-2.96 \pm j12.5$ and -6.1 .

The second system is

$$G_2(s)H_2(s) = \frac{10}{s(s+2)^2}$$

From the Bode plot of Figure 4, $\omega_{PC} = 2$ rad/sec, $GM = 1.6$ (+4.1dB), $\omega_{GC} = 1.56$ rad/sec, $PM = 14.2^\circ$. From the root locus of Figure 5 (with $K=1$) the nominal system is stable having closed-loop poles at $-0.17 \pm j1.65$ and -3.65 . The first system is stable with a positive dB(GM) while the second system is stable with a negative dB(GM). Both stable systems have positive PM.

Each system becomes unstable if the factor 10 changes by more than the limit imposed by the gain margin. The first system becomes unstable if the factor of 10 is multiplied 0.2 (-14 dB) which is more negative in dB than the allowed value of $GM = 0.3$ (-10.5 dB). The second system becomes unstable if the factor of 10 is multiplied 2, which is more than the allowed value of 1.6. The resulting unstable systems are

$$G_1(s)H_1(s) = \frac{2(s+10)^2}{s^2(s+2)}$$

$$G_2(s)H_2(s) = \frac{20}{s(s+2)^2}$$

System 1 is now unstable with $GM = 3.5$ dB and $PM = -7^\circ$ while System 2 is unstable with $GM = -1.9$ dB and $PM = -6.2^\circ$. The unstable systems both have negative values of PM, while the sign of dB(GM) is positive for one system and negative for the other. In the next section, it is noted that PM is also related to the damping ratio; hence, time response is also dependent on PM.

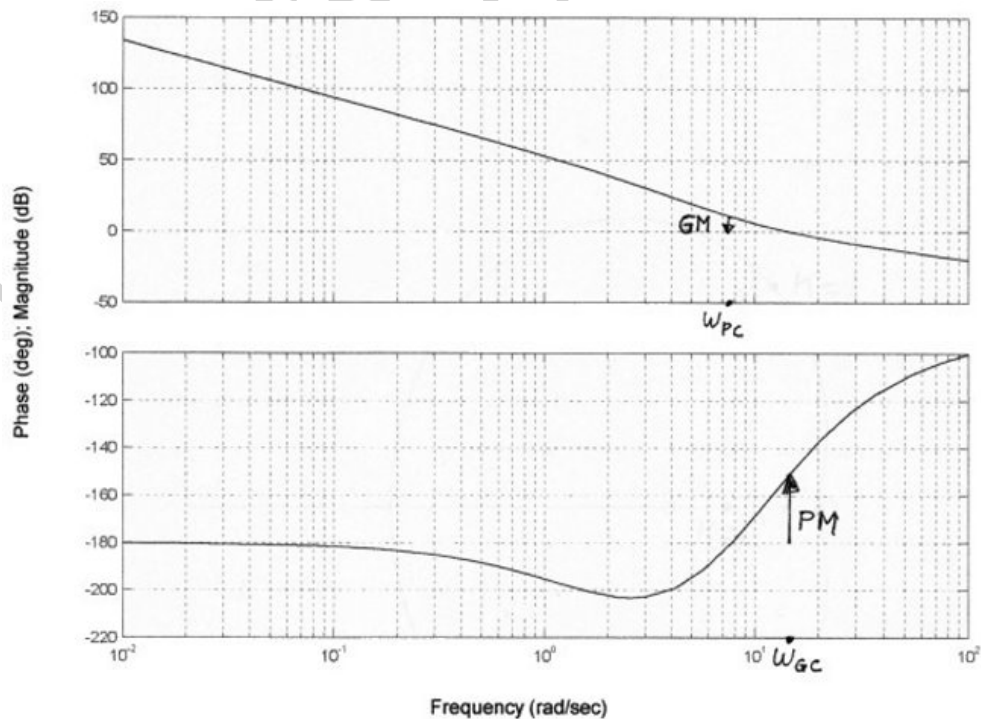


Figure 2 Bode Plot of $G_1(s)H_1(s)$

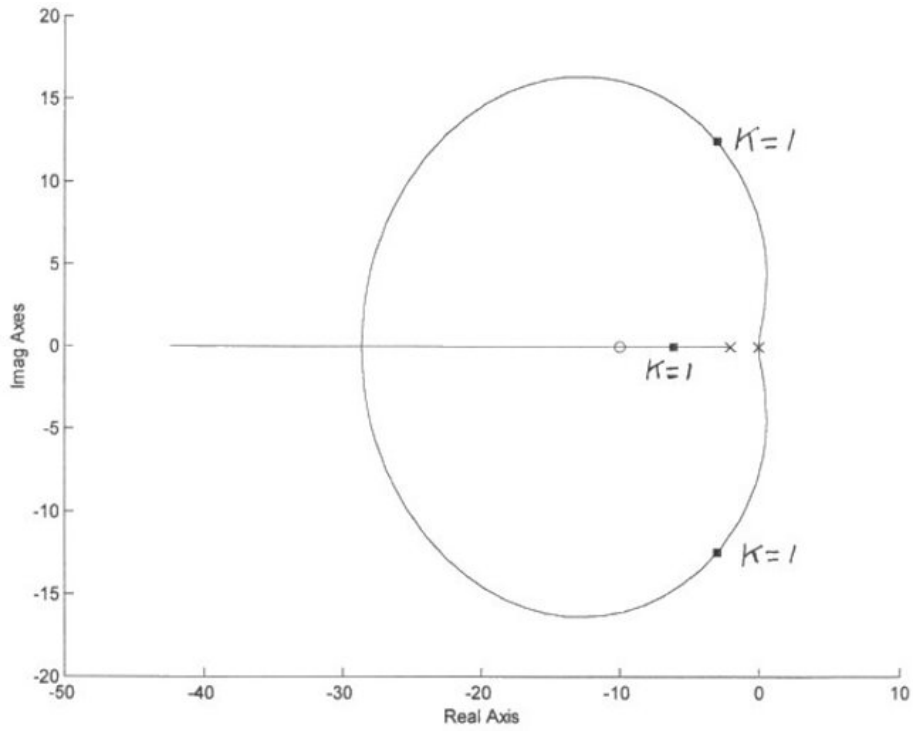


Figure 3 Root Locus of $G_1(s)H_1(s)$

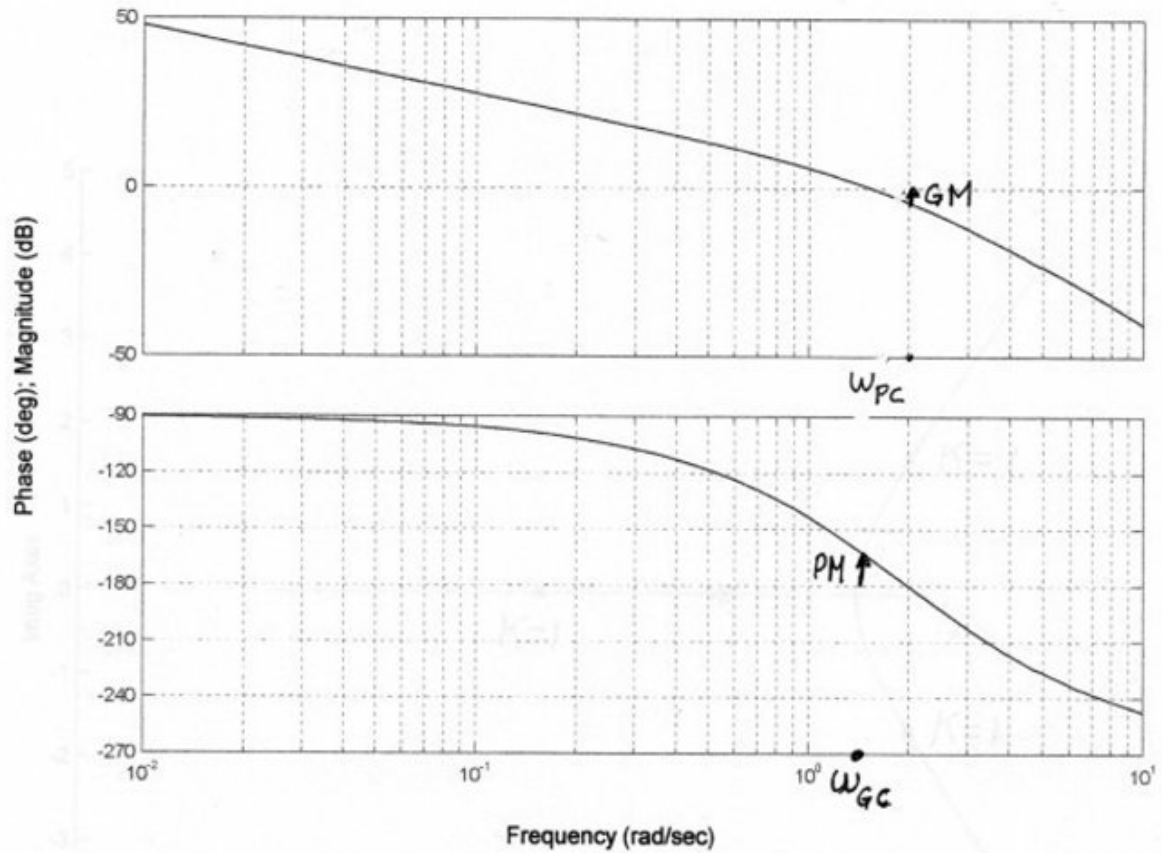
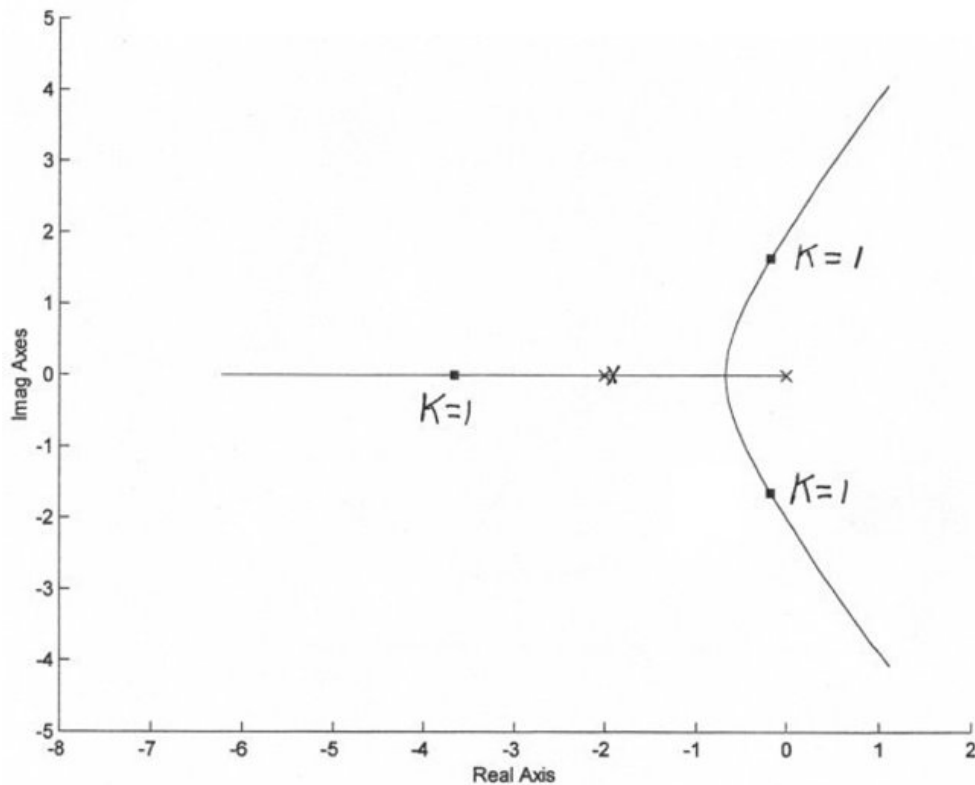


Figure 4 Bode Plot of $G_2(s)H_2(s)$

Figure 5 Root locus of $G_2(s)H_2(s)$

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Biographical Sketch

Dr. Raymond T Stefani is a Professor of electrical engineering at the California State University, Long Beach (California, USA). His teaching assignments include circuit analysis and control systems at both

undergraduate and graduate levels. His electrical engineering research includes classical and modern compensator design, stability analysis and observer design. His co-authored control system textbook is entering its fourth edition. Other research interests include statistical analysis of Olympic winning performances in athletics (track and field) swimming, weightlifting and rowing. He is also interested in rating and ranking systems. He has published a book chapter on predicting the outcome of sporting events.

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