IDENTIFICATION OF BLOCK –ORIENTED MODELS

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Contents

Summary

Because linear dynamic models provide the basis for so many useful applications (e.g., stability analysis, control systems design, dynamic model identification), they have been used extensively in applications ranging from electronic circuit theory to industrial process control to financial forecasting. Linear models are not adequate to all applications, but motivate interest in various types of nonlinear dynamic models. *Feedforward block oriented* models represent one of the simplest and best known classes of nonlinear models, and they have been widely adopted as a simple alternative to linear dynamic models.

These models consist of series, parallel, or combined series/parallel interconnections of linear dynamic models and static (i.e., memoryless) nonlinearities, the best-known examples of such models being the *Hammerstein* and *Wiener* models. The class of *feedback block-oriented models* is based on the same two types of component subsystems, but combined using feedback interconnections. As a consequence, these models exhibit a wider range of dynamic behavior, often including subtle input-dependent stability characteristics that are extremely undesirable in many applications, and they are generally more difficult to analyze than feedforward block-oriented

models. The best-known feedback block-oriented model is the *Lur'e model*, consisting of a single static nonlinearity connected as a feedback element around a linear dynamic system.

This chapter presents a broad overview of block-oriented models, including both the feedforward and feedback classes. The chapter includes discussions of the two types of component subsystems on which all block-oriented nonlinear models are based, and detailed discussions of the Hammerstein, Wiener, and Lur'e models. The relationship of Hammerstein and Wiener models to the larger class of Volterra models is also discussed, along with the qualitative behavioral differences between the feedforward and feedback block-oriented model classes.

1. Introduction

Block-Oriented models constitute a practically important class of mathematical models that are useful in describing the dynamic behavior of certain physical, biological, engineering, or other classes of systems. Historically, *linear dynamic models* have been extremely important in many different application areas because they are amenable to much mathematical analysis. Consequently, linear models serve as an effective basis for the characterization of important qualitative behavior like stability, for the design of feedback control systems, for the development of empirical models, among other applications.

Useful as these linear dynamic models are, however, they are not adequate to all applications, motivating substantial research efforts in the general area of nonlinear systems theory. This research activity has extended in many different directions, and many different classes of nonlinear dynamic models have evolved as a result. The class of block-oriented nonlinear models considered in this chapter is based on interconnections of linear dynamic models with *static* (or *memoryless*) nonlinearities. More specifically, this chapter considers two distinct subsets of this model class: the *feedforward block-oriented models*, obtained by restricting consideration to parallel or series interconnections, and the class of *feedback block-oriented models*, which also allow *feedback* interconnections.

Because they are the more popular in practice, the primary focus of this chapter is the feedforward block-oriented model class. The popularity of this model class arises in part from the fact that feedforward block-oriented models are in many respects the simplest and best-behaved extension of linear dynamic models into the realm of nonlinear systems. For example, feedforward block-oriented models based on *asymptotically stable* linear subsystems and *continuous* static nonlinearities are easily shown to be asymptotically stable themselves, regardless of the inputs considered. In marked contrast, feedback block –oriented systems can exhibit strongly input-dependent stability characteristics.

More generally, feedback block-oriented models exhibit a wider range of possible qualitative behavior than feedforward block-oriented models do, but this fact can be significant disadvantage in applications where the real-world system to be modeled exhibits mild but significant nonlinear dynamics. For example, biological systems like the fly photoreceptor that are inherently stable but which exhibit strongly nonlinear dependence on stimulus intensity have been frequently described using feedforward block- oriented models. Further, the qualitative behavior of feedback block-oriented models is more difficult to analyze than that of the feedforward class, as the stability differences just noted between these two models subclasses illustrates.

In particular, much more of our intuition about the behavior of linear dynamic models extends to feedforward block-oriented models than to any other popular nonlinear dynamic model class. Finally, because these models are most often identified form discrete-time data sequences, often as a basis for subsequent computer –based simulation, analysis, or control applications, this chapter restricts consideration to the discrete –time case.

The remainder of this chapter is organized as follows. Section 2 presents detailed discussion of the two fundamental components on which all block-oriented models are based: linear dynamic models and static nonlinearities. Next, Sections 3 and 4 present detailed discussions of Hammerstein models and Wiener models, respectively, the two best-known members of the feedforward block-oriented model class. Section 6 briefly considers the general range of qualitative behavior this model class is capable of exhibiting.

Next, Section 7 discusses the feedback block-oriented model class, taking the *Lur'e model* as a prototype and considering some of the important consequences of allowing feedback interconnections. Section 8 presents a brief overview of some of the practical issues that arise in the process of fitting block-oriented and other nonlinear dynamic models to observed input/output data. Finally, Section 9 concludes this chapter with a recap of the key ideas and a brief discussion of some extensions and related topics. More detailed treatments of the topics discussed here may be found in the bibliography at the end of this chapter.

2. The Building Blocks.

The following discussions briefly describe the basic building blocks for all of the block-oriented models considered in this chapter: linear dynamic subsystems and static nonlinearities.

2.1. Linear Dynamic Subsystems

The general class of linear models may be defined behaviorally as those models $\mathcal{L}$ satisfying the *principle of superposition*:

$$\mathcal{L}\left[au_k + b\right] = a\mathcal{L}\left[u_k\right] + b\mathcal{L}\left[v_k\right] \quad . \tag{1}$$

In this description, $\{u_k\}$ and $\{v_k\}$ represent two input sequences and *a* and *b* are any two real numbers. Most popular linear systems-and all of the linear systems considered in this chapter-are also *time-invariant*, meaning that if the input sequence is shifted by *j* time units from $\{u_k\}$ to $\{u_{k-j}\}$, the response $\{y_k\}$ is shifted by the same amount, to $\{y_{k-j}\}$. It is a standard result that any linear, time –invariant (or LTI)model is completely characterized by its *impulse response*, which is the sequence $\{h_i\}$ generated in response to the impulse input, $\delta_k = 1$ for k=0 and $\delta_k = 0$ for $k \neq 0$. In particular, given the impulse response, the response of the linear model to an arbitrary input sequence $\{u_k\}$ is given by the *discrete convolution relation*:

$$y_k = \sum_{i=\infty}^{\infty} h_i u_{k-i} \tag{2}$$

Generally, the linear dynamic models used to describe real-world systems are also causal, meaning that the impulse response coefficients $\{h_i\}$ are identically zero for i<0; this relation means that the causal systems cannot respond to future values of the input sequence, a reasonable requirement when modeling physical systems.

Although it does provide a complete description of LTI system behavior, a significant practical disadvantage of the impulse response $\{h_i\}$ is that it constitutes an infinite sequences of numbers. To overcome this difficulty, it is useful to introduce three finite parameterizations that are popular in practice. *Autoregressive moving average models* are defined by the difference equation.

$$y_k = \sum_{i=1}^{p} a_i y_{k-i} + \sum_{i=0}^{q} b_i u_{k-i}, \tag{3}$$

for some set of constants $\{a_i\}$ and $\{b_i\}$; the integers p and q appearing in this equation are called order parameters and this model structure will be denoted *ARMA* (p,q) for convenience. It is worth noting that certain linear dynamic phenomena involving unusually slow decays (specifically, non-exponential decays) or long –range correlations (e.g., 1/f noise phenomena) cannot be described by *ARMA*(p,q) models for any finite p and q, but most popular linear models do belong to this class. Taking p=0 in this representation corresponds to omitting the first sum from Eq. (3)and leads to the second important linear models class considered here: *the finite impulse response* or *FIR* models. The name for this model class derives form the fact that only a finite number of the impulse response coefficients are nonzero for such models: $h_i = b_i$ for $0 \le i \le q$ and $h_i = 0$ otherwise. Finally, the third description considered here is the transfer function, obtained by taking the z-transform of the *ARMA* (p,q) model representation:

$$H(z) = \frac{\sum_{i=0}^{q} b_i z^{-i}}{1 - \sum_{i=1}^{p} a_i z^{-i}} \tag{4}$$

This model may be viewed most simply as a rewriting of Eq. (3)in terms of the unit delay operator z^{-1}, but it is also closely related to the frequency response of the linear system, $H(\omega)$, which may be obtained by evaluating $H(z)$ at $z = e^{i\omega T}$ where T is the time between successive samples u_k and u_{k+1}.

2.2. Static Nonlinearities

The static nonlinearities on which block-oriented models are based are simply functions mapping one real number into another. One of the most popular classes of functions is the polynomial class:

$$g(x) = \sum_{\ell=0}^{r} \alpha_\ell x^\ell, \quad x^0 \equiv 1. \tag{5}$$

The study of these functions dates back at least as far as the ancient Greeks, so the variety of available analytical results is enormous. In addition, polynomials are easy to evaluate and are extremely well-behaved mathematically, being smooth (i.e. infinitely differentiable), continuous and linearly dependent on the coefficients α_ℓ.The *Weierstrass approximation* theorem establishes that any *continuous* function $f(x)$ may be approximated arbitrarily well on any compact (i.e., closed and bounded)subset of the real line by a polynomial of sufficient degree *r*. Further, it can be shown that the class of feedforward models(i.e. those involving only series and parallel interconnections) based on linear FIR subsystems and polynomial nonlinearities is equivalent to the class of *finite Volterra models*, described by the input –output representation

$$y_k = y_0 + \sum_{n=1}^{N} v_M^n(k) \tag{6}$$

$$v_M^n(k) = \sum_{i_1=0}^{M} \cdots \sum_{i_n=0}^{M} \alpha_n\left(i_{1,\ldots,}i_n\right) u_{k-i_1} \cdots u_{k-i_n}$$

Models of this class can approximate the dynamics of any *fading memory system* with arbitrary accuracy by taking *M* and *N* sufficiently large, a result that may be viewed as a dynamic extension of the Weierstrass approximation theorem. The fading memory class includes the finite-dimensional linear *ARMA* (*p,q*) systems discussed in Section 2.1 and is characterized by a weak dependence on events in the distant past, in contrast to some of the feedback block-oriented model structures discussed in Section 7**,** which can exhibit strong dependence on initial conditions for all later times *k*(e.g., chaotic impulse responses).

Functions that can be implemented with *artificial neural networks* (ANN' s)have also become quite popular, due in part to another extension of the Weierstrass approximation theorem: any continuous mapping $f : R^m \to R^n$ between finite dimensional Euclidean spaces R^m and R^n may be approximated arbitrarily well on any concept set by an ANN of sufficient complexity. An important difference between ANN' s and polynomials is that polynomials are unbounded on the real line *R*, whereas any ANN function is bounded due to the saturation nonlinearity (“squashing function”)on which the network is based. This observation has important practical consequences for the feedback block-oriented structures considered in Section 7

The canonical piecewise –linear (CPWL) functions are defined by

$$g(x) = a_0 + a_1 x + \sum_{j=1}^{n} \left[b_j \left| x - x_j \right| + c_j \operatorname{sgn}\left(x - x_j \right) \right], \qquad (7)$$

where $\{x_j\}$ corresponds to a set of knots, or points at which the local linear functions change slope, and a_0, a_1, b_j and c_j are real –valued constants; these functions are continuous at x_j if and only if $c_j = 0$. Like polynomials and ANN’ s it has been shown that CPWL functions can approximate any continuous function with arbitrary accuracy on any compact set, provided the number of knots n is sufficiently large. Further, this result applies to multidimensional mappings $g : R^m \to R^n$, exactly as in the case of ANN functions. Also, note that this class of functions represents the simplest special case of the class of spline functions, which are defined by polynomials on the intervals $[x_n, x_{n+1}]$ between successive knots.

More generally, given an arbitrary basis set $\{\phi_\ell(x)\}$, it is often useful to consider linear combinations of the form

$$g(x) = \sum_{\ell=0}^{r} \alpha_\ell \phi_\ell(x) \qquad (8)$$

Taking $\phi_\ell(x) = x^\ell$ yields the class of polynomials, and if the knots $\{x_n\}$ defining the CPWL functions are fixed, these functions may also be represented as in Eq. (7).Conversely, this representation is not possible for ANN functions, which depend nonlinearly on the network parameters.

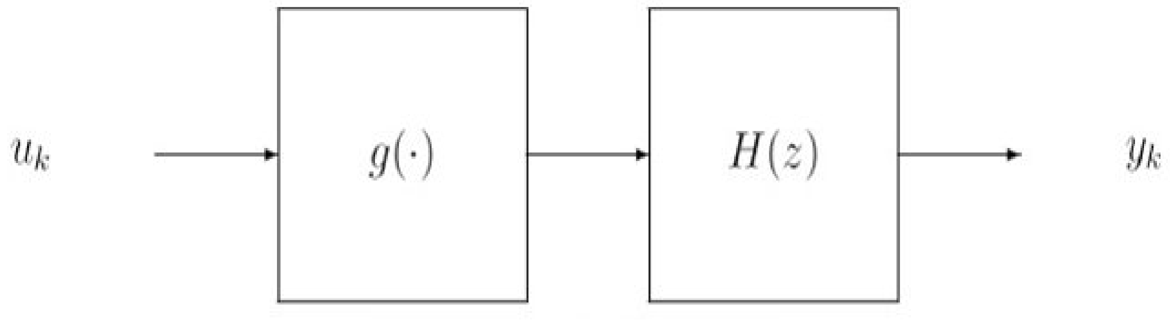

Figure 1: The Hammerstein model structure

3. Hammerstein Models.

One of the most popular block-oriented model structures is the Hammerstein model, consisting of the series connection of a static nonlinearity followed by a linear dynamic subsystems as show in the block diagram in Fig. 1. if we adopt the impulse –response representation for the linear subsystems, we immediately obtain the following input/output description of the Hammerstein model:

$$y_k = \sum_{i=0}^{\infty} h_i g\left(u_{k-i}\right) \tag{9}$$

Alternatively, given an ARMA(p,q) representation for the linear subsystem, the Hammerstein model may be represented more simply as

$$y_k = \sum_{i=1}^{p} a_i y_{k-i} + \sum_{i=0}^{q} b_i g\left(u_{k-i}\right). \tag{10}$$

If the steady-state gain of the linear subsystem is constrained to be 1, then

$$\sum_{i=1}^{p} a_i + \sum_{i=0}^{q} b_i = 1, \tag{11}$$

and the steady –state behavior of the Hammerstein model is entirely determined by the static nonlinearity: $y_s = g\left(u_s\right)$, where u_s is the steady –state input value and y_s is the corresponding steady-state output value. Hence, if the steady-state characterization is known for the system of interest, the nonlinear function g(.) is determined and the model identification problem reduces to that of determining the linear model coefficients a_i and b_i, subject to the constraint (11).

One context in which this model structure arises naturally is the square-law detector, used in recovering the information encoded in amplitude modulated communications signals. This system consists of the nonlinearity $g\left(x\right) = x^2$ followed in series by a linear lowpass filter, obtained through a suitable choice of the linear dynamic model coefficients a_i and b_i. For the FIR case p=0, substituting $g\left(x\right) = x^2$ into Eq. (9)leads to the Volterra model defined by Eq.(6).with $M = q, N = 2$,and all $\alpha_n\left(i_1,...,i_n\right)$=0 unless $i_1 = \cdots = i_n$, in which case $\alpha\left(i,\cdots,i\right)$ is proportional to the linear model coefficient b_i. This idea has been applied in modeling the response of fly photoreceptors to light pulse stimuli. Specifically, second –order Volterra models fit to the observed stimulus –response data have been observed to approximate this diagonal form. Refitting this experimental data directly to a Hammerstein model permits the use of higher-order polynomial nonlinearities, giving significantly better models. In particular, one disadvantage of the Volterra representation is the large number of parameters required in the general case, restricting most applications to N=2 or N=3. In contrast, the Hammerstein model involves only a few parameters, permitting consideration of much higher-order polynomials. Hammerstein models are also popular in modeling mechanical or chemical engineering systems. One reason for the popularity of the Hammerstein model is that it does not greatly complicate the problem of controller design, relative to that for linear dynamic models. For example, *model predictive* control (MPC) is a model –based control strategy that has become extremely popular in industrial practice: the underlying idea is to formulate a performance optimization problem to determine the control input sequence $\{u_k\}$ that forces the output sequences $\{y_k\}$ to follow a desired trajectory subject to various practically-motivated constraints. Generally, this strategy employs linear FIR models, but extension has been developed recently for Hammerstein models.

Generally, the static nonlinearity *g*(.) is not known a *priori* and must be determined empirically. A particularly popular algorithm for this case is that of Narendra and Gallman, which proceeds as follows. Assuming the nonlinearity is parameterized as in Eq. (9) and the linear subsystems is described by an *ARMA* (*p,q*) model , the overall model may be written as

$$y_k = \sum_{i=1}^{p} a_i y_{k-i} + \sum_{i=0}^{q} \sum_{j=0}^{r} b_i c_j \psi_j \left(u_{k-i}\right) \qquad (12)$$

Initially, the coefficients c_j are fixed at some preliminary estimates c_j^0,yielding an unconstrained identification problem for the linear model parameters a_i and b_i, which may be solved by standard methods like least squares. These parameters are then fixed at their estimated values and the parameters c_j are re-estimated from the available input/output data.

This process is iterated to convergence, which can fail to occur if the initial parameter estimates are too far from their values; in practice, however, this algorithm has usually been found to work reasonably well and convergence has been established for the case of linear FIR subsystems, *provided* $\{u_k\}$ *is an independent , identically distributed* (*i,i,d*) *random sequence* (*i.e.*, *"white noise"*). This observation emphasizes the role of the input sequence in empirical model identification, a point revisited in Section 8.

Finally, it is important to note that the basic Hammerstein model structure contains an inherent overparameterization, which must be addressed in any empirical model identification procedure. This point may be most easily seen by considering the impulse response representation (9): if all of the impulse response coefficients $\{h_i\}$ are multiplied by an arbitrary nonzero constant λ and the function *g*(.)is multiplied by $1/\lambda$, there is no net change in the input/output behavior of the resulting Hammerstein model. To obtain unique model parameter estimates, it is necessary to impose a constraint. Most commonly, the constraint chosen is $b_0 = 1$ for simplicity, but the steady-state gain constraint given in Eq.(11)is an equally effective alternative.

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Biographical Sketch

Ronald K. Pearson received the B.S. degree in physics from the University of Arkansas at Monticello, USA, in 1973, the M.S.E.E. degree in solid state materials and devices from M.I.T. in 1975, and the PhD in control theory from M.I.T. in 1982. Dr. Pearson joined the DuPont Company in Wilmington, Delaware, USA after completing his PhD, where he remained until 1997, working in the areas of on-line process instrumentation, empirical dynamic modeling, and exploratory analysis of historical process data. From 1997 until 2001, Dr. Pearson was with the Automatic Control Laboratory at the ETH in Zuerich, where he was active in research and teaching in the areas of nonlinear discrete-time dynamic models, exploratory data analysis, and nonlinear digital filter design and analysis. Dr. Pearson then spent a year as a Visiting Professor with the Tampere International Center for Signal Processing at the Tampere University of Technology in Tampere, Finland, where he was active in teaching and research in the theory of nonlinear digital filters and where he became involved in the analysis of cDNA microarray data. In 2002, Dr. Pearson joined the Daniel Baugh Institute for Functional Genomics and Computational Biology at Thomas Jefferson University in Philadelphia, where he was involved in the application of a variety of mathematical and statistical analysis techniques to data from cDNA microarrays and other significant biological sources. Dr. Pearson is currently Senior Scientist with ProSanos Corporation in Harrisburg, Pennsylvania, USA, where he is involved in the exploratory analysis of retrospective clinical data.