# FUNDAMENTALS OF THE QUANTITATIVE FEEDBACK THEORY TECHNIQUE

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**Contents**

## Summary

The Quantitative Feedback Theory (QFT) design technique, which has the ability to bridge the gap between theory and the real-world control design problem, that is utilized in the design of MISO and MIMO robust multivariable control systems whose plants have structured parametric uncertainty is presented in this chapter. Achieving a successful robust design involves a number of steps: specification of the control problem, plant model data, theoretical control system design, implementation of the theoretical design, simulation, and system under actual operating conditions (involving the nonlinear plant). Thus, this chapter provides an overview of "using robust control system design to increase quality" in attempting to demonstrate the "Bridging the Gap" between control theory and the realities of a successful control system design. In facing the technological problems of the future, it is necessary that engineers of the future must be able to *bridge the gap*, i.e., this "Bridging the Gap" must be addressed to better prepare the engineers for the 21$^{st}$ century.

## 1 Introduction

### 1.1 Quantitative Feedback Theory (QFT)

The very complex control systems of the 20$^{th}$ century have acted as a catalyst in promoting progress and development in propelling society into the 21$^{st}$ century. The QFT design technique has the capability of meeting the control system design challenge of the 21$^{st}$ century. It is a very powerful design technique for the achievement of assigned performance tolerances over specified ranges of structured plant parameter uncertainties without and with control effector failures. It is a frequency domain design technique that utilizes the Nichols chart (NC) to achieve a desired robust design over the specified region of plant parameter uncertainties. The chapter presents an introduction to QFT analog and discrete design techniques for both multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) control systems. QFT CAD packages are readily available to expedite the design process. The purposes of this chapter are: (1) to provide a basic understanding of QFT, (2) to provide the minimum amount of mathematics necessary to achieve this understanding, (3) to discuss the basic design steps, and (4) to present a practical example.

### 1.2 The Control System design Process

The essential aspects of the control system design process are illustrated in Figure 1. These aspects present the factors that help in *bridging the gap* between theory and the real-world are addressed in the next paragraph. While accomplishing a practical control system design, the designer must keep in mind that the goal of the design process, besides achieving a satisfactory theoretical robust design, is to implement a nonlinear control system which meets the functional requirements. In other words, during the design process one must keep the real world in mind. For instance, in performing the simulations, one must be able to interpret the results obtained, based upon a knowledge of what can be reasonably expected of the plant that is being controlled. For example, in performing a time simulation of an aircraft's transient response to a pilot's maneuvering command to the flight control system, the *simulation run time* may need to be only 5 *s* since by that time a pilot would have instituted a new command signal. If within this 5 *s* window the performance specifications are satisfied, then it will be deemed that a successful design has been achieved. However, if the performance of interest is the steady-state response, then the simulation run-time must be considerably longer. Another real-world factor is *control authority allocation*, that is, the manner in which the available control power is assigned to the control effectors.

This allocation must be based upon a thorough knowledge of the plant that is being controlled and the conditions under which the plant will operate. Position saturation and even more dramatically, rate saturation of the output effectors will significantly affect the achievement of the functional requirements. Linear and nonlinear simulations are very helpful in early evaluation of the controlled system, but if the system is to operate in the real word, hardware-in-the-loop and system tests must be performed to check for unmodeled effects not taken into account during the design and implementation phases. In order to be a successful control system designer, an individual must be fully cognizant of the role corresponding to each aspect illustrated in Figure 1.

*Bridging the gap*, as illustrated in Figure 1, is enhanced by the transparency of the metrics depicted by the oval items in the interior of the QFT design process. A key element of QFT is embedding the performance specifications, at the onset of the design process. This establishes design goals that enhance and expedite the achievement of a successful design. Another important element is the creation of templates at various frequencies. The size of the template indicates whether or not a robust design is achievable. If a robust design is not achievable, then the templates can be used as a metric in the reformation of the control design problem. Another element of the QFT design process, is the ability to concurrently analyze frequency responses of the J linear-time invariant (LTI) plants that represent the nonlinear dynamical system throughout its operating environment. This gives the designer insight into the behavior of the system.

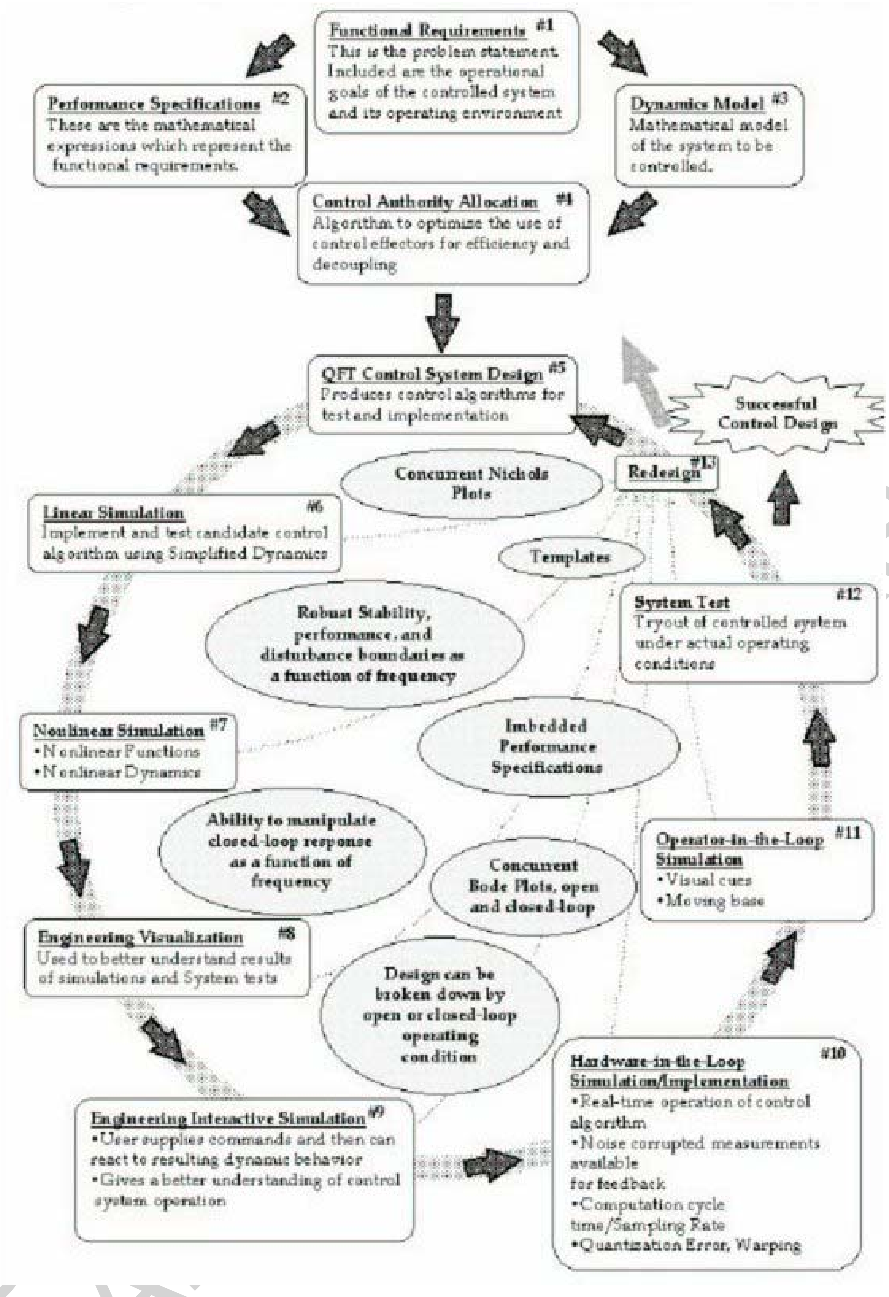


Figure 1 The QFT control system design process: bridging the gap.

The designer can use this insight for such things as picking out key frequencies to use during the design process, as an indicator of potential problems such as nonminimum phase behavior, and as a tool to compare the nonlinear system with the desired performance boundaries. The next element of QFT consists of the design boundaries. During the actual loop shaping process, the designer uses boundaries plotted on the Nichols chart. These boundaries are only guidelines and the designer can exercise engineering judgement to determine if all the boundaries are critical or if some of the boundaries are not important. For example, based on knowledge of the real world system, the designer may determine that meeting performance boundaries below a certain frequency is not important, but it is important to meet the disturbance rejection boundaries below that frequency. Once the initial design has been accomplished, all of the J loop transmission functions can be plotted on a Nichols chart in order to analyze the results of applying the designed compensator (controller) to the nonlinear system. This gives the designer a first look at any areas of the design that may present problems during simulation and implementation. The last two elements of the QFT design process that help *bridging the gap* is the relation of the controlled system's behavior to the frequency domain design and the operating condition. These relationships enable the designer to better analyze simulation or system test results for problems in the control design. To obtain a successful control design, the controlled system must meet all of the requirements during simulation and system test. If the controlled system fails any of the simulation or system tests, then, using the design elements of QFT, the designer can trace that failure back through the design process and make necessary adjustments to the design. QFT provides many metrics that provide the link between the control design process and real world implementation; this is the *transparency of QFT*.

In designing a feedback control system, it is desired to utilize a technique that:

a. Addresses all known plant variations up front.
b. Incorporates information on the desired output tolerances.
c. Maintains reasonably low loop gain (reduce the "cost of feedback").

The QFT technique, as indicated in Figure 1, involves the first two items The last item is important in order to avoid the problems associated with high loop gains such as sensor noise amplification, saturation, and high frequency uncertainties.

### 1.3 What Can QFT Do

Assume that the characteristics of a plant, that is to be controlled over a specified region of operation, vary, that is, a plant with structured parameter uncertainty. An example of such a plant is given by the transfer function of a d-c servo motor, utilized as a position control device:

$$P_\iota(s) = \frac{\Theta_m(s)}{V_f(s)} = \frac{Ka}{s(s+a)} \tag{1}$$

where the parameters $K$ and $a$ vary, due to the operating scenario, over the following range: $K \in (K_{\min}, K_{\max})$ and $a \in (a_{\min}, a_{\max})$. Over the region of operation, in a position control system, the plant parameter variations are described by Figure 2. The shaded region in this figure represents the region of structured parametric uncertainty (region of plant uncertainty). The motor can be represented by six LTI transfer functions $P_\iota(\iota = 1, 2, ..., J)$ points indicated on the figure. The Bode plots for these six LTI plants are shown in Figure 3. This figure represents the range of variation of plant magnitude (dB) and phase over a specified frequency range. The bounds of this variation, for this example, can be described by six LTI plant transfer functions. By the application of QFT, for a MISO control system containing this plant, a single compensator and a prefilter may be designed to achieve a specified robust design.

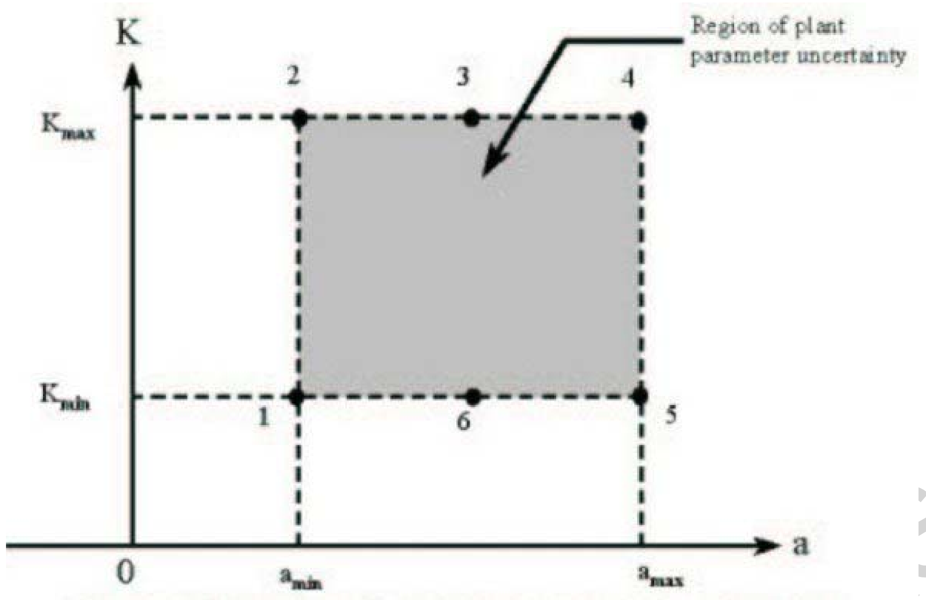


Figure 2 Region of plant parameter uncertainty.

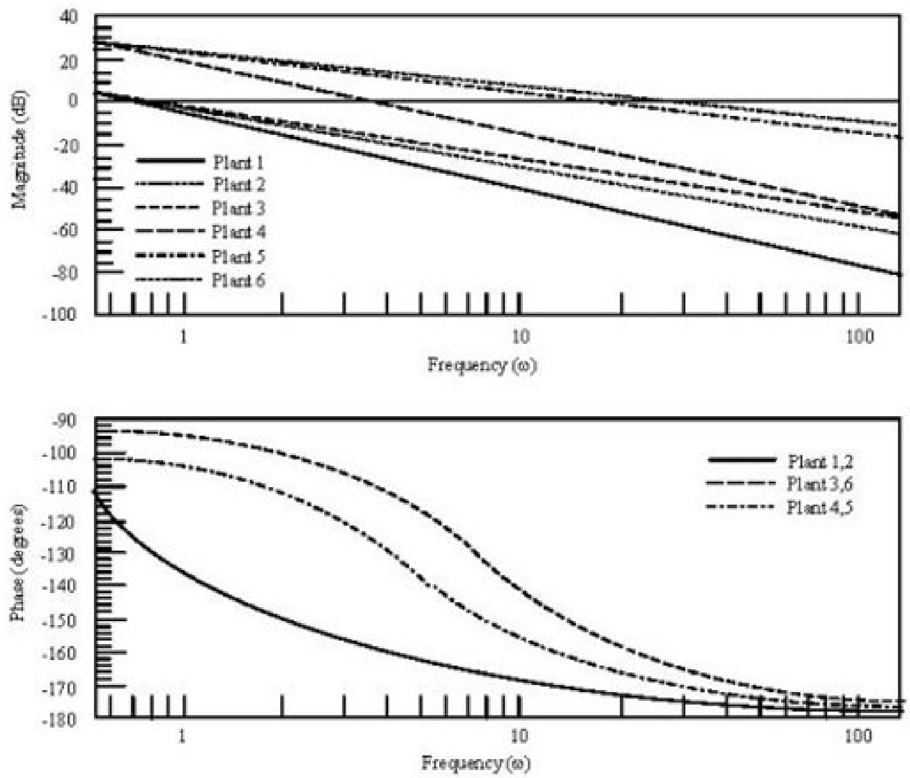


Figure 3 Bode plots of 6 LTI plants: the range of parameter uncertainty.

### 1.4 Benefits of QFT

The benefits of QFT may be summarized as follows:

a. The result is a robust design which is insensitive to plant variation.
b. There is one design for the full envelope (no need to verify plants inside templates).
c. Any design limitations are apparent up front.
d. In comparison to other multivariable design techniques there is less development time for a full envelope design.
e. One can determine what specifications are achievable early in the design process.
f. One can redesign for changes in the specifications quickly.
g. The structure of compensator (controller) is determined up front.

## 2. The MISO Analog Control System

### 2.1 Introduction

The mathematical proof that a mxm feedback control system can be represented by $m^2$ equivalent MISO feedback control systems is given in Section 4.2. A $3x3$ MIMO control system can be represented by the $m^9$ MISO equivalent loops shown in Figure 4. Thus, this and the next section present an introduction to the QFT technique by considering only a MISO feedback control system.

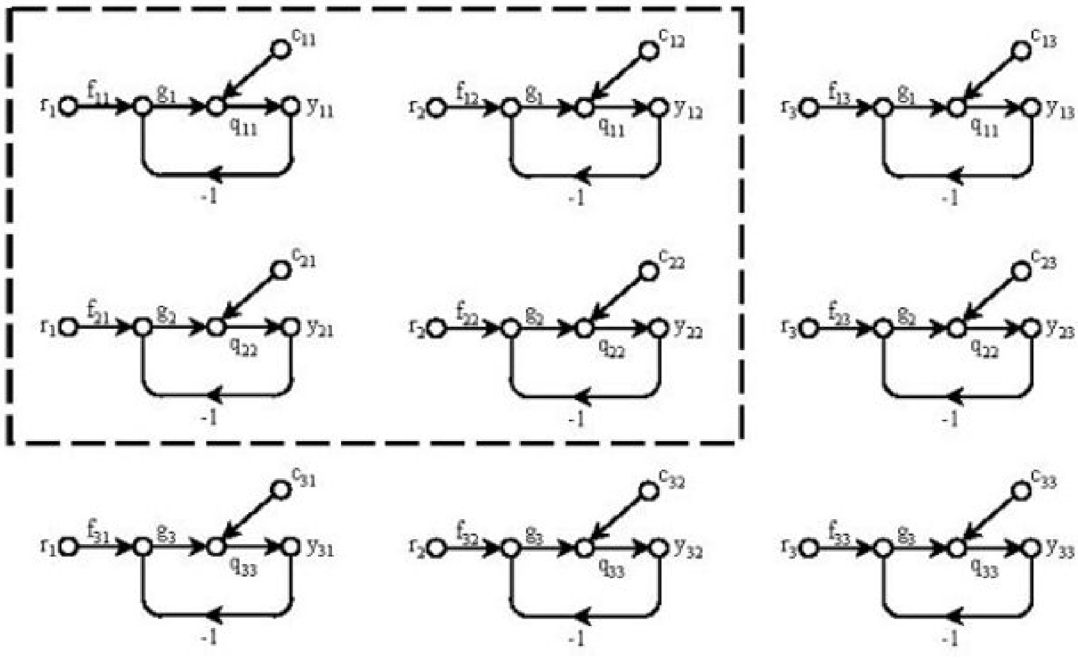


Figure 4 Effective MISO loops two-by-two (boxed in loops) and three-by-three (all 9 loops).

### 2.2 MISO System

The overview of the MISO QFT design technique is presented in terms of the minimum-phase (m.p.) linear time-invariant (LTI) MISO system of Figure 5. The control ratios for tracking $(D_1 = D_2 = 0)$ and for disturbance rejection $(R = 0)$ are, respectively:

$$T_R(s) = \frac{F(s)G(s)P(s)}{1+G(s)P(s)} = \frac{F(s)L(s)}{1+L(s)} = F(s)T(s) \, with \, d_1(t) = d_2(t) = 0 \tag{2}$$

$$T_{D_1} = \frac{P(s)}{1+G(s)P(s)} = \frac{P}{1+L} \, with \, r(t) = d_2(t) = 0 \tag{3}$$

$$T_{D_2} = \frac{1}{1+G(s)P(s)} = \frac{1}{1+L} \, with \, r(t) = d_1(t) = 0 \tag{4}$$

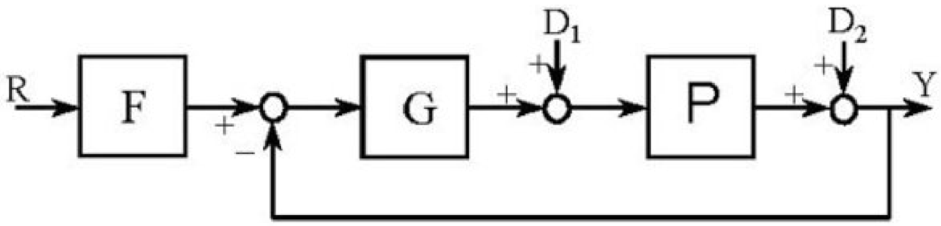


Figure 5 A feedback structure.

The design objective is to design the prefilter $F(s)$) and the compensator $G(s)$ so the specified robust design is achieved for the given region of plant parameter uncertainty. The design procedure to accomplish this objective is as follows:

Step 1: Synthesize the desired tracking model.
Step 2: Synthesize the desired disturbance model.
Step 3: Specify the J LTI plant models that define the boundary of the region of plant parameter uncertainty.
Step 4: Obtain plant templates, at specified frequencies, that pictorially describe the region of plant parameter uncertainty on the NC.
Step 5: Select the nominal plant transfer function $P_o(s)$.
Step 6: Determine the stability contour (U-contour) on the NC.
Steps 7-9: Determine the disturbance, tracking, and optimal bounds on the NC.
Step 10: Synthesize the nominal loop transmission function $L_o(s) = G(s)P_o(s)$ that satisfies all the bounds and the stability contour.
Step 11: Based upon Steps 1 through 10 synthesize the prefilter $F(s)$.
Step 12: Simulate the system in order to obtain the time response data for each of the J plants.

The following sections illustrate this design procedure.

**2.3 Synthesize Tracking Models**

The tracking performance specifications, based upon satisfying some or all of the step forcing function figures of merit for underdamped $(M_p, t_p, t_s, t_r, K_m)$ and overdamped

$(t_s, t_r, K_m)$ responses, respectively, for a simple-second system, are depicted in Figure 6(a). The Bode plots corresponding to the time responses $y(t)_U$ [Eq. (3)] and $y(t)_L$ [Eq. (4)] in Figure 6(b) represent the upper bound $B_U$ and lower bound $B_L$, respectively, of the acceptable performance area; i.e., an acceptable response $y(t)$ must lie between these bounds.

Note that for the m.p. plants, only the tolerance on $|\mathbf{T}_R(jw_i)|$ need be satisfied for a satisfactory design. For nonminimum-phase (n.m.p.) plants, tolerances on $\angle\mathbf{T}_R(jw_i)$ must also be specified and satisfied in the design process.[3,4] It is desirable to synthesize the tracking control ratios

$$T_{R_U}(s) = \frac{(w_n^2/a)(s+a)}{s^2 + 2\varsigma_n s + w_n^2} = \frac{(w_n^2/a)(s-z_1)}{(s-\sigma_1)(s-\sigma_2)} \tag{5}$$

$$T_{R_L}(s) = \frac{K}{(s+a_1)(s+a_2)(s+a_3)} = \frac{K}{(s-\sigma_1)(s-\sigma_2)(s-\sigma_3)} \tag{6}$$

that correspond to the upper and lower bounds $T_{R_U}$ and $T_{R_L}$, respectively, so that $\delta_R(jw_i) = B_U - B_L$ increases as $w_i$ increases above the 0 dB crossing frequency of $T_{R_U}$. This characteristic of $\delta_R$ simplifies the process of synthesizing the optimal loop transmission function $L_o(s) = G(s)P_o(s)$.

This synthesis process requires the determination of the tracking bounds $\mathbf{B}_R(jw_i)$ that are obtained based upon $\delta_R(jw_i)$. The achievement of the desired performance specification is based upon the frequency bandwidth $BW, 0 < w \le w_h$, which is determined by the intersection of the -12 dB line and the $B_U$ curve in Figure 6(b).

**2.4 Disturbance Model**

The simplest disturbance control ratio model specification is $|\mathbf{T}_D(jw)| = |\mathbf{Y}(jw)/\mathbf{D}(jw)| \le \alpha_P$ a constant [the maximum magnitude of the output based upon a unit step disturbance input ($D_1$ of Figure 5)]. Thus the frequency domain disturbance specification is Log magnitude $(Lm)\mathbf{T}_D(jw) \le Lm\,\alpha_p$ over the desired specified bandwidth (BW) $0 \le w \le w_h$ as defined in Figure 6(b). Thus the disturbance specification is represented by only an upper bound on the NC over the specified BW.

### 2.5 J LTI Plant Models

Consider the simple plant of Eq. (1) where $K \varepsilon \{1,10\}$ and $a\ \varepsilon\ \{1,10\}$, is used to illustrate the MISO QFT design procedure. The region of plant parameter uncertainty is illustrated by Figure 7. This region of uncertainty may be described by J LTI plants, where $\iota = 1, 2, ..., J$. These plants lie on the boundary of this region of uncertainty (see Figure 2). That is, the boundary points 1, 2, 3, 4, 5, & 6 are utilized to obtain six LTI plant models that adequately define the region of plant parameter uncertainty.

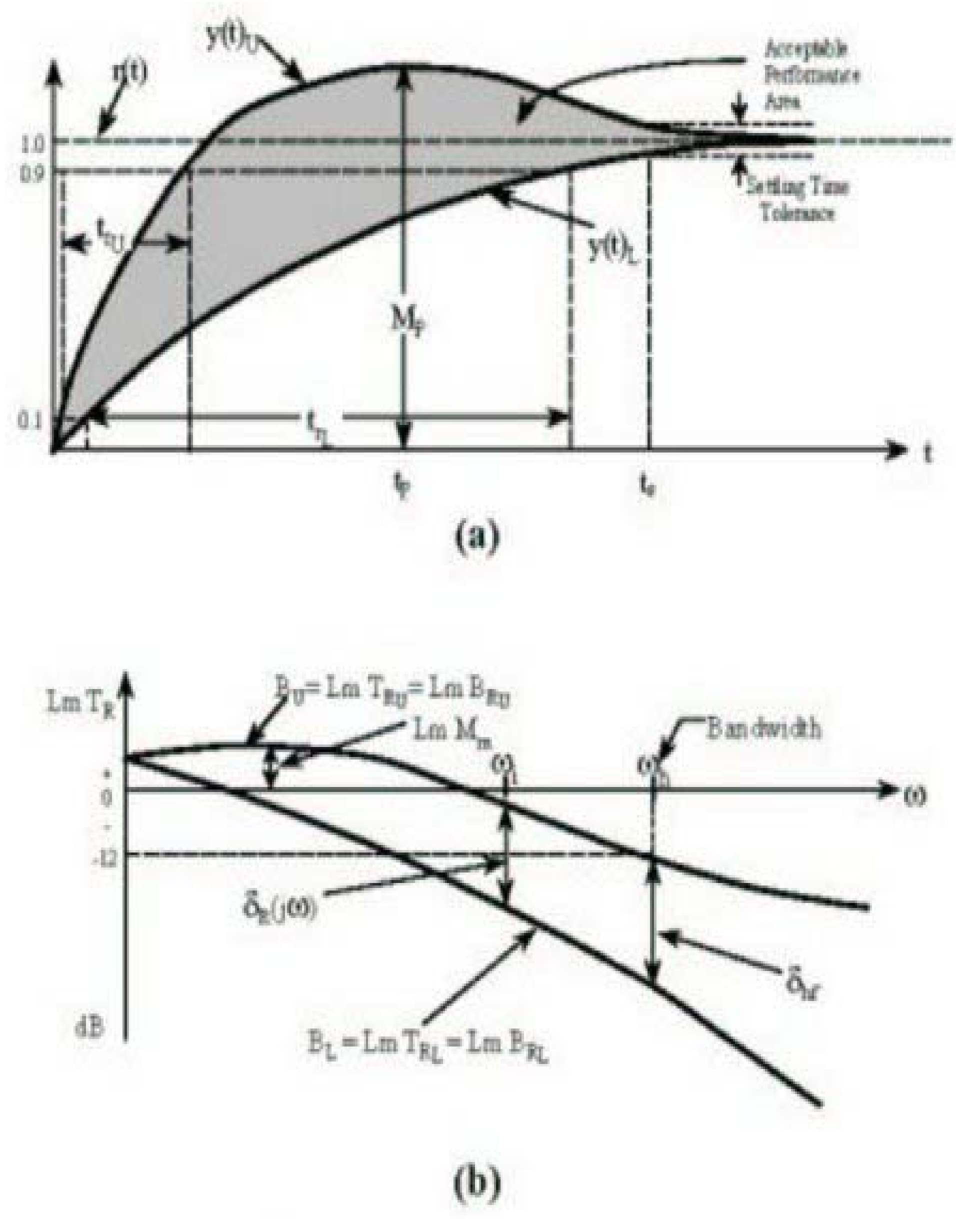


Figure 6 Desired system performance specifications: (a) time domain response specifications; (b) frequency domain response specifications.

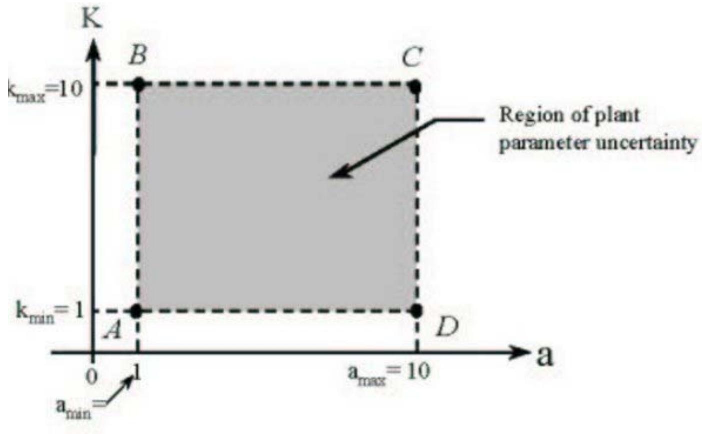


Figure 7: Region of plant parameter uncertainty.

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**Bibliography**

Arnold, P. B., I. M. Horowitz, and C. H. Houpis (1985). YF-16CCV Flight Control System Reconfiguration Design Using Quantitative Feedback Theory *Proceedings of the National Aerospace and Electronics Conference* (NAECON) **1**. 578-585. [This presents a QFT design of a real world MIMO plant with structured uncertainty.]

Horowitz, I. M. (1973). Optimum Loop Transfer Function in Single-Loop Minimum Phase Feedback Systems. *Int. J. of Control* **22**, 97-113. [This work is one of the earliest papers on QFT.]

Horowitz, I. M. (1976). Synthesis of Feedback Systems with Non-Linear Time Uncertain Plants to Satisfy Quantitative Performance Specifications. *IEEE Proc.* **64** 123-130. [This work is one of the earliest papers dealing with non-linear time uncertain plants.]

Horowitz, I. M. (1979). Quantitative Synthesis of Uncertain Multiple-Input Multiple-Output Feedback Systems. *Int. J. of Control* **30**, 81-106. [This work is one of the earliest QFT papers dealing with MIMO systems.]

Horowitz, I. M. and Y. K. Liao (1986). "Quantitative Feedback Design for Sampled-Data System. *Int. J. of Control* **44** 665-675. [This work is one of the earliest papers applying QFT to digital control systems.]

Horowitz, I. M., and C. Loecher (1981). Design of a 3×3 Multivariable Feedback System with Large Plant Uncertainty. *Int.J. of Control* **33**, 677-699. [This work is one of the earliest QFT papers dealing with MIMO systems.]

Horowitz, I. M. and M. Sidi (1972). Synthesis of Feedback Systems with Large Plant Ignorance for Prescribed Time Domain Tolerances. *Int. J. of Control* **16**, 287-309. [This work is one of the earliest papers on QFT.]

Houpis, C. H. and P. R. Chandler, Editors (1992). Quantitative Feedback Theory Symposium Proceedings. WL-TR-92-3063, Wright Laboratories, Wright-Patterson AFB, OH. [A collection of QFT papers that were presented at the first QFT symposium held at Wright-Patterson Air Force Base, Ohio in August 1992.]

Houpis, C. H. and G. Lamont (1992). Digital Control Systems: Theory, Hardware, Software. New York: McGraw-Hill, NY, 2nd Ed. [The first published text book that presents the QFT design for sampled-data and digital control systems.]

Houpis, C. H. and S. J. Rasmussen (1999). QUANTITATIVE FEEDBACK THEORY Fundamentals and Applications. New York: Marcel Dekker. [The first textbook that presents the fundamentals of QFT for MISO and MINO systems with applications.]

Houpis, C. H. and R. R. Sating (1997). MIMO QFT CAD Package (Version 3). *Int. J. of Control* **7**, 533-550. [The first MINO QFT CAD package that was developed at the Air Force Institute of Technology at Wright-Patterson Air Force Base, Ohio.]

Keating, M. S., M. Pachter, and C. H. Houpis (1994). Damaged Aircraft Control System Design Using QFT. Ohio *Proceedings of the National Aerospace and Electronics Conference* (NAECON) **1** 621-628. [This presents a QFT design of a real world MIMO plant with structured uncertainty.]

Reynolds, O.R., M. Pachter, and C. H. Houpis (1994). Design of a Subsonic Flight Control System for the Vista F-16 Using Quantitative Feedback Theory *Proceedings of the American Control Conference* 350-354. [This presents a QFT design of a real world MIMO plant with structured uncertainty.]

Trosen, D. W., M. Pachter, and C. H. Houpis (1994). Formation Flight Control Automation. *Proceeding of the American Institute of Aeronautics and Astronautics* (AIAA) Conference, 1379-1404. [This presents a QFT design of a real world MIMO plant with structured uncertainty.]

## Biographical Sketch

**Dr. Constantine H. Houpis** is currently a Professor Emeritus at the Air Force Institute of Technology in Ohio, where he has worked since 1952. He received his BS and MS degrees in Electrical Engineering from the University of Illinois in 1947 and 1948, respectively. Dr. Houpis graduated from the University of Wyoming with a PhD in Electrical Engineering in 1971. He is an internationally recognized educator who has published extensively in the area of control theory. The textbooks that he has co-authored with Professor John J. D'Azzo are used in colleges and universities around the world. He has been very active in the area of Quantitative Feedback Theory (QFT) and as a Senior Research Associate in the Air Force Research Laboratory. As a Professor Emeritus, Dr. Houpis is still active in publishing and presenting papers at national and international technical conferences. He is a Fellow of the Institute of Electrical and Electronics Engineers (IEEE). Dr. Houpis was born in Lowell, MA, and is married to Mary Stephens of Weirton, W: VA. They have a son Harry and a daughter Angella, both living in Dayton, OH.