FAULT-TOLERANT CONTROL USING LMI DESIGN

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Contents

Summary

This chapter outlines some of the recent developments in the design of robust fault-tolerant control systems, based on the use of linear matrix inequalities (LMI). Starting from the assumption that the effects of faults can be expressed in Linear-Fractional-Transformation (LFT) forms, a fault-tolerant control systems design problem is formulated and solved via a linear matrix inequality (LMI)-based synthesis. In order to recover the convexity of the design problem while considering the robust performance and robust stability against faults and uncertainties simultaneously, a constrained optimization approach is used.

To utilize fully the potential of fault-tolerant control, faults should be promptly and reliably detected using a fault diagnosis system. This chapter also presents the use of LMI in the design of fuzzy observers capable of generating robust residuals for systems with uncertainty or non-linear systems. The fuzzy observer, based on the concept of Takagi-Sugeno fuzzy inference modeling, comprises a number of locally linear observers. It is shown that the estimated state vector is generated using a fuzzy fusion of

all local observer outputs. The stability as well as eigenvalue constraint conditions for the fuzzy observer design are presented and solved in the LMI framework.

1. Introduction

A fault-tolerant control system is designed to retain some of its control integrity in the event of a specified set of possible component faults or large changes in system operation (considered as faults). A component fault is refereed to a change in the operating behaviour of the component such that the new behaviour differs significantly from what is defined as nominal behaviour for that component. Common examples of such faults include output bias errors in a sensor and loss of actuator function. Fault-tolerant control involves automatic detection and identification of failed components prior to on-line control law reconfiguration in response to these decisions.

A fault-tolerant control system must have the ability to adjust *off-nominal* behaviour, which occurs during sensor, actuator, or other component faults. A fault-tolerant control system (see Fig.1) consists of three primary parts: a controller, a fault diagnosis scheme, and a control law reconfiguration mechanism. The control and diagnostic modules, which are designed separately, are linked through the reconfiguration module to achieve reliable control. After the diagnostic module detects and isolates a fault in specific component, the reconfiguration module specifies a reconfiguration strategy for the control module. The aim is to maintain satisfactory control performance and stability in the presence of the fault.

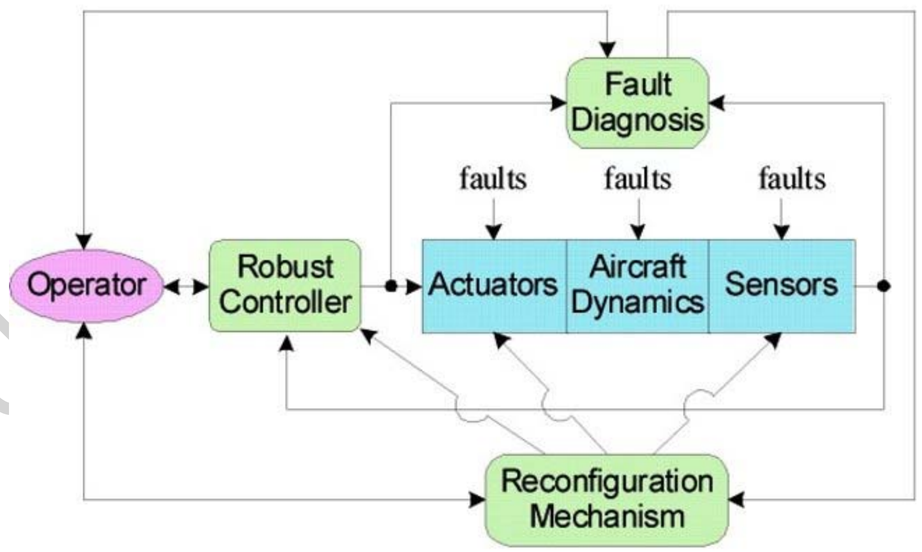

Figure 1: General scheme of an active fault tolerant control system

Key challenges in the fault-tolerant control problem are to design: (a) a sufficiently robust controller which is reconfigurable, (b) a robust fault diagnosis scheme and (c) a suitable reconfiguration mechanism. A fault-tolerant controller can engage suitably fixed or varied structure to perform its fault-tolerant functions. If the structure and/or parameters of a controller are adjusted on-line according to the real-time measurement of fault effects, the controller is known as an *active fault-tolerant controller*. This chapter describes the use of LMI design approaches for designing robust fault-tolerant controllers and robust fault diagnosis schemes. Section 2 describes the problem of fault-tolerant control system design using LMI. The robust fault diagnosis scheme for uncertain and non-linear systems using LMI is studied in Section 3.

2. Active Fault-Tolerant Control Systems Design Using LMI Design

To date a considerable amount of research has been conducted in the field of fault-tolerant control, although the developments have been scattered throughout the engineering literature. Here we focus on a realistic example of a robust fault-tolerant control problem formulated using LMI. With this formulation a satisfactory performance and guaranteed robust stability can be achieved simultaneously. We use the *Bounded Real Lemma* corresponding to the parameter-dependent controller structure. When compared with the robust control system design problem, the fault-tolerant control system design should consider the imperfect estimation of fault effects which are provided by the fault diagnosis mechanism.

The non-zero estimation error is due to exogenous disturbances and modeling uncertainty which are always present in flight control systems. In the fault-tolerant control system design, a set of performance specifications is presented. In this way both robust performance and robust stability with respect to modeling uncertainty can be considered. The simultaneous consideration of modeling uncertainty and system performance will introduce a rank constraint in the LMI formulation.

This usually leads to a non-convex optimization problem and its numerical tractability is discussed. A constrained optimization approach is used to build up an LMI formulation for control system synthesis which is based on the assumption that both fault effect factors and uncertainties can be of linear-fractional-transformation (LFT) parameter-dependence..

2.1. Fault-Tolerant Control System Formulation

During closed-loop system operation, the effects of a fault on the controlled system could lead to changes in signals and parameters. For small parameter and signal changes, a robust controller is sufficient to achieve a passive form of fault-tolerance. However, in many practical cases, changes in system parameters caused by faults are large. It can be impossible to stabilize a system with large system parameter changes, just by means of a fixed robust controller.

Here a fault detection and isolation (FDI) mechanism is engaged to provide diagnostic information which is used to supervise the controller reconfiguration or restructure via active fault-tolerance. The scheme for the fault-tolerant control system (FTCS) with supervision and FDI is given in Fig.2, where $\Delta(s)$ reflects uncertainties, $\xi(t)$ means the factors of fault effects, $P(s,\xi)$ denotes the plant model, $K(s,\hat{\xi})$ represents the feedback controller.

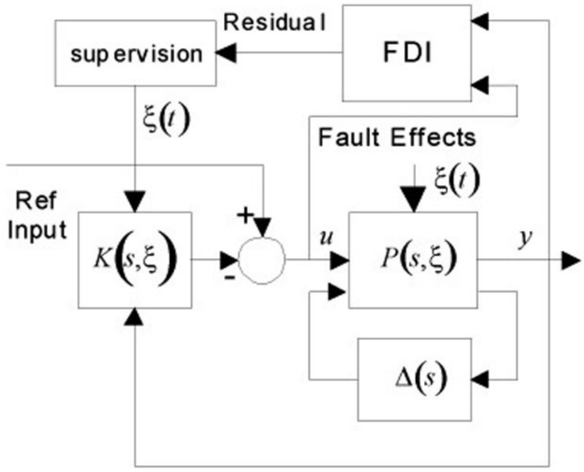

Figure 2: General scheme of fault-tolerant control system

The manifestation of fault varies in different systems, with different components, and with different kinds of faults. However, many categories of faults such as additive sensor/ actuator faults and some forms of multiplicative faults can be detected very effectively by a variety of model-based FDI approaches. In many cases, fault effects on system matrices can be of linear-fractional-transformation (LFT) parameter-dependence. Hence, a set of parameters termed *fault effects factors* can be defined and used to describe the effects of each fault. It is convenient to describe the fault effects in LFT forms as one of the following:

$$F_l(P,\Xi) \quad = \quad P_{11} + P_{12}\Xi(I - P_{22}\Xi)^{-1}P_{21} \tag{1}$$

$$F_u(P,\Xi) \quad = \quad P_{22} + P_{21}\Xi(I - P_{11}\Xi)^{-1}P_{12} \tag{2}$$

The sensor fault can be modeled in the system as an additive signal f_s which is described in Fig.3. By choosing the vector f_s correctly, we can then describe all sensor fault situations. When the sensors are “stuck at a particular value” (say at zero), the sensor measurement is zero and the fault vector is $f_s = -y$ .when there is a variation in the sensor scalar factors (multiplicative faults), the fault vector can be then designed as $f_s = \Delta y$, where Δ is the variation in the sensor scalar factor.

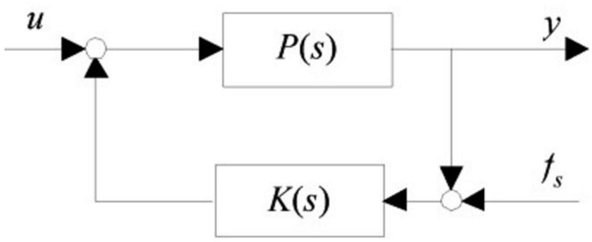

Figure 3: System with sensor fault

As shown in Fig.2, the FDI scheme and controller work together to achieve fault-tolerance. The controller should make the system robust enough to cope with small faults which cannot be easily detected by the FDI scheme. When the fault increases in magnitude, the FDI scheme starts to operate. That is to say, the FDI scheme detects and compensates the fault in a manner normally achievable using fault estimation. It is clear therefore that the controller in a fault-tolerant control system should be designed to be robust enough to cope with "smaller" faults. In this study, the fault is considered to be bounded, i.e. $\| f_S(s) \|_2 < \gamma$. The larger faults can be detected and compensated and fault effects which cannot be compensated by FDI are treated as the same as the bounded small faults.

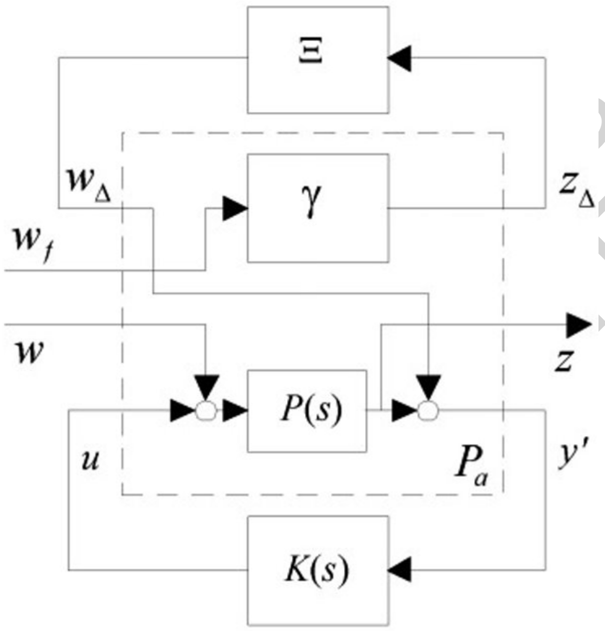

Figure 4: Scheme of equivalent augmented system (sensor faults)

To formulate the design problem, the equivalent augmented system which incorporates fault effects is expressed as shown in Fig. 4, where $\|\Xi\|_\infty < 1$.

Actuator faults can be treated in a similar way to sensor faults. The influence of actuator faults is illustrated in Fig. 5, and the equivalent augmented system is shown in Fig.6.

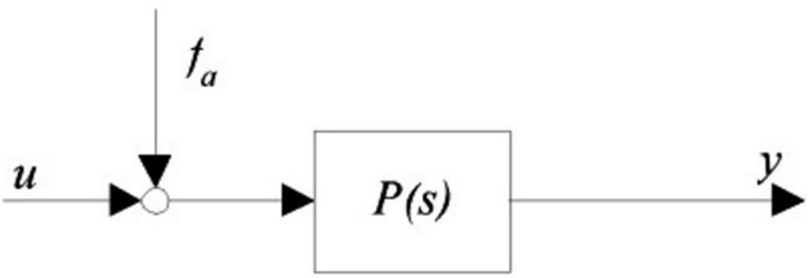

Figure 5: System with actuator faults

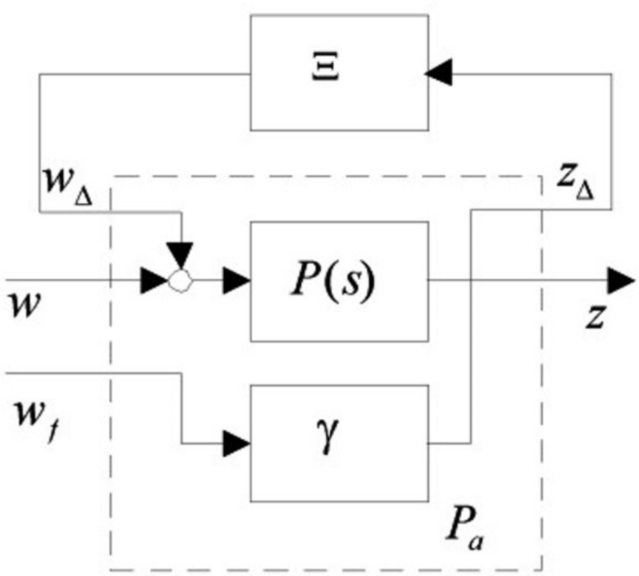

Figure 6: Equivalent augmented system scheme (actuator faults)

Parameter changes, caused by faults, are readily expressible in the form of a feedback connection, similar to the one used for parameter uncertainties.

Based on the above description, the fault-tolerant control system shown in Fig.7 can be re-structured in Fig.7. In Fig.7, the dashed lines represent the FDI signal paths, and the solid lines give the scheme of connection for the control system design.

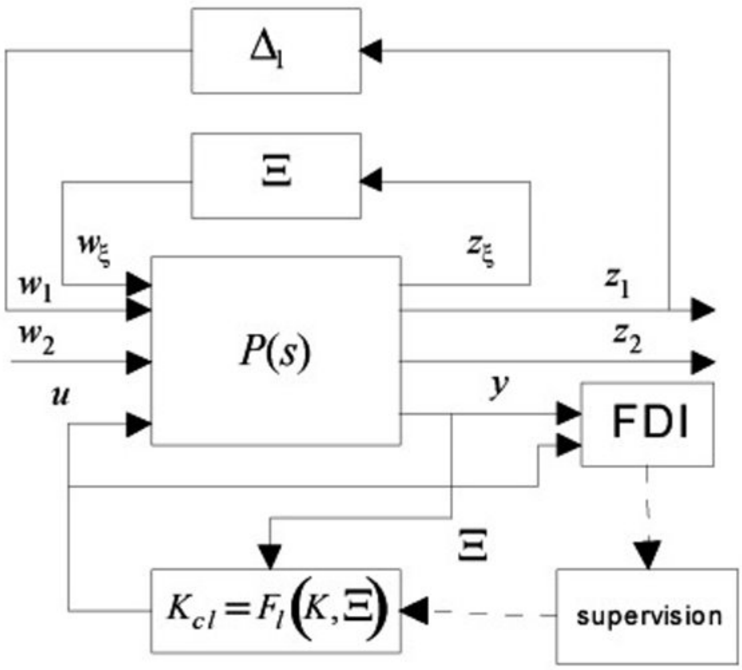

Figure 7: Signal connection scheme in fault-tolerant control system

In this scheme, on-line information provided by the FDI scheme is used to adjust the controller. For additive sensor and actuator faults, estimates of fault effect factors can be derived from the residuals generated by state estimation or parity equation approaches. For multiplicative faults, estimates can be generated by means of parameter estimation. In a practical situation, it is impossible to express fault effects precisely. All inaccuracies, modeling errors, and effects of non-linearity should be considered as uncertainties to achieve controller robustness with the assumption $\hat{\Xi} \approx \Xi$. As for uncertainties, it is well known that in many cases, it would be convenient to describe them in LFT forms.

In Fig. 7, $\Xi \in \Lambda_\xi$ is the fault effect factor matrix, $\Delta_1 \in \Lambda_\Delta$ is the disturbance matrix, and $\hat{\Xi} \in \Lambda_\xi$ is the matrix for estimated fault effect factors. Here, it is worth mentioning that the relationship between controller and fault effect factor matrix has already been defined as an LFT connection:

$$K_{cl} = F_l(K(s), \hat{\Xi}) \approx F_l(K(s), \Xi) \qquad (3)$$

Considering the use of H_∞ optimization for fault-tolerant controller synthesis, the problem now can be expressed as:

Problem: Find an internally stabilizing controller $K(s)$ such that the closed-loop system $T_{z_2 w_2}$ is internally stable for all fault factors $\|\Xi\|_\infty \leq 1$ and uncertainties $\|\Delta_1\|_\infty \leq 1$, and so that the H_∞ norm of closed-loop transfer function matrix satisfies

$$\left\|T_{z_2 w_2}(P, K, \Xi)\right\|_\infty < 1, \qquad (4)$$

for all $\|\Xi\|_\infty \leq 1$ and $\|\Delta_1\|_\infty \leq 1$.

Remarks: (a) The satisfaction of performance inequality (4) implies that the influence of external disturbances and fault effects on the system is minimized.

This is the performance robustness normally reserved for robust control. (b) It should be pointed out that the performance specification is defined for systems with faults. Ideally, this performance specification should be the same as in the fault-free operation if either the fault or fault compensation error are bounded.

In real applications it may not be possible for the fault-tolerant control system to achieve the same performance as the fault-free control system. A certain degree of performance degradation is acceptable if the system stability can be guaranteed.

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Biographical Sketches

Dr Jie Chen received BEng and MSc degrees in Control Systems Engineering from Beijing University of Aeronautics and Astronautics, China, in 1984 and 1987 respectively, and Dphil degree in Electronic Engineering from University of York, UK, in 1995. He joined the Department of Mechanical Engineering of Brunel University as a Lecturer of Aeronautical Engineering in February 1998. Before that, he worked in the University of Hull, UK as a Lecturer of Control Systems Engineering between July 1995 to January 1998. From March 1990 to September 1994, he worked as a Research Associate, in the University of York, UK, while he pursued his DPhil degree. From October 1994 to June 1995, he spent a short period in the University of Strathclyde, UK as Post-Doctoral Research Fellow. He has worked in the field of fault diagnosis and fault-tolerant control for many years and has published over 60 papers in international journals and conference proceedings on the subject. He is a member of IFAC Technical Committee: SAFEPROCESS. He was awarded jointly with R. J. Patton the 1997 IEE Kelvin Premium for a paper published in IEE Proceedings-D. His current research interests are: model-based fault diagnosis and applications to non-linear systems, robust and fault-tolerant control, neuro-fuzzy techniques for control and fault diagnosis.

Professor Ron Patton was born in Peru on the 8th March 1949 and was educated at Emmanuel Grammar School, Swansea and Sheffield University, graduating with BEng (1972) in Electronic and Electrical Engineering and Meng (1974), PhD (1980) degrees in Control Systems Engineering. He is Senior Member of the AIAA and IEEE and currently holds Membership of the IEE and is Chartered Engineer in the United Kingdom. Working in the hospital service in medical physics during 1968/1969 Ron was a founder member of the Electronics laboratory at the Royal Free Hospital, London. During 1972/1973 Ron worked as a telecommunications expert at the BBC Research Department, UK. He turned later to control systems and after pursuing PhD studies on non-linear dynamics in biology Ron then worked for GEC Electrical Projects, Rugby and Sheffield City Polytechnic on Kalman filtering in ``Dynamic Ship Positioning Control Systems". Ron became lecturer at Sheffield Hallam University in 1978 and moved as lecturer to the then new Electronics Department at York University in 1981 where he focused on fault diagnosis and aerospace control systems, with promotion to Senior Lecturer in 1987. In 1995 he was appointed to his present position of Professor of Control and Intelligent Systems Engineering in the University of Hull. Professor Patton is a well-known expert on the research topics of model-based fault diagnosis, fault-tolerant control and eigenstructure assignment design, having published 7 books covering these subjects and authored more than 270 papers in leading journals and international conferences. With Dr Jie Chen he was recipient of the IEE Kevin Premium award in 1997 for an IEE Proceedings D paper on stochastic approaches to robust fault diagnosis. During 1993/1994 he chaired the IEEE UK & R Ireland Region 8 Chapter on Control Systems and chaired the International programme Committees for UKACC CONTROL'98 and IFAC SAFEPROCESS'97. He has served on numerous conference committees in control engineering.

During 1996 to 2002 Ron served the International Federation of Automatic Control (IFAC) as chairman for the Technical Committee *SAFEPROCESS*, leading this field into one of the main technical activities of IFAC. He is Editor in *Fault-tolerant control (system supervision and diagnosis*) for the *Journal of Adaptive Control and Signal Processing* and serves on the editorial boards of *Control Engineering Practice*, *International Journal of Applied Maths. &Computing* and the I. Mech. E. *Journal of Systems & Control Engineering*}.