TRAJECTORY AND TASK PLANNING

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Summary

In general, the problems of robots can be divided into two subproblems of motion planning and motion control. A motion planning problem is solved where a geometric model is given. Furthermore, motion planning problems can be fundamentally divided
into path, trajectory, and task planning problems. These planning problems have many constraints concerning kinematics and dynamics of the robot as well as its environment. This paper introduces path, trajectory, and task planning methods for mobile robots and robot manipulators.

1. Introduction

Recently, robots have been seen in various fields. In general, robots can be divided into mobile robots and arm robots (robot manipulator). Lately, the mobile robots with manipulators including humanoid robots have been developed for improving the performance and flexibility. The main aim of mobile robots is to carry materials, products, tools and others, while the aim of robot manipulators is to handle them. The motion of a robot is basically constrained by its dynamics, kinematics, and environment. Because the robot is controlled using joints and wheels, the motion of a robot results from the motion of the controllable degrees of freedom (DOF). Basically, the problems of robots can be divided into two subproblems of motion planning and motion control. First, the motion planning is solved when a geometric model is given. Next, the motion control for a physical robot is done according to a planned trajectory. This paper focuses on the motion planning.

A robot receives a task from a human operator and performs the task in the workspace including a lot of obstacles such as humans, machining centers, and other robots. The robot should take into account the collision avoidance with the obstacles. Furthermore, the robot should generate its motion satisfying the spatial and temporal constraints for performing the task. There exist various methods for solving motion planning problems (Donald et al. (1993), Fogel (1995), Goldberg (1989), Bertsekas (1999), Ecker and Kupferschmid (1998), Canny (1988), Brooks (1983), Paul (1981), Russell and Norvig (1995), Fukuda and Kubota (1999a,b), Fukuda et al. (1997), Lozano-Perez. (1981), Davidor (1991)). The motion planning problems for performing given tasks can be fundamentally divided into path planning problems, trajectory planning problems, and task planning problems (Figure 1).

Here we define these planning problems as follows. The path planning problem requires generating the shortest path for the robot from a given starting point to a target point while satisfying the spatial constraints. The trajectory planning problem requires generating a trajectory that satisfies the time constraints. The task planning problem requires finding a sequence of primitive motion commands for solving a given task. In fact, each definition of these problems is conceptually differentiated from other planning problems, but each planning problem might share some elements of other planning problems. Furthermore, Kinodynamic motion planning has been proposed
(Russell and Norvig (1995)). Kinodynamic motion planning attempts to solve a robot motion problem subject to kinematic constraints, such as joint limits and collision avoidance, and dynamic constraints, such as modulus bounds on velocity, acceleration, and force, simultaneously. In the following, we introduce path, trajectory, task planning, and their optimization methods.

2. Path Planning for Mobile Robots

The path planning problem is one of the most fundamental problems in robotics. In general, a physical motion planning problem is transformed into geometrical path planning problem. Accordingly, a robot is represented as a point in an appropriate search space, because the motion of a particle mass is easy to describe. Therefore, a path can be represented as a route from a point to another on the search space. The path planning problems are close related with the collision avoidance problems.

![Figure 1: Task, trajectory, and path planning for robotic motions](image)

Actually, the path planning problem is to find a path connecting some points for avoiding the collision with obstacles in a workspace. A general approach for the path planning is shown as follows. First, a two-dimensional map around the robot including the starting and target points is built. An obstacle in the workspace is represented as an approximated polygonal object. The size of a polygonal object is larger than the real size of the object. If the size of a robot is added to that of a polygonal object, a robot can be represented as a point in the workspace map, not a polygon, because the size of every polygonal object includes the size of the robot (Figure 2). This method can simplify the workspace map with any accuracy. However, the workspace map is still continuous. In
order to reduce the search space size, the workspace map can be transformed into various types of search spaces from the visual point of view. A search space is built by cell decomposition methods, Skeletonization (roadmap) methods, and/or artificial potential field methods (Fogel (1995), Goldberg (1989), Bertsekas (1999), Ecker and Kupferschmid (1998), Canny (1988)). In the cell decomposition methods, a two-dimensional workspace is basically divided into a finite number of cells. The skeletonization method transforms the workspace into the set of vertices and paths that enable a graph search. In the artificial potential field method, a robot moves based on attractive force from the target point and repulsive force from the obstacles in the workspace. Next, we must consider search algorithms for optimizing paths in the built map. Generally, the path planning can be classified into combinatorial optimization in the discrete search space and numerical optimization in the continuous space. The detail of optimization algorithms is explained later. In the following, we discuss how to build a map (search space) and how to solve the path planning problem.

![Image: Polygonal workspace approximated from a real environment](image-url)

**Figure 2:** Polygonal workspace approximated from a real environment

### 2.1. Map Building

It takes much time to search a continuous workspace map generated by a polygonal approximation method. Therefore, the workspace map should be reduced into finite discrete search space. In a cell decomposition method, a two-dimensional workspace is often divided into $M \times N$ rectangular cells. Generally, a cell is represented by geometrically simple shape (Figure 3). If the least size of a cell is larger than the size of the robot, we don’t need to take into account the size of the robot in the search space. It is a general problem to choose the resolution of decomposition. A feasible path might not be found if the size of cells is large, while the search space becomes big if the size of cells is small. The size of cells should be adaptively chosen according to the state of search.
Skeletonization methods directly generate intermediate points and paths, while cell decomposition methods generate collision-free space. In the skeletonization methods, collision-free paths are basically generated according to the polygonal objects approximated in a workspace. Visibility graph consists of edges connecting visible pairs of vertices of the polygonal objects (Figure 4). In the visibility graph, the shortest path between two points can be generated easily by selecting edges (see Figure 4). However, it is dangerous for a mobile robot to move along the generated path, because the path is adjacent to the vertices of the polygonal objects. To overcome this problem, a Maklink graph can be used to generate a safe path. This method can be considered as one of the approximated Voronoi diagrams. In the Maklink graph, a candidate point is represented as a middle point between two vertices, and a path is generated by connecting some intermediate points (Figure 5). Although the generated path is safe, it might not be the shortest.

Figure 3: A cell decomposition method

Figure 4: A visibility graph based on the polygonal objects
In the above methods, the collision-free space is simply distinguished from obstacles in a workspace, but the distances from each point to obstacles and target point are not considered. Therefore, artificial potential field methods have been applied for path planning. In this method, a path is generated according to attractive force from the target point and repulsive force from the obstacles in the workspace. Basically, an ideal potential value is defined on each point of the search space, and a steepest decent approach is used to generate a path to reach a target point from a starting point. However, a potential function might have local minima. A point $q$ is a local minimum when $f(q) < f(q')$ where $f$ is a potential function; $q'$ are neighbor points of the point $q$; and $q'$ is not $q$. Various methods have been proposed for overcoming this problem so far. For example, the robot might be able to escape the local minima by reorganizing the potential function composed of attractive force and repulsive force, but the generated path might not be the globally optimal. Accordingly, it is difficult to generate a potential function suitable to the search in a given workspace.

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**Figure 5**: A Maklink graph

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Biographical Sketches

Toshio Fukuda graduated from Waseda University in 1971 and received the Master of Engineering degree and Dr. Eng. from the University of Tokyo in 1973 and 1977, respectively. Meanwhile, he studied at the graduate school of Yale University from 1973 to 1975. In 1977, he joined the National Mechanical Engineering Laboratory and became Visiting Research Fellow at the University of Stuttgart from 1979 to 1980. He joined the Science University of Tokyo in 1982, and then joined Nagoya University in 1989. Currently, he is Professor of Department of Micro System Engineering and Department of Mechano-Informatics and Systems, Nagoya University, Japan, mainly engaging in the research fields of intelligent robotic system, cellular robotic system, mechatronics and micro robotics. He is an author of six books, editing five books and has published over 1,000 technical papers in micro system, robotics, mechatronics and automation areas. He was awarded IEEE Fellow, SICE Fellow (1995), IEEE Eugene Mittlemann Award (1997), Banki Donat Medal form Polytechnic University of Budapest, Hungary (1997), Medal from City of Sartillo, Mexico (1998), IEEE Millennium Medal (2000) and JSME Fellow (2001). He is the Vice President of IEEE IES (1990 - 1999), IEEE Neural Network Council Secretary (1992 -1993), IFSA Vice President (1997 - ), IEEE Robotics and Automation Society President (1998 - 1999), current Editor-in-Chief, IEEE / ASME Transactions on Mechatronics (2000 - ), current IEEE Division X Director (2001-), and current IEEE Nanotechnology Council President (2002-)

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