MODELING FLOWS IN COLLAPSIBLE TUBES

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Contents

Summary

This chapter summarizes some recent modeling work of flows in flexible or collapsible vessels, with particular focus on a prototype problem concerning self-excited oscillations in the Starling Resistor. Although self-excited oscillations in collapsible tube flows have been extensively studied, our understanding of their origins and mechanisms is still far from complete. The paper starts briefly with the background to the problem, and introduces some mathematical and numerical approaches based on one-dimensional, two-dimensional, and three-dimensional models. The summary is not intended to be exhaustive but is designed to offer a flavor of the research in this area, and is inevitably focused on the work familiar to this author.

1. Introduction

In physiological fluid mechanics, blood flow inside large vessels may collapse and experience interesting self-excited oscillations under a negative transmural (internal minus external) pressure. These vessels are thus collapsible tubes. Flows in collapsible tubes form a significant branch of the biological and physiological applications of internal flow. Veins above the level of the heart can collapse as the transmural pressure P_{tm} reduces as external muscles squeeze (Wild et al, 1977, Pedley, 1980). Intramyocardial coronary blood arteries collapse during heart contraction in systole (Guiot et al., 1990). Similar behavior is seen in the branchial arteries compressed by a sphygmomanometer cuff (Bertram and Ribreau, 1989), in the large airways during forced expiration (Shapiro, 1977, Kamm and Pedley, 1989), and in the urethra during micturition (Griffiths 1989).

These, and the closely related problems, have been studied by various research groups over the last 40 years, ranging from flow in giraffe jugular vein to peristaltic pumping, and to flow over compliant surfaces (Shapiro, 1977, Brook and Pedley, 2002, Ku, 1997, Elad et al., 1989, Dia et al., 1999, Tang et al., 1999, Tutty and Pedley, 1993, Elad et al., 1987, Gavriely et al., 1982, Kamm and Shapiro, 1979, Davies and Carpenter, 1997, Cancelli and Pedley, 1985, Carpenter and Pedley 2003, Carew and Pedley, 1997, Jensen, 1990, Rast, 1994, Luo and Pedley, 1996, Grotberg and Gavriely, 1989, Hazel and Heil, 2003, Heil and Pedley, 1995, Guneratne and Pedley, 2006), to mention just a few. Some earlier reviews are given by Kamm and Pedley (1989), Pedley and Luo (1998), Bertram (2004), and Grotberg and Jensen (2004).

In this chapter, we shall limit our attention to studies that form a sub-set of applications concerning the flow in large vessels, focusing on the phenomena observed from the Starling Resistor.

2. The Starling Resistor and the Tube Law

The Starling resistor is a commonly used bench-top apparatus for studying flow in collapsible tubes, as depicted in Figure 1.

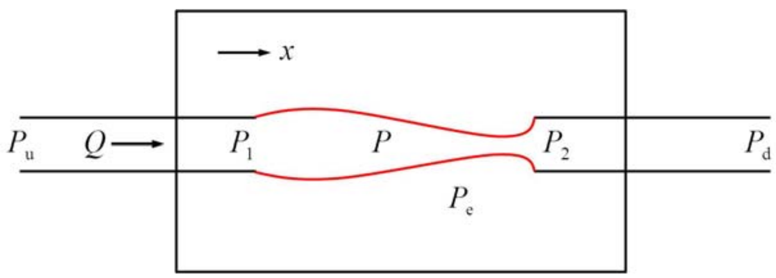

Figure 1. Sketch of a Starling resistor. p_1, Q are pressure and flow rate upstream of the collapsible segment; p_e is the external pressure, p_2 is the pressure downstream, p_u is the total pressure far upstream and p_d is the pressure far downstream. In the absence of the upstream and downstream tube resistances, $p_1 = p_u$, and $p_2 = p_d$.

It was first used in a study of cardiac functions by Knowlton and Starling (1912) to predict the collapse of a tube. The apparatus consists of a collapsible rubber tube fixed at both ends inside a chamber where the external pressure p_e can be adjusted independently. Fluid is held upstream in a reservoir from which it passes through a rigid tube into the collapsible segment in the chamber and out into a downstream reservoir. Resistors downstream are in place to control pressure and flow at the entrance and exit of the collapsible segment. When flow is driven through the elastic tube section, the transmural pressure, $p - p_e$, can become sufficiently negative due to the Bernoulli effect, that the tube collapses, partially or fully, with reduced cross-sectional area A. The relationship between $p - p_e$ and A is known as the “Tube law".

Unlike fluid flowing through a rigid tube, here more combinations of control parameters are possible. For example, one may specify the pressure head $(p_1 - p_2)$, or flow rate

Q, while keeping downstream transmural pressure $p_2 - p_e$ constant. These are sometimes referred to as the "pressure-driven system", or "flow-driven system", respectively (Liu et al., 2012). Alternatively, one can study the pressure- or flow-driven systems while keeping the upstream transmural pressure $p_1 - p_e$ fixed. Each of these settings determines a specific system with its own unique characteristics. The commonly observed "flow limitation" (Gavriely et al., 1989) and "pressure-drop limitation" (Bertram and Castles, 1999), are interesting phenomena corresponding to specific configurations of these settings. If we increase $p_1 - p_2$ while keeping $p_1 - p_e$ constant, then at some point the flow rate cannot be increased further, and this is called "flow limitation". Likewise, if we increase Q while keeping $p_2 - p_e$ constant, then soon or later the pressure-drop will stop increasing; this is "pressure-drop limitation". In fact, pressure-drop limitation can even show up as "negative effort dependence", whereby the flow rate increase is accompanied by pressure drop decrease (Gavriely et al., 1984, Gavriely and Grotberg, 1988, Luo and Pedley, 2000).

Over the last 30 years, Bertram and co–workers have conducted a sequence of extensive experimental studies of this system (Bertram, 1980, 1995, Bertram and Castles, 1999, Bertram and Chen, 2000, Bertram and Elliott, 2003, Bertram et al., 2001, Bertram and Tscherry, 2006). One of the interesting phenomena observed in these experiments is self-excited oscillation, when a spontaneous fluctuation in pressure, flow and cross-sectional area of the tube occurs (Conrad, 1969, Bertram, 1986, Low et al., 1997). It is important to recognize that pressure drop may not be the only contributing factor involved in the collapse. The length and rigidity of the tube also play a significant effect on collapse and subsequent self-excited oscillations (Bertram, 1980).

Despite the seemly straightforward experimental setup, the nature of the self-excited oscillations was proven to be rather difficult to explain. However, over the years, there have been some significant advances in the analytical and simulations. In the following, we provide a brief summary of the progress made to date, which ranges from early lumped parameter models to fully three dimensional simulations, with particular attention to several recent approaches developed by this and other groups in the last few years. Some of the earliest developments were lumped parameter, or zero-dimensional models such as (Conrad, 1969, Shapiro, 1977, Bertram and Pedley, 1983) which focused on describing pressure and flow as function of cross-sectional area only, at the narrowest point. This simplification enables one to obtain a 2nd or 3rd order ODEs, depending on inlet boundary conditions. These models can be used successfully to produce some self-excited oscillations. The models highlight the importance of energy dissipation in order to sustain the self-excited oscillations, and show that the flow-driven system (which can be described by 2nd order ODE) is more stable than the pressure-driven system (which is often described by 3rd order ODE). However, in general these models cannot incorporate many real mechanical features. Their inability to capture wave propagation is a fundamental limitation which prompts the development of 1D models.

3. One-Dimensional Models

A simple one-dimensional model for a steady flow is described by Shapiro (1997)

$$\frac{d}{dx}(uA) = 0, \tag{1}$$

$$u\frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx} - R(A,u)u, \tag{2}$$

$$p - p_{\mathrm{e}} = P(A), \tag{3}$$

where A is the cross-sectional area, u is the velocity, p is the pressure, x is the longitudinal coordinate, $Ru(R > 0)$ represents the resistance, and $P(A)$ describes the tube law.

Combining (1) and (2) , we have

$$\frac{dA}{dx} = -\frac{RuA}{c^2 - u^2},$$

where $c = \left(\frac{A}{\rho}\frac{dP}{dA}\right)^{1/2}$ is the speed of propagation of long, small-amplitude pressure waves.

These equations are exactly analogous to those for free-surface flow in shallow water channel. It is immediately clear that the steady model breaks down when u approaches c. This is known as “choking", analogous to a hydraulic jump in shallow water in a channel flow. A number of researchers have gone further to advocate that the fluid velocity becoming comparable to the wave speed, i.e. the occurrence of flow-limitation, is the major mechanism for the onset of self-excited oscillations. However, later experiments by Bertram and Raymond (1991) and computations by Luo and Pedley (2000) cast doubtover a causal link between choking and self-excited oscillations.

Indeed, it is now believed the “chocking" mechanism is not responsible for self-excited oscillations in the Starling resistor, since the length of tube used is too short for choking to occur, and the 1-D model fails to describe the downstream (tube re-opening) conditions (Pedley and Luo, 1998).

To address this issue, a modified 1-D model was developed to include tension, T, and energy dissipation (Cancelli and Pedley, 1985, Jensen and Pedley, 1989, Luo and Pedley, 1995), with the governing equations

$$\frac{d}{dx}(uA) = 0, \tag{4}$$

$$\chi u\frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx} - R(A,u)u, \tag{5}$$

$$p - p_e = P(A) - T\frac{d^2 A}{dx^2}, \tag{6}$$

where χ is the dissipation constant $(0 < \chi < 1)$, (6) is the updated tube law, and P(A) is empirically determined.

One simple form of P(A) is (Jensen and Pedley, 1989):

$$P(A) = \begin{cases} K_{\mathrm{p}}\left(1 - \alpha^{-3/2}\right) & \text{for } \alpha < 1, \\ K_{\mathrm{p}}\left(\alpha - 1\right) & \text{for } \alpha > 1, \end{cases}$$

where $\alpha = A/A_0$, and K_{p} is a constant. Using this model, Jensen and Pedley (1989) showed that, as long as there is energy loss in the system i.e. $\chi < 1$, then a steady solution exists for all positive values of flow rate and tension T. Since energy loss is inevitable, this suggests that the breakdown of the steady flow model is not caused by choking, but must arise through the global instabilities of the steady solutions. Indeed, this has been proved by (Luo and Pedley, 1996), among others, using two-dimensional models. More advanced 1-D approaches were used by (Pedley and Luo, 1998, Stewart, 2009), based on a long wavelength assumption. In this case, the mass and momentum equations are integrated across the two-dimensional channel height h, to give

$$h_t + (Uh)_x = 0, \tag{7}$$

$$U_t + UU_x + \frac{1}{h}\left(\int_0^h u^2 dy\right)_x = -p_x + \frac{1}{hRe}\left[u_y\right]_0^h, \tag{8}$$

$$p - p_{\mathrm{e}} = Th_{xx}\left(1 + h_x^2\right)^{-3/2}, \tag{9}$$

where T is the membrane tension, p_{e} is the (constant) external pressure, U is the average velocity across the channel, u represents the velocity fluctuation across the channel, and Re is the Reynolds number, defined as the ratio between the inertia and the viscous forces. Using the Karman-Pohlhausen approximation with a specific velocity profile, Pedley and Luo (1998) solved these equations and explored various assumptions of relating the pressure drop to flow separation downstream of the narrowest cross-sectional area. Essentially, they showed that these 1-D models cannot predict the full strength of the energy loss seen in the 2-D models when the wall deformation is severe.

On the other parameter region, in the limit of the high-Reynolds number region and when the channel deformation is small, Stewart (2009) was able to use this type of 1-D systems to capture the mode-1 “sloshing" oscillations initially identified by Jensen and Heil (2003) using an asymptotic analysis (where mode-i indicates that the perturbation profile contains i humps). By further analyzing the energy budget for high-Reynolds-number and pressure-driven systems, they showed that the energy budget behaves differently in mode-1 and mode-2 oscillations. For mode-1 oscillations about the uniform base state, the time-averaged net kinetic energy flux into the system is positive, therefore the kinetic energy is extracted from the mean flow and is dissipated by the oscillations. However, for mode-2 neutral oscillations, the time-averaged net kinetic energy flux into the system is negative, suggesting a different physical mechanism. Recently, Xu et al. (2013, 2014) considered a 1-D model under a uniform base state for a flow-driven (flux-driven) system, which is identical to the model used by Stewart et al. (2009, 2010) except for the boundary conditions. Using asymptotic analysis, they revealed that when the downstream length is comparable to the membrane length, the system becomes unstable when a Hopf and transcritical bifurcation arise simultaneously, giving rise to mode-2 perturbations (i.e. membrane displacements with two extrema). However, when the downstream length is much longer than the membrane length, there is an independent mechanism of instability that is intrinsically coupled to flow in the downstream rigid segment, and is promoted by a 1:1 resonant interaction between two modes. These studies provided new insight to the nature of self-excited oscillations that occurring in the system. However, due to the assumptions introduced, further two and three-dimensional computational studies are necessary to assess the wider relevance of the instability mechanisms identified therein.

4. Two-Dimensional Models

Due to the complexity of three-dimensional models and high computational requirements, two-dimensional models are used extensively to understand the mechanisms of the rich dynamic behavior observed in the Starling Resistor system. A widely used two-dimensional approach was introduced by Pedley (1992), in which the fluid mechanics is based on a lubrication theory. This is also known as the “Fluid-membrane" model, which consists of a channel flow with two rigid parallel planes, with the upper wall replaced by a thin membrane of length L. The membrane has no bending stiffness and inertia but is under longitudinal tension T, as shown in Figure 2. This system is simpler than 3D models but in principle can be realised experimentally.

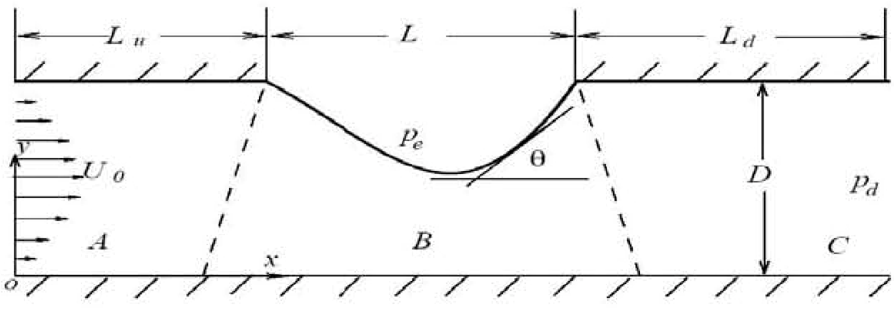

Figure 2. Sketch of the two-dimensional model.

The fluid mechanics in this model was later improved by using the Stokes equations (Lowe and Pedley, 1995), and then the Navier-Stokes equations (Luo and Pedley, 1995), so that the steady governing equations of the system are

$$\mathrm{Re}\, u_{i,j}\left(u_j\right) = -p_i + u_{i,jj}, \quad (10)$$

$$u_{j,j} = 0, \tag{11}$$

$$p_e - p = \kappa T \tag{12}$$

Where Re is the Reynolds number, and the coordinates are scaled with the rigid channel height D, the velocity components u_i are scaled with the inlet velocity U_0, tension T is scaled with μU_0, pressure p is scaled as $p = \bar{p}D/\mu U_0$, and κ is the curvature calculated from the channel height h under the elastic section:

$$\kappa = h''\left(1 + h'^2\right)^{-3/2}.$$

These equations were solved by (Rast, 1994, Lowe and Pedley, 1995, Luo and Pedley, 1995) using the finite element methods. At the low Reynolds number, the solutions agree very well with the results using the lubrication theory, and importantly, solutions exist for almost any positive values of the membrane tension. The only limitation comes from the numerical scheme, which fails to converge if tension is too small. For a Reynolds number (Re) of up to a few hundred, Shim and Kamm (2002), and Rast (1994) predicted steady membrane configurations similar to those of 1D models, but showed more fluid flow details, such as flow separation downstream of the collapsed section, long–wavelength nonlinear standing waves downstream of the constriction, and vortex– shedding eddies along both walls.

Treating the flexible segment as a membrane and assuming $Re \gg 1$, Guneratne and Pedley (2006) used interactive boundary-layer theory to describe steady flows: when the transmural pressure downstream ~ 0 and the membrane tension T is reduced from an initially large value, the system exhibits an increasing number of static eigenmodes arising via a static divergence instability; nonzero values of p_e break the symmetry of the solution structure so that as T falls one passes through regions of parameter space exhibiting single, multiple, or no steady solutions.

Huang et al. (2001) assumed that the membrane has inertia, damping, and relatively low tension. This enabled him to analyze the linearized Navier–Stokes equations, and he showed that the system exhibits both static divergence (at sufficiently low tension) and flutter (dependent on the membrane inertia), which are sensitive to the choice of upstream and downstream boundary conditions.

Using the arbitrary Lagrangian-Eulerian (ALE) approach, Luo and Pedley (1996) embarked on unsteady modeling of a fully coupled nonlinear system:

$$\frac{\partial u_i}{\partial t} + u_{i,j}\left(u_j - u_j^{\mathrm{A}}\right) = -p_{,i} + \frac{1}{Re}u_{i,jj}, \tag{13}$$

$$u_{j,j} = 0, \tag{14}$$

where all others variables are the same as in (10)(11) but now the pressure p in (12) is replaced by the normal stress component which is scaled with the dynamic pressure head ρU_0^2, and T is scaled with $\rho U_0^2 D$. Note u_j^A is the grid velocity which is in general non-zero. In the extreme cases when $u_j^A = 0$ or u_j, the equations are described in the Lagrangian or Eulerian frame of reference. These are solved with a Petrov-Galerkin finite element scheme coupled with a spine method. For six-node triangle elements, it can be shown that in this approach the geometric conservation law is exactly satisfied (Liu et al., 2012). Luo and Pedley (1996) showed how these steady flows can become unstable to self-excited oscillations if Re is sufficiently high or the membrane tension sufficiently low. The membrane oscillations are found to be closely associated with the downstream propagating waves in the inviscid core flow beyond the constriction. These resemble the vorticity waves or large-amplitude TS waves described previously by Stephanoff et al. (1983), Ralph and Pedley (1988). Luo and Pedley (1996,1998) also found that the dissipation primarily occurs in the viscous boundary layers on the channel walls upstream of the constriction, and not in the downstream separated flow zones as was previously expected, which is illustrated by Figure 3.

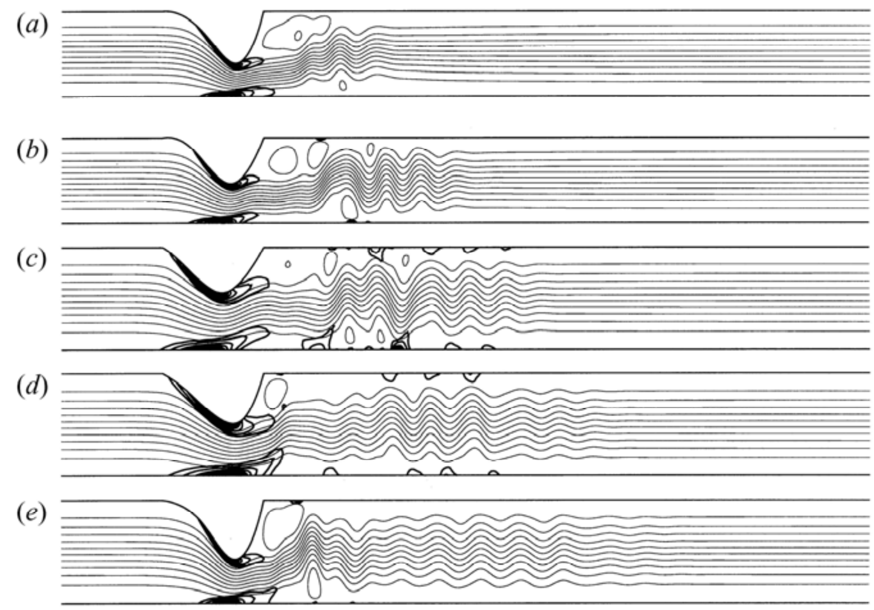

Figure 3. Snap shots of instantaneous streamlines (lighter lines) and energy dissipation contours (darker lines) generated in the self-excited oscillations by Luo and Pedley (1998).

In addition, Luo and Pedley (1998) showed how introducing inertia in the membrane allows an additional high-frequency flutter mode to grow. In a subsequent study (Luo and Pedley, 2000), they found existence of multiple solutions for a given set of control parameters, and how the primary instability is sensitive to the choice of boundary conditions. The system is more stable when the upstream flux, rather than the pressure drop, is prescribed. Using interactive boundary-layer equations, Pihler-Puzovic and Pedley (2013) discovered that for high-Reynolds-number flow in a two-dimensional collapsible tube, a unique steady solution exists when the pressure is fixed precisely at the downstream end of the membrane, but there are multiple states possible if the pressure is specified further downstream. They also found that no self-excited

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Biographical Sketches

Xiaoyu Luo received her first degree in Theoretical Solid Mechanics at Xi'an Jiaotong University, China, where she also completed her MSc and PhD in Fluid Mechanics and Biomechanics. She has been working in the UK since 1992 and became a Professor of Applied Mathematics at the University of Glasgow in 2008. Her main research interests are in fluid-structure interaction and soft tissue mechanics, with applications to modeling of heart, heart valves, artery, gallbladders and several other physiological problems. She has published over 70 papers in the international journals. She is a fellow of Royal Society of Edinburgh, IMechE, member of the EPSRC college, a conjunct Professor of Xi'an Jiaotong University and Northwest Polytechnic University, China, and in the editorial boards of four international journals.