FUNDAMENTALS OF THE HEAT TRANSFER THEORY

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Contents

Summary

In designing heat engines of flying vehicles and a number of other facilities it is necessary to take into account heat transfer processes. In some cases these processes become determining ones in choosing a design, for example, the making of thermal shielding of combustion chambers of gas turbines, nozzles of jet engines, etc.

1. Types of Heat Transfer

By convection the complex heat transfer process is divided into a number of simpler processes: heat conduction, convection and radiation. Each of these heat transfer processes obeys its laws.

Heat conduction is the process of molecular heat transfer by microparticles (molecules, atoms, ions, etc.) in a medium with a non-uniform temperature distribution.

Convection is the process of heat transfer by displacing the macroscopic elements of a medium (molar volumes).

Radiation is the process of heat transfer from one body to another by electromagnetic waves (or quanta).

In technological facilities heat is as a rule transferred by two or three ways at a time. Such a combined process is referred to as heat transfer. The heat transfer process conditioned by the simultaneous action of convection and heat conduction is called convective heat transfer. The particular case of this process is heat transfer representing convective heat exchange between a moving medium and its interface with another medium, solid body, liquid or gas.

2. Investigation Method of Heat Transfer

The phenomenological method is mainly used for studying the heat transfer processes in power plants. It is based on applying the basic laws of physics and some additional hypotheses on the course of thermal gas-dynamic processes. As a result of the use of this method differential or integral heat conduction equations are obtained. In the simple cases, they are being solved analytically or numerically. In more complex cases, the method of similarity or dimensions is used to obtain similarity numbers, a relationship between which is established as a result of the experimental study of the process. The phenomenological method of investigating the heat transfer processes is based on the following principles:

- the substance that takes part in heat transfer is considered as a continuous medium. Its molecular structure and also the microscopic mechanism of heat transfer are not viewed but are taken into consideration by introducing the quantities that are responsible for the physical properties of substances (heat conduction, viscosity, heat capacity, density, etc.);

- to construct a mathematical description of the heat transfer process the first law of thermodynamics, the conservation law of substance and the conservation law of momentum are used. Fourier’s law and Fick’s law are adopted to set up a closed system of differential equations.

In deriving the energy equation Fourier’s law is used: the vector of the heat flux density due to heat conduction at a given point and at a given time moment is directly proportional to that of the temperature gradient at this very point and at this very time $q = -\lambda \mathrm{grad} T$ where q is the heat flux density determined as the amount of heat transferred per unit time from unit surface; $\mathrm{grad} T = dT/dn$ is the rate of threshold temperature variation to an isothermal surface at a given body point and at a given time moment; λ is the thermal conductivity of substance serving as its physical characteristic; T - temperature; n – normal to surface.

Fourier’s law is the particular case of the common law of energy flux transfer and is rigorous only if the substance is homogeneous in every respect except temperature. In an inhomogeneous medium the heat transfer process can be conditioned not only by molecular heat conduction but also by diffusion of substance. The substance diffusion in the inhomogeneous medium can arise under the action of the concentration gradient of substances (concentration diffusion), pressure gradient (barodiffusion) and temperature gradient (thermodiffusion). The influence of baro-and thermodiffusion on heat transfer is small as against that of concentration diffusion and can be neglected in the majority of cases. In this case, for a binary gas mixture we have: $q = -\lambda \mathrm{grad} T + \left(i_1 - i_2\right) g_1$ where the subscripts 1 and 2 refer to the corresponding components of the binary mixture, i is the enthalpy, g is the density of the diffusion flow of the mass of the component of the mixture representing the amount of the substance of the mixture component transferred per unit time from unit surface.

The second term in this equation is responsible for heat transfer with diffusing substance. When the gradients of other potentials are absent, the diffusion flow of substance in the binary mixture is determined according to Fick's law: $g_i = -\rho D_i \mathrm{grad} C_i$ where $C_i = \rho_i / \rho$ is the concentration of the *i*-th component, ρ_i is the density of the *i*-th component, $\rho = \sum_{i=1}^{n} \rho_i$ is the mixture density, D_i is the diffusion coefficient. This relation can be used for the mixtures composed of a small number of components, whose binary diffusion coefficients for all pairs of substances are the same. Knowing the composition of the gas mixture it is possible to determine its physical properties such as the mixture enthalpy $i = \sum_{i=1}^{n} C_i i_i$, the mixture mean heat capacity $C_\mathrm{p} = \sum_{i=1}^{n} C_i C_{\mathrm{p}i}$, the thermal conductivity $\lambda = 0.5\left[\sum_{i=1}^{n} x_i \lambda_i + \left(\sum_{i=1}^{n} x_i / \lambda_i\right)^{-1}\right]$.

In deriving the motion equation Newton's law is used: shear stress of friction between two layers of a linearly moving liquid is directly proportional to the velocity gradient normal to the motion direction. For example, in the case of the plane moving with a velocity *n* parallel to the plane OXZ that is parallel to the OX-axis, the friction shear stress is equal to $\tau_{\mathrm{xz}} = \mu \frac{\partial u}{\partial y}$, where μ is the dynamic viscosity coefficient; along with this coefficient use is made of the kinematic viscosity coefficient that is determined as the relation $\nu = \mu / \rho$.

Newton's law for three-dimensional flows assumes a linear dependence of the liquid strain velocity and is mathematically written as:

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \qquad \text{for} \quad j \neq i$$

$$\tau_{ij} = 2\mu \frac{\partial u_i}{\partial x_j} - \frac{2}{3} \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right) \qquad \text{for} \quad i = j.$$

Here the coordinates (*x*, *y*, *z*) are designated through x_i(i=1, 2, 3), u_i is the projection of the liquid velocity onto the x_2 -axis.

3. Differential Equations and Uniqueness Conditions

The theoretical study of heat transfer amounts is first of all associated with determining a temperature field *T* (*t*, *x*, *y*, *z*), a velocity field $\overline{u}(t, x, y, z)$, a pressure field *p* (*t*, *x*, *y*, *z*), and a concentration field of substance (*t*, *x*, *y*, *z*). Knowing these quantities and also the temperature and pressure dependences of the physical properties of substance, it is possible to determine all quantities that characterize heat transfer (heat flux, hydraulic resistance, etc.). In order to determine the above-mentioned influences the following equations are used.

Energy equation $\rho \frac{di}{dt} = \mathrm{div}\left(\lambda \mathrm{grad} T\right) + \mathrm{div}\left(\sum \rho D : \mathrm{grad} C_i\right) + \frac{dP}{dt} + q_v$ (1)

Motion equation $\rho \frac{d\bar{u}}{dt} = \bar{R} - \mathrm{grad} P + \mu \nabla^2 u + \frac{1}{3}\mu \mathrm{grad}\left(\mathrm{div}\bar{u}\right)$ (2)

Diffusion equation $\rho \frac{u}{dt} - \mathrm{div}\left(\sum \rho D : \mathrm{grad} C_i\right) + w_i = 0$ (3)

Continuity equation $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$ (4)

Here $i = \sum c_i i_i$ is the enthalpy of the mixture, q_{v} is the volume density of internal heat sources, $\bar{u}$ is the velocity vector, *F* is the stress vector of the volume force, w_i is the velocity of flowrate (or of formation) of the mass of the *i*-th substance per unit volume of the chemically reacting medium.

The total derivatives in the above equations for a moving medium are equal to: $\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$ where *u*, *v*, *w* are the velocity projections of the moving medium onto the *x*, *y*, *z*. -axes, respectively.

In the motion equation $F = g\beta\Delta T$ is the lifting force (Archimedean buoyancy force) where *g* is the gravitational acceleration, β is the volume expansion coefficient, $\Delta T = T - T_{\mathrm{w}}$ is the temperature drop with respect to the surface temperature T_{w}.

For the thermal and hydrodynamic processes to be analyzed, the above equations must be supplemented by the uniqueness conditions involving the physical, geometrical, boundary and initial conditions. The physical conditions specify the values of the physical properties of liquid (density, viscosity, heat capacity, thermal conductivity, diffusion coefficient, etc.) and their temperature and density dependences. Among the geometrical conditions is the shape and sizes of a region where heat transfer is realized, for example, the shape and sizes of a body in the gas flow, the shape and sizes of channels where heat carrier moves. The initial conditions characterize the temperature, velocity and pressure distributions of a medium at the initial time moment. If the heat transfer process is steady-state, then the initial conditions drop out of consideration. The boundary conditions characterize the laws of thermal and hydrodynamic interaction of the surface of a considered body with the surroundings. The medium velocity over the body surface is taken equal to zero as the moving medium adheres to the surface. When the injection or suction of liquid (coolant) is present over the surface of the considered body, it is necessary to assign the velocity of its injection or suction. For the energy equation the boundary conditions can be prescribed in the form of the temperature distribution over the body surface (boundary conditions of the first kind) or in the form of the heat flux distribution over the body surface (boundary conditions of the second kind).

4. Simplified Equations

The boundary layer method is the most efficient procedure to solve the above-mentioned system of equations. The essence of the boundary layer method is as follows. In external flow past the body surface, by convention it is possible to select two regions: the region near the surface in which the action of viscosity and heat conduction (boundary layer) much manifests itself and the external flow region (far from the surface) in which it may be with a sufficient accuracy considered that the liquid is ideal (non-viscous and non-heat conducting). The dynamic, thermal and diffusion boundary layers are distinguished and are not equal in the general case (Figure 1) In the case of flow past a flat surface, when the gravitational forces and chemical reactions are absent, the system of the equations for a steady-state flat boundary layer is written in the form:

energy equation $\rho u c_p \frac{\partial T}{\partial x} + \rho v c_p \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + q_v$ (5)

motion equation $\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{\partial P}{\partial x}$ (6)

continuity equation $\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0$ (7)

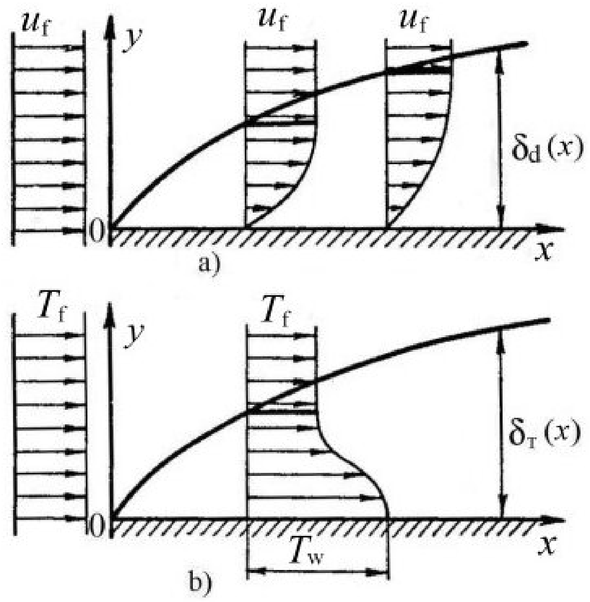

Figure 1: Dynamic (a) and shadow (b) boundary layers near the plate surface in the liquid flow.

At large velocities (at the Mach numbers M>0.7) it is necessary to take account of the heat release in the boundary layer due to the kinetic energy dissipation of the gas flow $\mu(\partial u \ /\partial y)^2$ and the work of pressure forces $u(\partial p/\partial x)$, i.e., $q_v = \mu\left(\frac{\partial u}{\partial y}\right)^2 + u\frac{\partial P}{\partial x}$.

The energy equation for the boundary layer at large velocities of flow past the surface with respect to a stagnation temperature $T_0 = T + \left(u^2/2C_p\right)$ is written in the following form:

$$\rho u c_p \frac{\partial T_0}{\partial x} + \rho v c_p \frac{\partial T_0}{\partial y} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T_0}{\partial y}\right) + \frac{\partial}{\partial y}\left[\mu\left(1 - \frac{1}{P_r}\right)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right] \tag{8}$$

In the chemically reacting boundary layer it is necessary to taken into consideration heat transfer by heat diffusion and release (or absorption) due to chemical transformations. The energy equation in this case can be presented as:

$$\rho u \frac{\partial I_0}{\partial x} + \rho v \frac{\partial I_0}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\lambda}{c_p}\frac{\partial J_0}{\partial y}\right) + \frac{\partial}{\partial y}\left[\mu\left(1 - \frac{1}{P_r}\right)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right]. \tag{9}$$

Here $I_0 = I + \frac{u^2}{2}$ is the total stagnation enthalpy, $I = \Sigma C_i J_i$ is the total enthalpy of a mixture, $J_i = i_i + h_i$ is the total enthalpy of the i-th component incorporating the formation heat of the i-th component h_i due to chemical transformations. In these equations $Pr = \mu C_p / \lambda$ is the Prandtl number, $Le = \lambda / \rho D C_{\mathrm{p}}$ is the Lewis number. From the above-mentioned equations it follows that at Pr=1 and Le=1 the energy equations are written in the same form as in the cases of no chemical reactions. Hence, at small rates the heat transfer process is determined by the static temperature field; at large rates, by the temperature field and in the presence of chemical reactions, by the total enthalpy field.

5. Transition from Laminar to Turbulent Flow

In external liquid or gas flow past a body both laminar and turbulent boundary layers can develop on its surface. In laminar flow the liquid particles proceed quiet certain trajectories, all time keeping their motion in the direction of the mean velocity vector. As the liquid flow velocity increases, the laminar flow disintegrates and there appear velocity, temperature, pressure pulsations. Separate small volumes of liquid (moles) start moving across the flow and even in the opposite direction relative to the averaged motion. The transition from the laminar flow to the turbulent one occurs not at a point but over some section (Figure 2). Flow over this section is unstable and is called the transient one characterized by a periodic change of laminar and turbulent states. The flow shape in the boundary layer is judged from the value of the critical Reynolds numbers $Re_{\mathrm{cr}} = u_f x_{\mathrm{cr}} / \nu$ where x_{cr} is the coordinate along surface and is reckoned from its leading edge The value of the critical Reynolds number in the general case depends on many factors: the heat transfer intensity, the mean velocity along the surface, its roughness, the shape of the body leading edge, the turbulence degree of the external flow, etc.

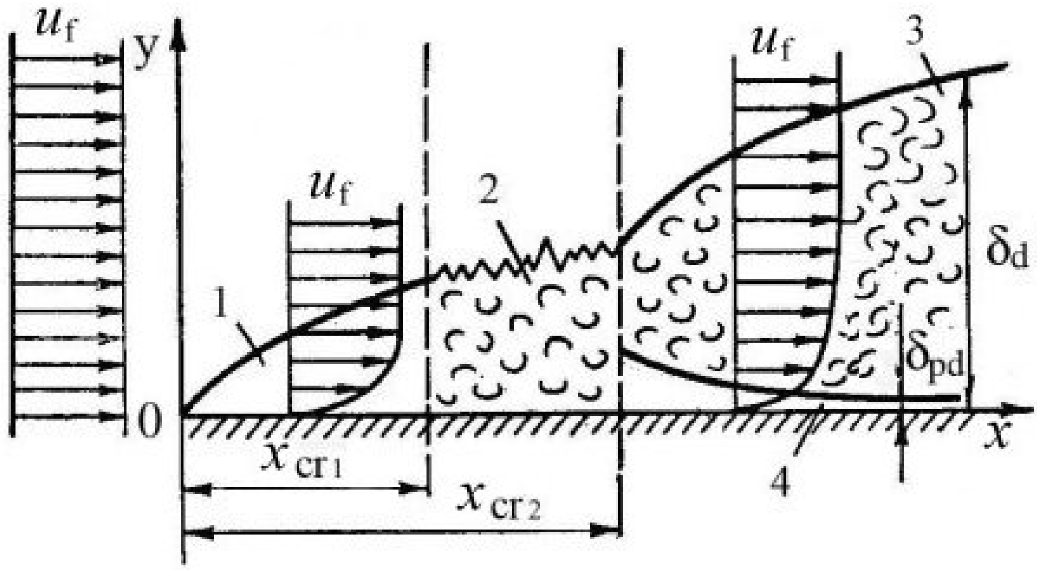

Figure 2 Development of the turbulent boundary layer on the flat plate: 1 – laminar flow region,
2 – transient flow region, 3 – turbulent flow region, 4 – viscous sublayer.

In the case of liquid or gas flow past a smooth plate at $M \to 0$ the lower limit of the critical Reynolds numbers (onset of disintegration of the laminar boundary layer) is $Re_{cr1} = u_f x_{cr1}/\nu = 10^5$ and the upper limit of the critical Reynolds numbers (steady turbulent flow formation) is $Re_{cr2} = u_f x_{cr2}/\nu \approx 10^6$. When the liquid moves in cylindrical channels, the channel diameter is used as a characteristic size, therefore, $Re_{cr1} \approx 2300$, and $Re_{cr2} = 10^4$. To calculate the critical value of the Reynolds number Re_{cr} in the case of flow past turbine and compressor grids the empirical relation is proposed:

$$Re_{cr1} = A\left(1+0.3M_0^{1.7}\right)\left(1+0.58M\right)^{0.6}\left(\frac{T_w}{T_t}\right)^{-2.3} \qquad (10)$$

Here M_0 is the Mach number at the point of the minimum of the pressure on the profile loop, the coefficient A is determined depending on the turbulence degree of the external flow. $A = 3.1 \cdot 10^6$ for $\varepsilon \le 0.12\%$; $A = 0{,}71 \cdot 10^6 \cdot \varepsilon^{-0{,}7}$ for $\varepsilon = 0{,}12 - 1\%$ $A = 0{,}71 \cdot 10^6 \cdot \varepsilon^{-1.76}$ for $\varepsilon = 1 - 3\%$.

The length of the transient flow region $\Delta l = x_{cr2} - \tilde{o}_{cr1}$ in the zero-pressure gradient flow can be estimated according to the relation: $Re_{cr_{ms}} = \frac{u_f \Delta_l}{\nu} = 16.8 Re_{cr}^{0.8}$.

Under the action of turbulent pulsations heat and momentum transfer within the turbulent flow regime increases as compared to the laminar one. In the steady-state turbulent motion of liquid (or gas) its mean parameters for some time interval remain constant. True or current parameters (velocity, temperature, pressure) of the flow continuously differ from this mean value both in quantity and in direction. Owing to this, the current velocity, temperature, pressure can be presented as a sum of the mean values of these quantities and pulsation components. Averaging the equations of energy and motion over time yields the Reynolds equation for the character of the averaged turbulent liquid flow. The averaged Reynolds equations enable one to present the turbulent flow equations in the form of the equations for laminar flow in which by thermal conductivity and viscosity coefficient is understood a sum of molecular and turbulent components of viscosity and heat conduction, i.e., $\lambda_{\Sigma} = \mu + \mu_{\mathrm{T}}$; $\mu_{\Sigma} = \mu + \mu_{\mathrm{T}}$. For the turbulent thermal conductivity λ_{T} and the viscosity coefficient μ_T to be determined, both algebraic and one-and two-parameter differential semi-empirical turbulence models can be adopted. These problems are detailed in (Avduevsky, V.S. (1992)).

6. Heat Transfer Coefficient and Friction Resistance

In engineering practice it is important to know in each particular case the regularities of heat interaction between the surface of a considered body and its surrounding medium (heat transfer processes), i.e., it is necessary to know how to determine a surface temperature and a heat flux to the surface. In order to find these parameters there is a need to know a temperature field in the moving medium that in the general case can be determined from solving the above-given heat transfer equations. In the general case, such a method of determining the temperature field in the moving medium is rather an intricate problem and in some cases it is practically unresolved. To solve engineering problems it is important to know a relationship between the surface temperature T_{w}, the medium temperature T_{f} and the heat flux density q_{w} (Figure 1). For a chemically inhomogeneous medium this dependence is usually presented in the form of Newton's formula $q_{\mathrm{w}} = \alpha\left(T_{\mathrm{w}} - T_{\mathrm{f}}\right)$ where α is the heat transfer coefficient.

The heat transfer coefficient is not a constant quantity but depends on the geometrical, hydrodynamic and thermal characteristics of a considered system. For the heat transfer coefficient to be found theoretically, it is necessary to know the temperature field in the moving medium, i.e., the system of the convective heat transfer equations must be solved. When the temperature field $\alpha = -\left(\lambda \frac{\partial T}{\partial n}\right)_{\mathrm{w}} / \left(T_{\mathrm{w}} - T_f\right)$ - where $\partial T / \partial n$ is the temperature gradient normal to the surface.

In the majority of practical cases, because of the complexity of the system of the convective heat transfer equations it is not easy to obtain their exact solution. In these cases, the similarity theory is used and the heat transfer coefficient is found from experiments.

There is a need to distinguish between the heat transfer process in external flow past bodies (external heat transfer problem) and the heat transfer process in channels

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Biographical Sketch

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He is a well-known specialist in the heat-mass transfer and space thermo techniques. He conducted the investigations of a heat transfer in the oscillating flows, porous systems, jet systems, system cooling of power plant. He is specialized in effective methods of the heat transfer intensification and highly productive methods of systems cooling calculation.

He is author of more then 250 published works, concluding monograph: Heat Transfer in the Power Installations of Spacecrafts (1975), Heat and Hydrodynamic Processes in Oscillating Flows (1977). Heat transfer in aviation engines (1985). Fundamentals of Heat Transfer in Aviation and Rocket-space Technics (1992, in cooperation with other authors). Thermal protection of turbine blade (1996).