

## NONPARAMETRIC SYSTEM IDENTIFICATION

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### Summary

This article presents a survey of various methods for nonparametric identification of nonlinear systems. Nonparametric identification methods are those that measure Wiener kernels or Volterra kernels, since an output of a nonlinear system can be described by the convolution integral of Wiener or Volterra kernels and the system input. Section 1 highlights the representation methods of nonlinear systems by kernels including mutual relationships between Wiener kernels and Volterra kernels.

Section 2 describes identification methods for Wiener kernels including Wiener's orthogonal expansion method and Lee-Schetzen's correlation method. Section 3 covers the identification method of Volterra kernels including Hooper-Gyftopoulos' correlation method, Watanabe-Stark's orthonormal basis function method, and Kashiwagi-Sun's M-sequence correlation method. For each method, comments are made on the computational load from a practical application point of view.

### 1. Introduction

System identification methods are divided into two groups: parametric and nonparametric. Parametric methods identify system model with an underlying mathematical structure that is associated with a coefficient set or parameters, whereas nonparametric methods model a system directly with its responses.

Let us take an example in a linear system. A method for obtaining a transfer function of a system is a parametric method. The system parameters in this case are coefficients of the transfer function, and the number of parameters is less than or equal to  $2n + 1$  where  $n$  is the order of the system. In the same way, a state equation or a difference equation method belongs to the parametric group. In contrast, a method for obtaining an impulse response, step response or frequency response of the system belongs to the nonparametric group.

Similarly, identification methods for nonlinear systems are also divided into the parametric and nonparametric groups. Nonparametric methods of nonlinear system identification include those system representation methods using Volterra kernels or Wiener kernels. Hence a nonparametric method for nonlinear system identification usually means a method for obtaining Volterra kernels or Wiener kernels.

In this article, identification methods for obtaining Volterra kernel or Wiener kernel of a nonlinear control system are described in detail. (For parametric methods of nonlinear system identification, see articles under the topic entitled *Identification of Nonlinear Systems*.)

## 2. Representation of Nonlinear Systems

Let  $u(t)$  be an input to a nonlinear system and  $y(t)$  be its output. Then the output  $y(t)$  can be in general written as follows.

$$y(t) = \sum_{i=0}^{\infty} H_i[u(t)] \quad (1)$$

where

$$H_i[u(t)] = \int_0^{\infty} \int_0^{\infty} \dots \int_0^{\infty} h_i(\tau_1, \tau_2, \dots, \tau_i) u(t - \tau_1) u(t - \tau_2) \dots u(t - \tau_i) d\tau_1 d\tau_2 \dots d\tau_i \quad (2)$$

and  $h_i(\tau_1, \tau_2, \dots, \tau_i)$  is called the Volterra kernel of the  $i$ -th order.

The basis of Eq. (1) is due to Frechet (1910) who showed that any continuous functional can be represented by a series of functionals of integer order whose convergence is uniform on all compact sets of continuous functions. Hence Eq. (1) applies to those nonlinear systems whose output is continuous for a continuous input. Therefore we should note that Eq. (1) does not apply to those nonlinear systems that have a multi-valued nonlinearity like hysteresis, backlash or an on-fading memory. However we should also note that most nonlinear systems are considered to be representable by Eq. (1).

Another representation method for nonlinear system via kernel method, when the input  $u(t)$  is a white Gaussian signal, is the Wiener kernel method as shown in the following equation.

$$y(t) = \sum_{n=0}^{\infty} G_n[k_n; u(t)] \quad (3)$$

where  $G_n$  is called the Wiener G-functional, with orthogonal properties given by the next two equations.

$$\overline{G_m[k_m; u(t)] G_n[k_n; u(t)]} = 0, \quad \text{for } m \neq n \quad (4)$$

$$\overline{H_m[u(t)] G_n[k_n; u(t)]} = 0, \quad \text{for } m < n \quad (5)$$

Here  $k_n$  is called the  $n$ -th order Wiener kernel and the bar over  $G_m G_n$  denotes the time average over the interval  $(-\infty, \infty)$ . Eq. (4) shows that Wiener G-functionals are orthogonal to each other. Equation (5) shows that  $G_n[k_n; u(t)]$  is orthogonal to  $H_m[u(t)]$  when  $m < n$ . The first four terms of the Wiener G-functional are

$$\begin{aligned} G_0[k_0; u(t)] &= k_0 \\ G_1[k_1; u(t)] &= \int_{-\infty}^{\infty} k_1(\tau_1) u(t - \tau_1) d\tau_1 \\ G_2[k_2; u(t)] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k_2(\tau_1, \tau_2) u(t - \tau_1) u(t - \tau_2) d\tau_1 d\tau_2 \\ &\quad - A \int_{-\infty}^{\infty} k_2(\tau_1, \tau_1) d\tau_1 \\ G_3[k_3; u(t)] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k_3(\tau_1, \tau_2, \tau_3) u(t - \tau_1) \\ &\quad u(t - \tau_2) u(t - \tau_3) d\tau_1 d\tau_2 d\tau_3 \\ &\quad + \int_{-\infty}^{\infty} k_{1(3)}(\tau_1) u(t - \tau_1) d\tau_1 \end{aligned} \quad (6)$$

Here  $k_{1(3)}(\tau_1)$  is called the derived Wiener kernel from the third Wiener kernel  $k_3(\tau_1, \tau_2, \tau_3)$  given by the next equation.

$$k_{1(3)}(\tau_1) = -3A \int_{-\infty}^{\infty} k_3(\tau_1, \tau_2, \tau_2) d\tau_2 \quad (7)$$

In this way,  $k_{i(j)}$  denotes the  $i$ -th degree kernel derived from the  $j$ -th degree kernel.

$A$  is the power of the input white Gaussian signal. We should note that the Wiener G-functional representation of a nonlinear system was originally developed for white Gaussian input, but it is also applicable to a non-white or non-Gaussian input by the use of a prewhitening technique.

Given the input white Gaussian signal, what is the relationship between the Volterra kernel  $h_i(\tau_1, \tau_2, \dots, \tau_i)$  in Equation (1) and the Wiener kernel  $k_i(\tau_1, \tau_2, \dots, \tau_i)$  in Eq. (3)? When we compare Eq. (1) and Eq. (3), we get

$$\begin{aligned} h_0 &= k_0 + k_{0(2)} + k_{0(4)} + \dots \\ h_1 &= k_1 + k_{1(3)} + k_{1(5)} + \dots \\ h_2 &= k_2 + k_{2(4)} + \dots \\ h_3 &= k_3 + k_{3(5)} + \dots \end{aligned} \quad (8)$$

Therefore when we wish to identify a nonlinear system using a white Gaussian input by a kernel method, obtaining Volterra kernels is equivalent to obtaining Wiener kernels, since these two representations have one to one correspondence.

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### Biographical Sketch

Professor **Hiroshi Kashiwagi** was born in 1939 in Kumamoto, Japan. He received BE, ME and PhD degrees in measurement and control engineering from the University of Tokyo, Japan, in 1962, 1964, and 1967 respectively.

In 1967, he became an Associate Professor at Kumamoto University. From 1973 to 1974, he served as a visiting Associate Professor at Purdue University, Indiana, USA. He became a Professor at Kumamoto University in 1976. He served as a member of Board of Trustees of SICE (Society of Instrument and Control Engineers, Japan), General Chairman of SICE 1992 held in Kumamoto, General Chairman of ICAUTO 1995 (International Conference of Automation) held in Indore, India, in 1995, and General Chairman of APCCM 1998, 2000, and 2002 (Asia Pacific Conference on Control and Measurement) held in China in 1998, 2000, and 2002, respectively.

In 1994, he was awarded SICE Fellow for his contributions to the field of measurement and control engineering through his various academic activities. He also received the Gold Medal Prize at ICAUTO 1995 held in India. In 1997, he was awarded “Best Book Award” from SICE for his new book entitled

“M-sequence and its Application” written in Japanese and published in 1996 by Shoukoudo Publishing Co. in Japan. In 1999, he received the “Best Paper Award” from SICE for his paper “M-transform and its application to system identification”.

His research interests include signal processing and applications, especially pseudorandom sequence and its applications to measurement and control engineering including nonlinear system identification.

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