LIE BRACKET

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Contents

Summary

The Lie bracket is a map which assigns a third tangent vector field to two given tangent vector fields, all defined on an abstract manifold. Therefore, this chapter starts with an introduction into the concept of n-dimensional abstract manifolds and special pairs of manifolds, called bundles. These constructions are necessary to derive the tangent bundle of a manifold, the container of all tangent vector fields. The Lie derivative of a smooth function is a linear map that respects the Leibniz rule; its commutator is the Lie bracket. But the Lie bracket is also a differential operator that acts on vector fields, that tells us whether flows on manifolds commute, that helps to find hidden constraints in systems of linear partial differential equations. The last property is contained in the theorem of Frobenius. After the discussion of these topics and their significance for control, a short example concerning the controllability of a nonlinear system shows how the developed mathematical machinery helps to solve problems in nonlinear control.

1. Introduction

When engineers investigate systems which are described by a set of ordinary differential equations, they have to face the problem to find a suitable space which contains the state and the input of the system. Unfortunately, the choice of a Euclidean space is not possible in general, and one has to pass to a more abstract concept. E.g. one cannot describe all rotations in the 3-dimensional space globally by 3 coordinates, although a local description is always possible. Abstract manifolds overcome this problem, since they behave locally like a piece of an Euclidean space, but they take into a count the global nonlinear behavior. This approach implies that one has to extend the calculus from the Euclidean space to manifolds. Choosing special coordinates for a problem allows us to perform all the investigations in the usual way, but one has to face the problem that a canonical choice of coordinates does not exist. Roughly speaking, all the calculations must be meaningful for any coordinate system. This idea leads to a coordinate free representation of dynamic systems. It will turn out that the systems under consideration can be represented by the help of special mathematical objects, called tangent vector fields on abstract manifolds. This implies that differential geometry becomes the main tool for the description and the design of nonlinear systems.

The Lie bracket is a map which assigns two tangent vector fields to a third one. The importance of this map follows from the fact that this map appears in problems which do not seem to be linked to each other. E.g. the Lie bracket measures the change of a vector field moving along a curve, it is the commutator of two differential operators, it describes the hidden constraints in systems of linear partial differential equations, it measures the lack of commutability of special maps called flows, etc.

For control purpose, the Lie bracket comes into play, if we look for nonlinear state- and/or input transformations. To get a feeling for this problem, we take a look at the simple *n*-dimensional linear time invariant system

$$\dot{x}_j = \sum_{j=1}^{n} a_j^i x^j , \quad i = 1,...,n ,$$

with $a_j^i \in \mathbb{R}$. Let us consider the state transform

$$z^i = \sum_{j=1}^{n} T_j^i x^j , \qquad (1)$$

$T_j^i \in \mathbb{R}$. By taking the time derivative of (1) we get, of course

$$z^i = \sum_{j=1}^{n} T_j^i \dot{x}^j . \qquad (2)$$

The important observation is that we can choose the transformation (2) for the derivatives of the coordinates freely and derive (1), the transformation for the coordinates, in a trivial manner. Let us consider the nonlinear system

$$\dot{x}^i = f^i(x), \quad i = 1,...,n$$

with smooth functions f^i together with the nonlinear state transform

$$z^i = \varphi^i(x) . \qquad (3)$$

Taking again the time derivative of (3) we get

$$\dot{z}^i = \sum_{j=1}^{n} T_j^i(x)\dot{x}^j , \tag{4}$$

where the functions T_j^i meet

$$T_j^i(x) = \frac{\partial}{\partial x^j}\varphi^i(x) . \tag{5}$$

Here, it is not possible to choose the functions T_j^i in (4) freely, since the functions T_j^i must satisfy (5). Therefore, given (4) we have to check, if is possible to find (3). Additionally, we have to solve a system of partial differential equations to derive φ^i from T_j^i. Now, the Lie-bracket allows us to construct admissible choices for T_j^i and tells us, whether these partial differential equations have a non trivial solution.

This chapter is organized as follows. Section 2 gives an introduction to the concept of abstract manifolds and special pairs of manifolds, called bundles, and develops the basics for doing calculus on manifolds. Section 3 presents the Lie bracket and shows that this bracket belongs to a family of differential operators that operate on geometric objects defined on a manifold. Although the Lie bracket is an important map the importance for control is strictly related to the theorem of Frobenius which is presented in Section 4. A short application to the problem of controllability of nonlinear systems is given in Section 5. Finally, this chapter finishes with some remarks concerning differential geometry and the literature.

2. Basics of Manifolds and Bundles

Curves and surfaces in the Euclidean space were studied since the earliest days of geometry. However, the discoveries of Gauss profoundly altered the course of differential geometry and pointed the way to the concept of an abstract manifold. Therefore, we give the basic definitions and present some essential results concerning *n*-dimensional abstract manifolds. One can construct more complex manifold from simpler ones. Fibered manifolds are an import case; it will turn out that the generalizations of the tangent plane of a surface, the so called tangent bundle is a special fibered manifold. Therefore a short introduction to fibered manifolds and bundles is also given in Section 2.1. The subsection after the next presents the concept of tangent bundles, tangent vectors for abstract manifolds and introduces important maps on manifolds, called flows.

2.1. Manifolds

A manifold is, roughly speaking, the generalization of an *n*-dimensional smooth surface in the space $\mathbb{R}^m$ with $m \geq n$. Although one can show that every smooth *n*-dimensional manifold can be embedded in $\mathbb{R}^{2n+1}$, which was proven by Hassler Whitney, we will give a more abstract definition of a smooth manifold that avoids the reference to any embedding in the space $\mathbb{R}^k$ for some k.

Definition 1 *A smooth n*-dimensional manifold *is a set* $\mathcal{M}$, *together with a countable collection of subsets* $\mathcal{U}_\alpha \subset \mathcal{M}$, *the* coordinate charts, *and one-to-one maps* $\phi_\alpha : \mathcal{U}_\alpha \to V_\alpha$ *onto connected open subsets* $V_\alpha \subset \mathbb{R}^\nu$, *the local* coordinate maps, *such that the following properties are satisfied:*

1. *The* coordinate charts *cover* $\mathcal{M}$, *or* $\mathcal{M} = \cup_\alpha \mathcal{U}_\alpha$ *is met.*

2. *.Let* $\mathcal{U}$ *denote the intersection* $\mathcal{U}_\alpha \cap \mathcal{U}_\beta = \mathcal{U} \neq \{\}$ *.The composite map* $\phi_\beta \circ \phi_\alpha^{-1} : \phi_\alpha(\mathcal{U}) \to \phi_\beta(\mathcal{U})$ *is a smooth* (*infinitely differentiable*) *function*.

3. *For any pair of different points* $p \in \mathcal{U}_\alpha, q \in \mathcal{U}_\beta, p \neq q$ *there exist open subsets* W_α, W_β *such that* $\phi_\alpha(p) \in W_\alpha \subset V_\alpha$, $\phi_\beta(q) \in W_\beta \subset V_\beta$ *and* $\phi_\alpha^{-1}(W_\alpha) \cap \phi_\beta(W_\beta) = \{\}$ *are met*.

Some facts are worth mentioning at this stage. The coordinate charts allow us to define a topology for $\mathcal{M}$ by declaring the sets $\phi_\alpha^{-1}(W)$ to be open for any open $W \subset \phi_\alpha(\mathcal{U}_\alpha) \subset \mathbb{R}^n$. In terms of this topology, the third requirement in the definition above says that $\mathcal{M}$ has the Hausdorff separation property. The degree of differentiability of the transition functions $\phi_\beta \circ \phi_\alpha^{-1}$ determines the degree of smoothness of the manifold $\mathcal{M}$. For the sake of simplicity, we will consider only *smooth manifolds* here. Besides the basic coordinate charts $\phi_\alpha : \mathcal{U}_\alpha \to V_\alpha$, one can add additional coordinate charts $\phi : \mathcal{U} \to V \subset \mathbb{R}^n$ subject to the requirement that the transition functions $\phi \circ \phi_\alpha^{-1}, \phi_\alpha \circ \phi^{-1}$ are smooth for any α on the overlap $\phi_\alpha(\mathcal{U} \cap \mathcal{U}_\alpha)$. In this case the chart $\phi : \mathcal{U} \to V$ is said to be *compatible* with the basic charts. Furthermore, the *maximal collection* of all *compatible charts* is called an *atlas* of $\mathcal{M}$.

The simplest case of an *n*-dimensional manifold is the space $\mathbb{R}^n$. Let $x = (x^1, ..., x^n)$ be a coordinate system of $\mathbb{R}^n$ and let φ^i be the *coordinate functions* of $\mathbb{R}^n$ which map a point $q \in \mathbb{R}^n$ into $\mathbb{R}$ by $x^i = \varphi^i(q)$. Given a *n*-dimensional manifold $\mathcal{M}$ with coordinate charts $\mathcal{U}_\alpha$ and *coordinate maps* ϕ_α, then the composite mapping $\varphi^i \circ \phi_\alpha$ maps $\mathcal{M}$ into $\mathbb{R}$ by $x^i = \varphi^i \circ \phi_\alpha(p)$. We call the functions $x^i = \varphi^i \circ \phi_\alpha(p)$ *coordinate functions* of $\mathcal{M}$. Although we denote different objects, here the coordinates and the coordinate functions, by the same symbol, there should be no confusions. Furthermore, this allows us to dispense the explicit reference to a local coordinate chart. We will say, $x = (x^1, ..., x^n)$ is a local coordinate system of $\mathcal{M}$, which is an abbreviation that there is a local coordinate map $\phi_\alpha : \mathcal{U}_\alpha \to V_\alpha$ with an open subset $V_\alpha \subset \mathbb{R}^n$ and a coordinate chart $\mathcal{U}_\alpha$ such that each $p \in \mathcal{U}_\alpha$ has the local representation $x^i = \phi^i_\alpha(p)$. Of course, we know the representation of p in any other compatible chart by the conditions of definition 1.

Let us consider the *n*-dimensional unit spheres $\mathbb{S}^n$, then it is straightforward to see that the *n*-dimensional spheres

$$\sum_{i=1}^{n+1} (x^i)^2 = 1$$

are smooth manifolds embedded in $\mathbb{R}^{n+1}$, since the sets $\mathcal{U}_i = \mathbb{S}^n \cap \{x \in \mathbb{R}^{n+1} \mid x^i > 0\}$, $\mathcal{U}_{1+n+i} = \mathbb{S}^n \cap \{x \in \mathbb{R}^{n+1} \mid x^i < 0\}$ are *coordinate charts* with the projections on the planes $x^i = 0$ as coordinate maps ϕ_i, ϕ_{1+n+i}. Whereas the 2-dimensional cone

$$(x^1)^2 - (x^2)^2 - (x^3)^2 = 0$$

embedded in $\mathbb{R}^3$ is no manifold, because the origin does not have a neighborhood $\mathcal{U}_\alpha$, which can be mapped one-to-one onto an open subset of $\mathbb{R}^2$. A less trivial example of a smooth *n*-dimensional manifold is the set of all lines through the origin 0 in $\mathbb{R}^{n+1}$, which is called the real projective space $\mathbb{RP}^n$.

Having the concept of manifolds at our disposal, we may consider functions $f : \mathcal{M} \to \mathbb{R}$ on the *n*-dimensional smooth manifold $\mathcal{M}$. Let $\mathcal{U}_\alpha$ be a coordinate chart with coordinate map ϕ_α, then $\overline{f}(x) = f(\phi_\alpha^{-1}(x))$ is a map $\overline{f} : \phi_\alpha(\mathcal{U}_\alpha) \to \mathbb{R}$. We call f differentiable (smooth) at $p \in \mathcal{U}_\alpha$, if and only if (iff) $\overline{f}$ is differentiable (smooth) at $\phi_\alpha(p)$. It is easy to see that this property does not depend on the choice of $\mathcal{U}_\alpha$. The important set of smooth functions on $\mathcal{M}$ is denoted by $C^\infty(\mathcal{M})$. Obviously, we have $\phi^i_\alpha C^\infty(\mathcal{M})$ for a smooth manifold. The reader is asked to reflect twice on this construction. Roughly speaking, we can do calculus like in $\mathbb{R}^n$, if we confine our calculations to the suitable charts and take care that the calculations are meaningful also in other charts. Now, we are ready to extend this idea to maps between manifolds.

Let $\mathcal{M}, \mathcal{N}$ be two smooth *m*- and *n*-dimensional manifolds. A map $f : \mathcal{M} \to \mathcal{N}$ is said to be smooth, iff its local representation is smooth for every coordinate chart. In other words, let $\mathcal{U}_\alpha, \mathcal{V}_\beta$ be charts of $\mathcal{M}$ and $\mathcal{N}$ with coordinate maps ϕ_α, ϕ_β such that, $f(\mathcal{U}_\alpha) \subset \mathcal{V}_\beta$ is met, then f is smooth if the composite map

$\overline{f} = \varphi_\beta \circ f \circ \phi_\alpha^{-1} : \phi_\alpha(\mathcal{U}_\alpha) \to \varphi_\beta(\mathcal{V}_\beta)$ is smooth. The smooth map f can be used to transfer a smooth function $g \in C^\infty(\mathcal{N})$ from $\mathcal{N}$ to $\mathcal{M}$ by

$$f^*(g) = g \circ f : \mathcal{M} \to \mathbb{R}. \tag{6}$$

The function $f^*(g) \in C^\infty(\mathcal{M})$ is also called the *pullback* of g by f. Only if $f : \mathcal{M} \to \mathcal{N}$ is a local *diffeomorphism*, f has a smooth inverse f^{-1} locally, then we can transfer functions from $\mathcal{M}$ to $\mathcal{N}$.

Let $x = (x^1, ..., x^m)$ and $y = (y^1, ..., y^n)$ be coordinate systems for the two smooth *m*- and *n*-dimensional manifolds $\mathcal{M}, \mathcal{N}$, then we may rewrite the map $f : \mathcal{M} \to \mathcal{N}$ as $y^i = \overline{f}^i(x)$ using these coordinates. Following the ideas above, we are able to define the *rank* of f as the rank of the Jacobian $[\partial_{x^i} \overline{f}^j]$. Again, the reader may convince himself/herself that this definition of the rank is independent of the choice of the charts. Additionally, the following theorem tells us that the maps f between manifolds with constant rank, also called *regular maps*, admit a very simple form.

Theorem 2 *Let $\mathcal{M}, \mathcal{N}$ be two smooth m-and n-dimensional manifolds. Let k be the rank of the regular map $f : \mathcal{M} \to \mathcal{N}$ at $p \in \mathcal{M}$, then there exists coordinates $x = (x^1, ..., x^m)$ near p and coordinates $y = (y^1, ..., y^n)$ near $f(p)$ such that f takes the canonical form*

$$y = \left(x^1, ..., x^k, \underbrace{0, ..., 0}_{n-k} \right)$$

in x, y.

This theorem is an easy consequence of the implicit function theorem, and will not be proven here.

One-to-one maps between manifolds can be used to parameterize submanifolds, like we parameterize curves and surfaces in $\mathbb{R}^3$. Let $\mathcal{N}$ be a smooth manifold, then a submanifold of $\mathcal{N}$ is a subset $\mathcal{M} \subset \mathcal{N}$ together with a manifold $\overline{\mathcal{M}}$ and a smooth one-to-one map $f : \overline{\mathcal{M}} \to \mathcal{M}$ with maximal rank. In particular the dimensions of $\mathcal{M}, \overline{\mathcal{M}}$ coincide and do not exceed the dimension of $\mathcal{N}$. Unfortunately, this definition of a submanifold admits some irregularities that we wish to avoid. Therefore, we define now the more restrictive regular submanifold.

Definition 3 *Let $\mathcal{N}$ be a smooth n-dimensional manifold and $\mathcal{M}$ be a submanifold of $\mathcal{N}$. We call $\mathcal{M}$ a regular m-dimensional* submanifold, *iff for each $p \in \mathcal{M}$ there exists a chart*

$\mathcal{U}, p \in \mathcal{U}$ *with the coordinate map* ϕ *such that in local coordinates* $x = (x^1, ..., x^n)$ *the conditions*

$$x^{m+i} = \phi^{m+i}(q) = 0, \quad i = 1, ..., n - m$$

are met for all $q \in \mathcal{U}$ *.*

The coordinates of definition 3 are also called flat coordinates. Furthermore, the parameterization of the submanifold is replaced by the natural inclusion.

2.1.1. Fibered Manifolds and Bundles

Let us consider the two smooth manifolds $\mathcal{M}, \mathcal{N}$ and a smooth map $f : \mathcal{M} \to \mathcal{N}$. Frequently, one considers the graph of f instead of f itself. The *graph* of f is the new function $\mathrm{gr}_f : \mathcal{M} \to \mathcal{M} \times \mathcal{N}$ defined by $\mathrm{gr}_f(p) = (p, f(p)), p \in \mathcal{M}$. The product $\mathcal{M} \times \mathcal{N}$ is called the *total space*. This set contains the domain $\mathcal{M}$ and the co-domain $\mathcal{N}$ of the map f, the domain is also called the *base space*. This idea can be extended to a new structure.

Definition 4 A fibered manifold is *a triple* $(\mathcal{E}, \mathcal{B}, \pi)$ *with the manifolds* $\mathcal{E}$, $\dim(\mathcal{E}) = m + n, \mathcal{B}$, $\dim(\mathcal{B}) = m$ *and a map* $\pi : \mathcal{E} \to \mathcal{B}$ *that is onto with rank m. The manifold* $\mathcal{E}$ *is called the* total space, *the map* π *the* projection *and the manifold* $\mathcal{B}$ *the* base space. *The subset* $\mathcal{F}_p = \pi^{-1}(p)$ *of* $\mathcal{E}$ *is called the* fiber *over* $p \in \mathcal{B}$.

In many cases the idea of a fibered manifold without any additional restriction is slightly too general. E.g. different fibers may have totally different topological structures. This problem may be resolved by insisting that the fibered manifold look rather like a product of manifolds and the resulting object is called a *bundle*.

Definition 5 *A* fibered manifold $(\mathcal{E}, \mathcal{B}, \pi)$ *is a bundle, iff there exists a manifold* $\mathcal{F}$ *, called the typical fiber, and a map* $\Psi : \pi^{-1}(\mathcal{U}_p) \to \mathcal{U}_p \times \mathcal{F}$ *defined on a neighborhood* $\mathcal{U}_p$ *of* $p \in \mathcal{B}$ *such that* $\mathrm{pr}_1 \circ \Psi = \pi(\mathrm{pr}_1(p, q) = p, q \in \mathcal{F}_p)$ *and* $\mathcal{F}, \mathcal{F}_p$ *are diffeomorphic for all* p *.*

On a bundle we can introduce *adapted coordinates* (x, u) at least locally, where $x^i, i = 1, ..., m$ are coordinates of the base $\mathcal{B}$ and $u^\alpha, \alpha = 1, ..., n$ are coordinates for the typical fiber. We get even a simpler picture, if we look at x as the independent and u as the dependent coordinates. Obviously, we have $\pi(x, u) = x$. A fibered manifold $(\mathcal{E}, \mathcal{B}, \pi)$ which is diffeomorphic to the bundle $(\mathcal{B} \times \mathcal{F}, \mathcal{B}, \mathrm{pr}_1)$ is called *trivial*. The well known Möbius band can be represented by a fibered manifold which is, of course, nontrivial. If the typical fiber of a bundle is a linear space, then the bundle is called vector bundle. An important example of a vector bundle is, e.g. the tangent bundle

$\mathcal{T}(\mathcal{M})$ of the manifold $\mathcal{M}$ which will be introduced in the following subsection.

This subsection started with the graph of a map between manifolds. Now, we are ready to give the adequate definition for fibered manifolds.

Definition 6 *Let* $(\mathcal{E},\mathcal{B},\pi)$ *be a fibered manifold. A map* $\sigma:\mathcal{B}\to\mathcal{E}$ *is called a* section *of* π, *if it satisfies* $\pi\circ\sigma=\mathrm{id}_{\mathcal{B}}$ *on its domain with* $\mathrm{id}_{\mathcal{B}}$ *as the identity map on* $\mathcal{B}$. *The set of all sections of* π *will be denoted by* $\Gamma(\pi)$.

It is worth mentioning that we do not require that a section is globally defined. Furthermore, there are manifolds that do not admit global smooth sections, which are zero nowhere. Let us look at the unit sphere $\mathbb{S}^2\subset\mathbb{R}^3$, then one can imagine that it is impossible to assign a non zero tangent vector to any point of $\mathbb{S}^2$ in a smooth way.

This fact is expressed by the Hairy Ball theorem that states, simply spoken, you cannot comb a hairy ball in a smooth way, or more precisely that any smooth tangent vector field on $\mathbb{S}^2$ must vanish somewhere.

Like we considered maps between manifolds, we can introduce maps for bundles that preserve the bundle structure.

Definition 7 *Let* $(\mathcal{E},\mathcal{B},\pi),(\overline{\mathcal{E}},\overline{\mathcal{B}},\overline{\pi})$ *be two bundles. A* bundle map *is a pair* $f=(f_{\mathcal{B}},f_{\mathcal{E}})$ *of maps* $f_{\mathcal{B}}:\mathcal{B}\to\overline{\mathcal{B}}, f_{\mathcal{E}}:\mathcal{E}\to\overline{\mathcal{E}}$ *such that* $\overline{\pi}\circ f_{\mathcal{E}}=f_{\mathcal{B}}\circ\pi$ *is met on the domain of f.*

Let (x,u) and $(\overline{x},\overline{u})$ be adapted coordinates of the bundles $(\mathcal{E},\mathcal{B},\pi),(\overline{\mathcal{E}},\overline{\mathcal{B}},\overline{\pi})$, then a bundle map f has the local representation

$$\overline{x}=\overline{f}_{\mathcal{B}}(x),\quad (\overline{x},\overline{u})=\overline{f}_{\mathcal{E}}(x,u),$$

where $\overline{f}_{\mathcal{B}},\overline{f}_{\mathcal{E}}$ denote the representation of $(f_{\mathcal{B}},f_{\mathcal{E}})$ in the adapted coordinates. From this representation it is easy to see that a bundle map $f=(f_{\mathcal{B}},f_{\mathcal{E}})$ allows us to transfer a section $\sigma\in\Gamma(\pi)$ to a section $\overline{\sigma}\in\Gamma(\overline{\pi})$ by

$$f_*(\sigma)=\overline{\sigma}=f_{\mathcal{E}}\circ\sigma\circ f_{\mathcal{B}}^{-1}, \qquad (7)$$

iff $f_{\mathcal{B}}$ is a diffeomorphism. We call $f_*(\sigma)$ also the *pushforward* of σ by f.

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Biographical Sketch

Prof. Kurt Schlacher Kurt Schlacher was born in Graz, Austria, on the 16*th* of August in the year 1955. 1973 he started to study electrical engineering at the Technical University of Graz, and finished 1979 with the diploma degree cum laude. In the year 1980 he did his obligatory national service. In the year 1981 he joined the department of automatic control at the Technical University of Graz, where he received his Ph.D. cum laude in the year 1984 and his habilitation for automatic control in the year 1990. In 1992 he moved to Linz at the Johannes Kepler University, Austria, where he got the position of a full professor for Automatic Control that he holds presently. Apart from several academic positions, he serves as an Associate Editor of the IEEE Transactions on Control Systems Technology. He is also member of the scientific committees of the following journals: IFAC International Journal of Automation Austria, Automatisierungstechnik (Oldenbourg-Verlag, Germany), as well as member of the IFAC Technical Committees on Control Design and Mechatronics. Since 2002 he is member of the IFAC council and of EUCA council. Furthermore, he is head of the Christian Doppler Laboratory for Automatic Control of Mechatronic Systems in Steel Industries. His main interests are modeling and control of nonlinear systems with respect to industrial applications applying differential geometric and computer algebra based methods. He is author of more than 80 publications published in national and international proceedings and journals, as well as co-author of the book Digitale Regelkreise (Oldenbourg-Verlag) together with Prof. Hofer and Prof. Gausch.