CONTROL OF BIFURCATIONS

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Contents

Summary

Bifurcation control generally refers to the design of a controller that can modify the bifurcating properties of a given nonlinear system, so as to achieve some desired dynamical behavior. Typical bifurcation control objectives include, but are not limited to, delaying the onset of an inherent bifurcation, introducing a new bifurcation at a preferred parameter value, changing the parameter value of an existing bifurcation

point, modifying the shape or type of a bifurcation chain, stabilizing a bifurcation point or a bifurcated solution, monitoring the multiplicity, amplitude, and/or frequency of some limit cycles emerging from bifurcations, optimizing the system performance near a bifurcation point, creating a particular bifurcation purposefully, or even a combination of some of these objectives. This chapter introduces this challenging and yet stimulating and promising field of research, putting the main subject of bifurcation control into perspective.

1. Introduction

Bifurcation Control, in general terms, refers to the task of designing a controller that can modify the bifurcating properties of a given nonlinear system, thereby achieving some desired dynamical behavior. Typical bifurcation control objectives include delaying the onset of an inherent bifurcation, introducing a new bifurcation at a preferred parameter value, changing the parameter value of an existing bifurcation point, modifying the shape or type of a bifurcation chain, stabilizing a bifurcation point or a bifurcated solution, monitoring the multiplicity, amplitude, and/or frequency of some limit cycles emerging from bifurcation, optimizing the system performance near a bifurcation point, creating a particular bifurcation purposefully, or even a combination of some of these objectives.

Bifurcation control not only is important in its own right, as further discussed in this chapter, but also suggests an effective strategy for chaos control, since bifurcation and chaos are usually "twins" and, in particular, period-doubling bifurcation is a typical route to chaos in many nonlinear dynamical systems.

It is now known that bifurcations can be controlled via various methods. Some representative approaches employ linear or nonlinear state-feedback controls e.g., use time-delayed feedback, apply a washout-filter-aided dynamic feedback controller, employ harmonic balance approximations, utilize quadratic invariants in normal forms, and so on. Some of these effective methods will be briefly introduced and described in the present chapter, along with a few closely related topics, as well as some potential real-world applications of bifurcation control and its implications to other areas of dynamical and control systems.

This chapter is an updated and simplified version of the tutorial of Chen, Moiola and Wang (2000) (courtesy of World Scientific Pub. Co., Singapore) and offers an overview of the interesting but challenging, and yet quite promising field of research on bifurcation control.

2. Bifurcation Control – The New Challenge

To start with, it may be illuminating to consider one simple yet representative system — the discrete-time Logistic map — to see how bifurcation control is different from most classical systems control, and how difficult this kind of control tasks would be. This may help appreciate the technical challenge of the bifurcation control problems in general.

The classical Logistic map is described by

$$x_{k+1} = f(x_k, p) := p\, x_k\,(1 - x_k), \tag{1}$$

where $p > 0$ is a real variable parameter. Two equilibria of the map can be found by solving the algebraic equation $x = f(x, p)$, which are: $x^* = 0$ and $x^* = (p-1)/p$. Further examination of the Jacobian, $J = \partial f/\partial x = p - 2p\,x$, reveals that the stabilities of these equilibria depend on parameter p.

For $0 < p < 1$, the point $x^* = 0$ is stable. Starting from any bounded initial point, the iterated sequence will converge to zero as $k \to \infty$. However, for $1 < p < 3$, all nonzero initial points of the map converge to $x^* = (p-1)/p$ instead. The dynamical evolution of the system behavior, as p is gradually increased from 3.0 to 4.0 by small steps, is very complex and interesting, as shown in Fig. 1. As can be seen from the figure, at $p = 3$ a stable period-two orbit $\{x^1, x^2\}$ is born out of x^*, with

$$x^{1,2} = (1 + p \pm \sqrt{p^2 - 2p - 3})/(2p).$$

As p increases to the value of $1+\sqrt{6} = 3.44948\ldots$, each of these two points bifurcates out into two new points. These four points together constitute a period-four solution of the map (at $p = 1+\sqrt{6}$). As p continuously moves through a sequence of values: $3.44948\ldots$, $3.5644\ldots$, $\cdots$, an infinite series of bifurcations is created by such *period-doubling*, which eventually leads to chaos:

$$\text{period } 1 \to \text{period } 2 \to \text{period } 4 \to \cdots \text{period } 2^k \to \cdots \to \text{chaos}$$

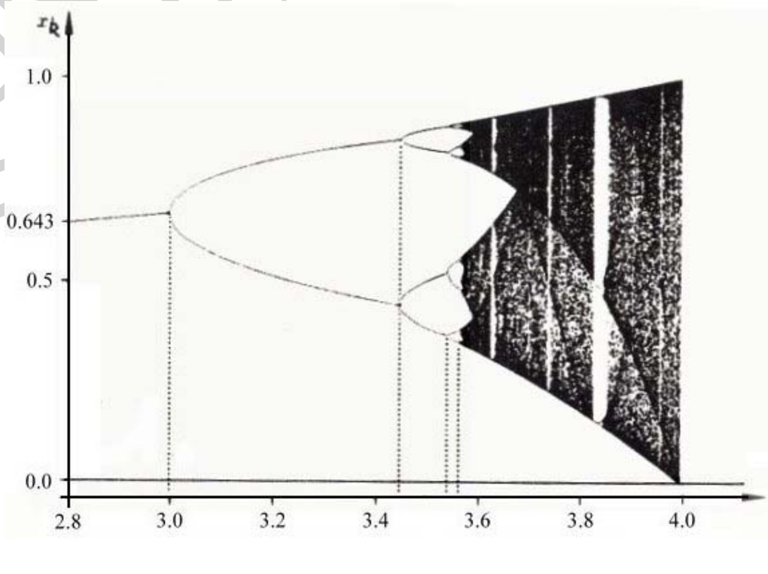

Figure 1: Period-doubling of the Logistic map.

At this point, several control problems may be raised: Is it possible (and, if so, how) to find a simple (say, linear) control sequence, $\{u_k\}$, such that the controlled Logistic system

$$x_{k+1} = F(x_k, p) = p\, x_k(1 - x_k) + u_k \tag{2}$$

can achieve, for instance, the following goals:

(i) The limiting chaotic behavior of the period-doubling bifurcation process is delayed, or completely suppressed.
(ii) The first or the second bifurcation is delayed to take place, or some bifurcations are changed either in form or in stability.
(iii) The asymptotic behavior of the system becomes chaotic (if chaos is beneficial), when the parameter p is currently not in the chaotic region.

Obviously, these are not typical objectives in conventional control theory, and may not be solved by classical stability-based feedback control methods.

3. Bifurcations in Control Systems

The example of system bifurcations discussed above is simple but illustrative. In fact, bifurcations can occur in many nonlinear dynamical systems, even in systems that are under feedback or adaptive controls. This perhaps comes as a surprise to control engineers, and may be counterintuitive; however, local instability and complex dynamical behavior can indeed result from such globally controlled systems – if adequate process information is not available for feedback or for parameter estimation. In these situations, one or more poles of the closed-loop transfer function of the linearized system may move to cross over the stability boundary, potentially leading to signal divergence as the control process continues. Sometimes, this may not lead to a global unboundedness in a complex nonlinear system, but rather, to self-excited oscillations, bifurcations, and even chaos.

Automatic Gain Control (AGC) loops provide a typical example of feedback control systems with bifurcation phenomena. AGCs are very popular in industrial applications, such as in most receivers of communication systems. Both the Video Graphics Array (VGA) and the detector are nonlinear, so that the AGC loop can have complex behavior such as homoclinic bifurcation leading to chaos. Its discrete version also has the common route of period-doubling bifurcations to chaos, similar to the Logistic map discussed above.

As a second simple example of feedback control systems, a single pendulum controlled by a linear proportional-derivative (PD) controller may also have various bifurcations. In fact, even a feedback system with a linear plant and a linear controller can produce bifurcations and chaos – if a simple nonlinearity (*e.g.*, saturation) exists somewhere in the loop.

Adaptive control systems are more likely to produce bifurcations than other control systems, due to the frequent changes of system stabilities. Different pathways that lead to estimator instability in a model-referenced adaptive control system can be identified. Similarly, rich bifurcation phenomena have been observed in discrete-time adaptive control systems.

Bifurcations are ubiquitous in physical systems, need not subject to control, such as the well-known example of power systems which generally have rich bifurcation phenomena. In particular, when the consumers' demands for power reach a peak, the dynamics of the service power network may move to its stability margin. This may yield oscillations and bifurcations, and quickly result in voltage collapse.

A typical double pendulum can also display bifurcation as well as chaotic motions. Some rotational mechanical systems also have similar behavior. A road vehicle under steering control can have Hopf bifurcation when it loses stability, which may also develop chaos and even hyperchaos. A hopping robot, even a simple two-degree-of-freedom flexible robot arm, can produce unusual vibrations and undergo period-doubling which leads to chaos. An aircraft rotating stall during flight, either below a critical speed or over a critical angle-of-attack, is caused by bifurcations. Dynamics of ships can exhibit bifurcations according to wave frequencies that are close to the natural frequency of the ship, creating oscillations and chaotic motions leading to ship capsize. Simple nonlinear circuits are rich sources of bifurcation phenomena. Many other systems have bifurcation properties, including cellular neural networks, laser machines, aero-engine compressors, weather, and biological population dynamics, to mention only those typical ones.

Given this background, it can be easily seen that controlling bifurcations will have tremendous impacts on many real-world applications. Meanwhile, it also provides new motivations to control theory development.

4. Preliminaries of Bifurcation Theory

This section first introduces some mathematical definitions of bifurcations.

It is convenient to consider a two-dimensional, parameterized, nonlinear dynamical system of the form

$$\begin{cases} \dot{x} = f(x, y; p) \\ \dot{y} = g(x, y; p), \end{cases} \tag{3}$$

where p is a real variable system parameter.

Let $(x^*, y^*) = (x^*(p_0), y^*(p_0))$ be an equilibrium of the system at $p = p_0$, satisfying both $f(x^*, y^*; p_0) = 0$ and $g(x^*, y^*; p_0) = 0$. If the equilibrium is stable (resp., unstable) for $p > p_0$ but unstable (resp., stable) for $p < p_0$, then there is a qualitative change of dynamical behavior. Here, p_0 is a *bifurcation value* of p, and (x^*, y^*, p_0) is a *bifurcation point* in the *parameter space* of coordinates $x - y - p$. A few examples are given below to distinguish several different but typical bifurcations.

4.1. Bifurcations in One-dimensional Systems

One-dimensional maps, used to define continuous-time systems here, are convenient to use for conceptual illustrations.

The one-dimensional system

$$\dot{x} = f(x; p) = p\,x - x^2$$

has two equilibria: $x_1^* = 0$ and $x_2^* = p$. If p is varied, then there are two equilibrium curves (see Fig. 2). Since the Jacobian of the system is $J = \partial f / \partial x|_{x=0} = p$, it is clear that for $p < p_0 = 0$ the equilibrium $x_1^* = 0$ is stable, but for $p > p_0 = 0$ it becomes unstable. Hence, $(x_1^*, p_0) = (0,0)$ is a bifurcation point. This is called a *transcritical bifurcation*. In this and the following figures, the solid curves indicate stable equilibria and the dashed curves, the unstable ones.

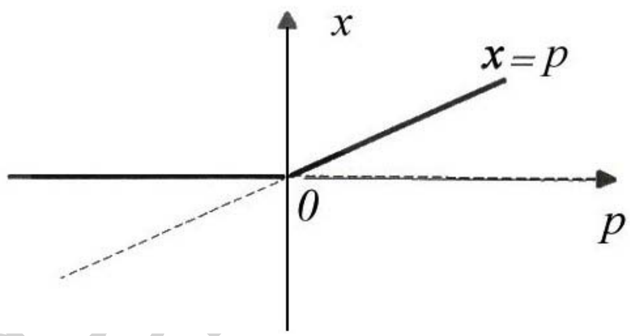

Figure 2: The transcritical bifurcation.

The one-dimensional system

$$\dot{x} = f(x; p) = p - x^2$$

has an equilibrium point, $x_1^* = 0$, at $p_0 = 0$, and an equilibrium curve, $(x^*)^2 = p$, at $p \geq 0$, where $x_2^* = \sqrt{p}$ is stable and $x_3^* = -\sqrt{p}$ is unstable for $p > p_0 = 0$. This is called a *saddle-node bifurcation* (see Fig. 3).

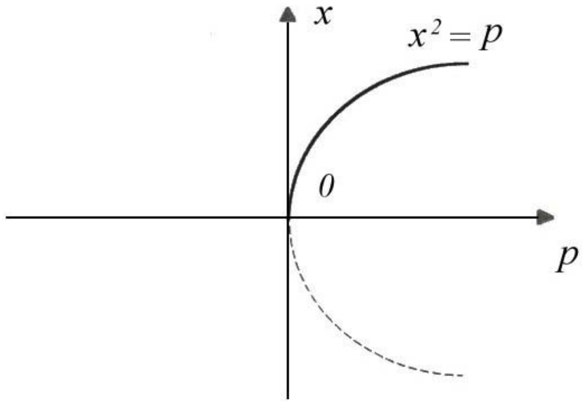

Figure 3: The saddle-node bifurcation.

The one-dimensional system

$$\dot{x} = f(x; p) = p\,x - x^3$$

has two equilibrium curves: one is $x_1^* = 0$ for all p and another is $(x^*)^2 = p$ for $p \geq 0$. Its Jacobian is $J = p - 3(x^*)^2$, so $x_1^* = 0$ is unstable for $p > p_0 = 0$ and stable for $p < p_0 = 0$. Also, the entire equilibrium curve $(x^*)^2 = p$ is stable for all $p > 0$ (because, at which the Jacobian is $J = -2p$). This is called a *pitchfork bifurcation*, and is depicted in Fig. 4.

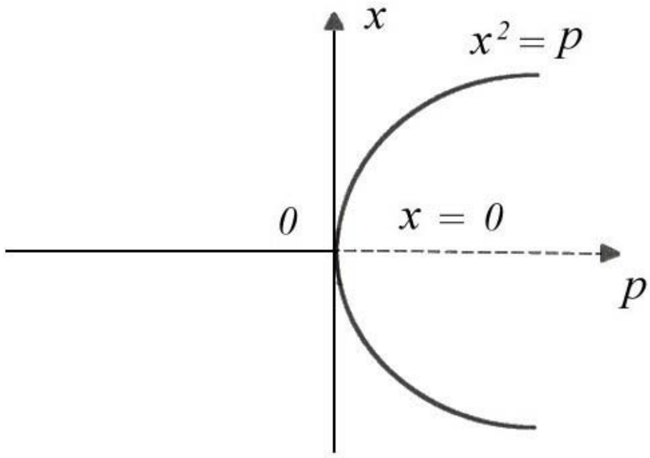

Figure 4: The pitchfork bifurcation.

The above-discussed bifurcation phenomena for one-dimensional parameterized nonlinear systems are usually referred to as *static bifurcations*. Analysis of such elementary static bifurcations by using a frequency domain approach is not only possible but quite efficient.

It should be noted that not all nonlinear dynamical systems have bifurcations, as can be verified by examining the following simple example:

$$\dot{x} = f(x;p) = p - x^3 .$$

This equation has an entire stable equilibrium curve, $x = p^{1/3}$, and does not have any bifurcation as the real parameter p is varied.

4.2. Hopf Bifurcation

In higher-dimensional systems (or maps), bifurcation phenomena are generally complex and complicated. For instance, in addition to the aforementioned static bifurcations, there is another important type of bifurcation existing in systems of higher dimensions — the *Hopf bifurcation*, classified as a *dynamic bifurcation*.

Hopf bifurcation occurs in the following scenario: As the parameter p is varied to pass a critical value p_0, the system Jacobian has one pair of complex conjugate eigenvalues moving from the left-half plane to the right, crossing the imaginary axis, while all the other eigenvalues remain to be stable. At the moment of crossing, the real parts of the two eigenvalues become zero, and the stability of the existing equilibrium changes from being stable to unstable. Also, at the moment of crossing, a limit cycle is born. These phenomena are supported by the following classical result (see Fig. 5):

Theorem (Poincaré-Andronov-Hopf)

Suppose that the two-dimensional system (3) *has a zero equilibrium,* $(x^*, y^*) = (0,0)$, *and that its associate Jacobian has a pair of purely imaginary eigenvalues,* $\lambda(p)$ *and* $\bar{\lambda}(p)$. *If*

$$\left. \frac{d\Re\{\lambda(p)\}}{d\,p} \right|_{p=p_0} > 0$$

where $\Re$ *denotes 'the real part of', then*

1. $p = p_0$ *is a bifurcation value of the system;*
2. *for close enough values* $p < p_0$, *the zero equilibrium is asymptotically stable;*
3. *for close enough values* $p > p_0$, *the zero equilibrium is unstable;*
4. *for close enough values* $p \neq p_0$, *the zero equilibrium is surrounded by a limit cycle of magnitude* $O(\sqrt{|p - p_0|})$.

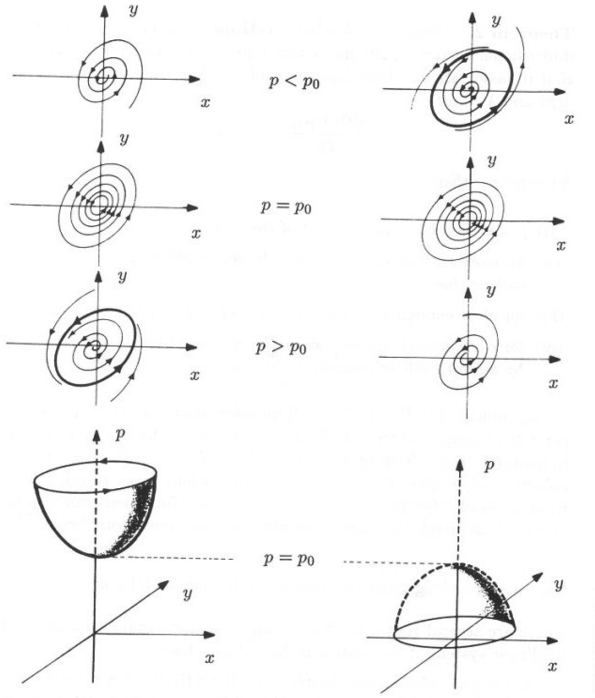

Figure 5: Two types of Hopf bifurcation in the phase plane.

As indicated in Fig. 5, the Hopf bifurcations are classified as *supercritical* (resp., *subcritical*) if the equilibrium is changed from stable to unstable (resp., from unstable to stable), where for the latter case the eigenvalues move from the right to the left. The same terminology of supercritical and subcritical bifurcations applies also to some other non-Hopf types of bifurcations.

For the discrete-time setting, consider a two-dimensional parameterized system:

$$\begin{cases} x_{k+1} = f(x_k, y_k; p) \\ y_{k+1} = g(x_k, y_k; p), \end{cases} \tag{4}$$

with a real variable parameter p and an equilibrium point (x^*, y^*), satisfying $x^* = f(x^*, y^*; p)$ and $y^* = g(x^*, y^*; p)$ simultaneously for all p. Let $J(p)$ be its Jacobian at this equilibrium, and $\lambda_{1,2}(p)$ be its eigenvalues, with $\lambda_2(p) = \bar{\lambda}_1(p)$. If

$$|\lambda_1(p^*)| = 1 \qquad \text{and} \qquad \left.\frac{\partial |\lambda_1(p)|}{\partial p}\right|_{p=p^*} > 0, \tag{5}$$

then the system undergoes a *Hopf bifurcation* at (x^*, y^*, p^*), in a way analogous to the continuous-time setting. Both supercritical and subcritical Hopf bifurcations exist in the discrete case, which can be determined via a sequence of coordinate transformations.

5. State-Feedback Control of Bifurcations

First, consider a one-dimensional discrete-time parameterized nonlinear control system of the form

$$x_{k+1} = F(x_k; p) := f(x_k; p) + u(x_k; p), \tag{6}$$

where p is a real variable parameter, $x_0 \in \mathbf{R}$ the initial state, and $u(\cdot)$ the state-feedback controller to be designed. The map $F: \mathbf{R} \to \mathbf{R}$ is autonomous, representing the dynamical behavior of the control system that can be visualized in the $x_k - x_{k+1}$ plane, called the *discrete phase plane*, where $k = 0, 1, 2, \cdots$.

Bifurcation analysis for this control system may be formulated as the following routine checking procedure for convenience in designing the controller through a trial-and-error process.

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Biographical Sketch

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