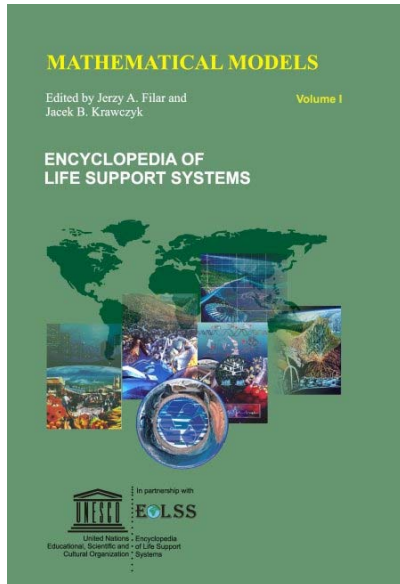


CONTENTS

MATHEMATICAL MODELS



Mathematical Models - Volume 1

No. of Pages: 438

ISBN: 978-1-84826-242-3 (eBook)

ISBN: 978-1-84826-695-7 (Print Volume)

Mathematical Models - Volume 2

No. of Pages: 498

ISBN: 978-1-84826-243-0 (eBook)

ISBN: 978-1-84826-696-4 (Print Volume)

Mathematical Models - Volume 3

No. of Pages: 400

ISBN: 978-1-84826-244-7 (eBook)

ISBN: 978-1-84826-697-1 (Print Volume)

For more information of e-book and Print Volume(s) order, please [click here](#)

Or contact : eolssunesco@gmail.com

CONTENTS

VOLUME I

Mathematical Models	1
Jerzy A. Filar, <i>Centre for Industrial and Applicable Mathematics, University of South Australia, Mawson Lakes, Australia</i>	

1. Introduction
2. Why Do We Resort to Mathematical Modeling of Life Support Systems?
3. What Kinds of Life Support Systems Can Be Described by Mathematical Models?
4. How Is Mathematical Modeling Done?
 - 4.1. Modeling on the Basis of Previously Established "Governing Equations"
 - 4.2. Extracting Models from Data
 - 4.3. Mathematical Computer Models
 - 4.3.1. Models Created with the Help of High-Level Languages
 - 4.3.2. Integrated Mathematical Computer Models
 - 4.3.3. Complex Adaptive Systems
 - 4.4. Hybrid Mathematical Models
 - 4.5. The Iterative Nature of Model Construction
5. Understanding Uncertainty Accompanying Mathematical Models
6. The Impact of the Information Technology "Revolution" on both the Practice and Uses of Mathematical Modeling
 - 6.1. Misinformed Use of Mathematical Modeling tools
 - 6.2. A Threat to the "Principle of Parsimony"
 - 6.3. Shadows of Computer Implementations

Basic Principles of Mathematical Modeling	27
Claude Elysee Lobry, <i>Institut National de Recherche Agronomique, Laboratoire de biométrie et analyse des systèmes, 9Place Viala, 34060 Montpellier Cedex, France</i>	

1. Introduction
 - 1.1. A Fashionable Word
 - 1.2. Modeling: A Complex Activity
 - 1.3. The Need for Mathematics
 - 1.4. Orientation of the Chapter
2. The Mathematical Concept of Dynamical System
 - 2.1. Deterministic Systems
 - 2.1.1. Dynamical System on a Finite Set
 - 2.1.2. Discrete Dynamical Systems on R
 - 2.1.3. Systems of Differential Equations
 - 2.1.4. Dynamical Systems on Manifolds
 - 2.1.5. Infinite Dimensional Systems
 - 2.1.6. Miscellanies
 - 2.2. Stochastic Dynamical Systems
 - 2.2.1. Markov Chain
 - 2.2.2. Random Walk and Wiener Process
 - 2.3. Discrete versus Continuous models
 - 2.3.1. Discrete versus Continuous Time Models
 - 2.3.2. Cellular Automata and Individual oriented Models
 - 2.3.3. Stochastic Differential Equations
3. Modeling in Automatic Control (Mathematical Systems theory)
 - 3.1. The deterministic input-output system
 - 3.1.1. Input-output Systems
 - 3.1.2. Feedback
 - 3.2. Examples of Deterministic Input-output Systems

- 3.2.1. An Artificial System: The Inverted Pendulum
- 3.2.2. A Natural System: Bacterial Growth
- 3.2.3. A Natural System: A Structured Population
- 3.2.4. Continuous Culture of Micro-organisms
- 3.2.5. Examples of Infinite-dimensional Systems
- 3.3. Mathematical Theory of Input-output Systems
 - 3.3.1. Controllability
 - 3.3.2. Observability
 - 3.3.3. Realization Theory
 - 3.3.4. Stabilization and Observers
- 4. Conclusion: Mathematical Models for what Purposes?
 - 4.1. Models for Understanding
 - 4.2. Models for Description and Prediction
 - 4.3. Models for Control

Classification of Models

88

Jean-Luc Gouze, *COMORE, INRIA, Sophia-Antipolis, France*
 Tewfik Sari, *Mulhouse University, France*

- 1. Discrete-time Models
 - 1.1. A Model for Cell Division
 - 1.2. Matrix and Leslie Models
 - 1.3. Nonlinear Discrete Models
- 2. Continuous-time Models
 - 2.1. Malthus's Model and Verhulst's Model
 - 2.2. The Chemostat
 - 2.3. Lotka-Volterra Equations for Predator-Prey Systems
 - 2.4. Lotka-Volterra Equations for Competing Species
 - 2.5. The General Lotka-Volterra Equation
 - 2.6. The Predator-Prey Model of Gause

Basic Methods of the Development and Analysis of Mathematical Models

101

Jean-Luc Gouze, *COMORE, INRIA, Sophia-Antipolis, France*
 Tewfik Sari, *Mulhouse University, France*

- 1. Discrete Time Models
 - 1.1. Making a Model
 - 1.2. The State Space: Basic Vocabulary
 - 1.3. Linear Discrete Equations
 - 1.3.1. The Homogeneous Constant Linear System
 - 1.3.2. The Homogeneous Time-varying Linear System
 - 1.3.3. The Non-homogeneous Linear System
 - 1.3.4. The Controlled Linear System
 - 1.3.5. Conversion to Matrix Linear Form
 - 1.4. Basic Study of the Homogeneous Constant Linear System
 - 1.5. Basic Study of the Non-homogeneous Constant Linear System
 - 1.6. Basic study of the homogeneous time-varying linear system
 - 1.7. Positive Linear Systems
 - 1.7.1. Basic properties of positive linear constant systems
 - 1.7.2. Basic Properties of Non-homogeneous Positive Linear Systems
 - 1.7.3. Various properties of positive linear systems
 - 1.8. Nonlinear discrete systems
 - 1.8.1. Useful Elements of the Study
 - 1.8.2. Stability
 - 1.8.3. Local Study around an Equilibrium
 - 1.8.4. Liapunov Functionals

- 1.8.5. The One-dimensional Example
- 1.8.6. Bifurcation with respect to a Parameter
- 2. Continuous time Models
 - 2.1. The Concept of Differential System
 - 2.1.1. Solutions of Differential Equations
 - 2.1.2. Continuous Dependence of Solutions, Stability
 - 2.2. Linearization
 - 2.2.1. Linear Systems
 - 2.2.2. Stability in the Linear Approximation
 - 2.2.3. The Chemostat with two competing species
 - 2.3. Autonomous Systems
 - 2.3.1. Lotka-Volterra Equations for Predator-Prey Systems
 - 2.3.2. Limit Sets
 - 2.3.3. Poincaré-Bendixon theory
 - 2.3.4. The Gause predator prey model with Holling-type interaction

Measurements in Mathematical Modeling and Data Processing

132

William Moran, *University of Melbourne, Australia*

Barbara La Scala, *University of Melbourne, Australia*

- 1. Introduction
- 2. Hypothesis Testing
 - 2.1. Overview of Signal Detection
 - 2.2. Bayes Detection
 - 2.2.1. Risk
 - 2.3. Neyman-Pearson Method
 - 2.4. Minimax Method
 - 2.5. Composite Testing
- 3. Sufficient Statistics
 - 3.1. Sufficient Statistics and Hypothesis Testing
 - 3.2. Invariance
- 4. Signal Detection
 - 4.1. Signal Detection Problems
 - 4.1.1. Detection of a Deterministic Signal in Independent Noise
 - 4.1.2. Gaussian Noise
 - 4.2. Coherent Signals in IID Noise
 - 4.3. Signal Selection
 - 4.4. Stochastic Signals
 - 4.5. Quadratic Detectors
- 5. Estimation Theory
 - 5.1. Cramér-Rao Lower Bound
 - 5.1.1. CRLB-Vector Parameter Case
 - 5.1.2. DC in Noise of Unknown Variance – CRLB
 - 5.1.3. Line Fitting – CRLB
 - 5.1.4. Gaussian Case – CRLB
 - 5.2. Sufficient Statistics
 - 5.3. Maximum Likelihood Estimation
 - 5.3.1. MLE for Exponentially Distributed Signals
 - 5.4. Bayesian Estimation
 - 5.4.1. Bayesian Minimum Mean Square Estimation
 - 5.4.2. Maximum A Posteriori Estimation

Controllability, Observability, and Stability of Mathematical Models

213

Abderrahman Iggidr, *INRIA (Ur Lorraine) and, University of Metz, France*

- 1. Introduction

2. Controllability
 - 2.1. What is Controllability?
 - 2.2. Controllability of Linear Systems
 - 2.3. Accessibility Criteria for Nonlinear Systems
3. Stability
 - 3.1. General Definitions
 - 3.2. Stability of Linear Systems
 - 3.3. Linearization and Stability of Nonlinear Systems
 - 3.4. Lyapunov Functions
 - 3.5. Limit Cycle
 - 3.6. Stabilization
 - 3.6.1. Sufficient Stabilizability Conditions
4. Observability
 - 4.1. Observability of Linear Systems
 - 4.2. Observability of Nonlinear Systems
 - 4.3. Examples from Life Support Systems
5. Observers
 - 5.1. Observers for Linear Systems
 - 5.2. Some Nonlinear Observers
 - 5.2.1. System with no Input
 - 5.2.2. Affine Control Systems
 - 5.2.3. Observers for a Class of Non-affine Control Systems
 - 5.2.4. Asymptotic Observers

Identification, Estimation, and Resolution of Mathematical Models

283

J.M. van den Hof, *Researcher at Statistics Netherlands, Voorburg, The Netherlands*

Jan H. van Schuppen, *Senior researcher, Centre for Mathematics and Computer Science, Amsterdam, and the Department of Mathematics, Vrije Universiteit, Amsterdam, The Netherlands*

1. Introduction
2. Problem of Identification
3. Identification Procedure
 - 3.1. Selection of Subclass of Dynamic Systems
 - 3.2. Data Collection
 - 3.3. Identifiability
 - 3.3.1. The Problem
 - 3.3.2. Observations
 - 3.3.3. Realization of Finite-dimensional Linear Systems
 - 3.3.4. Weak Stochastic Realization for Finite-dimensional Gaussian Systems
 - 3.3.5. Parameterization
 - 3.3.6. Identifiability from the Impulse Response Function
 - 3.3.7. Identifiability from Input-Output Time Series
 - 3.4. Approximation
 - 3.4.1. The Approximation Problem
 - 3.4.2. Approximation Criteria
 - 3.4.3. Approximation Theory
 - 3.4.4. Approximation Algorithms: Least-squares and Maximum Likelihood
 - 3.4.5. Approximation Algorithms: Subspace Identification Algorithm
 - 3.4.6. Order Estimation
 - 3.5. Evaluation
4. Identification for several other Classes of Dynamic Systems
5. Research Problems
6. Software for Identification

Mathematical Theory of Data Processing in Models (Data Assimilation Problems)

337

Arnold W. Heemink, *Delft University of Technology, 2600 GA Delft, The Netherlands*

1. Introduction
2. Variational Data Assimilation
 - 2.1. Data Assimilation Formulated as a Minimization Problem
 - 2.2. The Adjoint Model
 - 2.3. Discussion
3. Kalman Filtering
 - 3.1. The Linear Kalman Filter
 - 3.2. The Extended Kalman Filter
 - 3.3. Kalman Filtering for Large-scale Systems
 - 3.3.1. Square Root Filtering
 - 3.3.2. Ensemble Kalman Filter
 - 3.3.3. Reduced Rank Square Root Kalman Filter
 - 3.3.4. Discussion

Chaos and Cellular Automata	353
<i>Claude Elysee Lobry, Université de Nice, Faculté des Sciences, parc Valrose, 06000 NICE, France.</i>	

1. Chaos
 - 1.1. Introduction
 - 1.2. One Dimensional Discrete Chaotic Systems
 - 1.3. Two Dimensional Discrete Chaotic System
 - 1.4. The Cascade of Bifurcation
 - 1.5. Continuous Chaotic Systems
 - 1.6. The Story of Chaos in Dynamical Systems
2. Cellular Automata
 - 2.1. Introduction
 - 2.2. The S.E.I.R. Model for Epidemics
 - 2.3. Analysis of the I.R.N automaton
 - 2.3.1. Definitions
 - 2.3.2. Some simulations
 - 2.4. A partial Theory for the (I, R, N, G) Automaton
 - 2.5. The Need for a Theory when one uses Cellular Automata in Modeling
3. Conclusion

Index	373
--------------	------------

About EOLSS	379
--------------------	------------

VOLUME II

Mathematical Models in Hydrodynamics	1
<i>Claus Albrecht Wagner, Institute for Aerodynamics and Flow Technology, German Aerospace Center Göttingen, Germany</i>	

1. Introduction
2. Some Fundamentals
3. Direct Numerical Simulation
4. Statistical Turbulence Modeling
5. Large Eddy Simulation
 - 5.1. Eddy Viscosity Models
 - 5.2. Alternative Subgrid Scale Models

Mathematical Modeling of Flow in Watersheds and Rivers**23**

Vijay P. Singh, *Louisiana State University, Department of Civil and Environmental Engineering Baton Rouge, LA 70803-6405, USA*

1. Introduction
2. Flow in Watersheds and Channels
3. Governing Equations
 - 3.1. Surface Flow
 - 3.2. Unsaturated Flow
 - 3.3. Saturated Flow
 - 3.4. Initial and Boundary Conditions
4. Deterministic and Statistical Flow Modeling
5. Deterministic Modeling of Flow in Watersheds
 - 5.1. Infiltration and Soil Moisture
 - 5.2. Storm Runoff Generation
 - 5.3. Surface Runoff
 - 5.3.1. Surface Runoff Hydrograph
 - 5.3.2. Surface Runoff Volume
 - 5.4. Hydrologic Data
6. Deterministic Modeling of Flow in Channels
 - 6.1. Geometric Representation
 - 6.2. Governing Equations
 - 6.3. Sources and Sinks
 - 6.4. Initial and Boundary Conditions
 - 6.5. Simplifications of Governing Equations
 - 6.6. Applicability of Simplified Representations
 - 6.7. Solution of St. Venant Equations
 - 6.8. Data Acquisition
7. Statistical Modeling of Flow in Watersheds
 - 7.1. Empirical Analyses
 - 7.2. Phenomenological Analyses
 - 7.3. Stochastic Analyses
8. Emerging Technologies for Flow Modeling
 - 8.1. Artificial Neural Networks
 - 8.2. Fuzzy Logic
 - 8.3. Genetic Algorithms
 - 8.4. Combination Methods
9. Uncertainty Analysis
 - 9.1. Univariate Distributions from Impulse Response Functions
 - 9.2. Entropy-Based Univariate Probability Distributions
 - 9.3. Joint Probability Distributions
 - 9.4. Point Estimation Methods
 - 9.4.1. Rosenblueth's Method
 - 9.4.2. Harr's Method
 - 9.4.3. Li's Method
 - 9.4.4. Modified Rosenblueth's Method
 - 9.4.5. Characteristics of Point Estimation Methods
10. Hydrologic Design

Mathematical Models of Circulations in Oceans and Seas**61**

Aike Beckmann, *Alfred Wegener Institute for Polar and Marine Research, Bremerhaven, Germany;*
Division of Geophysics, Department of Physical Sciences, University of Helsinki, Helsinki, Finland
 Jens Schroter, *Alfred Wegener Institute for Polar and Marine Research, Bremerhaven, Germany*

1. Introduction
2. Areas of Model Application
 - 2.1. Elements of the Large-Scale Circulation

- 2.1.1. Thermohaline Circulation
- 2.1.2. Wind-driven Circulation
- 2.1.3. Global Fresh-Water Cycle and Sea Ice
- 2.1.4. Patterns and Modes of Climate Fluctuations
- 2.2. Important Small-Scale Processes
 - 2.2.1. Boundary Layers
 - 2.2.2. Turbulent Mixing and Convection
- 3. Approximate Systems of Equations
 - 3.1. Nonhydrostatic Primitive Equations
 - 3.2. Hydrostatic Primitive Equations
 - 3.3. Other Systems
 - 3.4. Initial and Boundary Conditions
- 4. Ocean Modeling Concepts
 - 4.1. Domain Size: Global - Basinscale – Regional
 - 4.2. Degree of Physical Realism: Idealized Study - Realistic Simulation
 - 4.3. Inclusion of Observational Data: Forward-Inverse
- 5. Numerical Aspects
 - 5.1. Scales and Resolution
 - 5.2. Horizontal Grids
 - 5.2.1. Finite Differences
 - 5.2.2. Finite Elements
 - 5.3. Vertical Coordinate Systems
 - 5.3.1. Geopotential Coordinates
 - 5.3.2. Terrain-following Coordinates
 - 5.3.3. Density Coordinates
 - 5.3.4. Unstructured and Hybrid Coordinates
 - 5.4. Advection Schemes
 - 5.5. Subgrid-scale Parameterizations
 - 5.5.1. Diffusive Effects
 - 5.5.2. Orientation of Mixing
 - 5.5.3. Nondiffusive Effects
 - 5.5.4. Parameterization of Convection
 - 5.5.5. Filtering
 - 5.6. Solution Procedures
 - 5.6.1. Explicit and Implicit Time Stepping
 - 5.6.2. Elliptic Systems
 - 5.7. Technical Aspects
- 6. The Quality of Model Results; Validation and Evaluation
- 7. Outlook

Wave Modeling at the Service of Security in Marine Environment

93

Eva Bauer, *Potsdam Institute for Climate Impact Research (PIK), Potsdam, Germany*

- 1. Introduction
 - 1.1. Classification of Waves
 - 1.2. History of Wave Modeling
 - 1.3. Outline of the Chapter
- 2. Physical Principles of Free Surface Waves
 - 2.1. Statistical and Spectral Characteristics
 - 2.2. Wave Kinematics in the Domain of Potential Theory
 - 2.3. Wave Kinematics in the Domain of Ray Theory
- 3. Forcing Functions for Wave Modeling
 - 3.1. Weak Nonlinear Wave-wave Interactions S_{nl}
 - 3.2. Wave Generation S_{in}
 - 3.3. Wave Dissipation S_{ds}
- 4. Present Applications of Wave Modeling
 - 4.1. Validation using Global Data

- 4.2. Synoptic-scale Wave Predictions
- 4.3. Coupled Wind and Wave Data Assimilation
- 5. Outlook

Mathematical Modeling of the Transport of Pollution in Water

117

Joachim Dippner, *Baltic Sea Research Institute Warnemünde, Seestr. 2, D-18119 Rostock, F.R. Germany*

- 1. Introduction
- 2. Phenomenology
- 3. Experiments
- 4. A Short Introduction to Turbulence Theory
 - 4.1. Fundamentals
 - 4.2. The Turbulent Closure Problem
 - 4.3. Turbulent Diffusion from Point Sources
 - 4.4. The Spectrum of Turbulence
- 5. Mathematical Modelling of the Transport of Pollution
 - 5.1. Analytical Solutions
 - 5.1.1. The Advection Equation
 - 5.1.2. The Diffusion Equation
 - 5.1.3. The Reaction Equation
 - 5.1.4. The Advection-Diffusion Equation
 - 5.2. Numerical Examples
 - 5.2.1. The Advection Equation
 - 5.2.2. The one-dimensional Diffusion Equation
- 6. An Alternative Approach: Lagrangian Tracer Technique (LTT)
 - 6.1. What is a Tracer?
 - 6.2. Lagrangian versus Eulerian View
 - 6.3. Simulation of Advection with LTT
 - 6.4. Simulation of turbulent mixing with LTT
- 7. Examples

Mathematical Models in Electric Power Systems

161

Prabha Kundur, *Powertech Labs Inc., Surrey, B.C., Canada*

Lei Wang, *Powertech Labs Inc., Surrey, B.C., Canada*

- 1. Introduction
- 2. Basic Concepts
 - 2.1. Basic Electrical Quantities
 - 2.2. Power in an AC Circuit
- 3. Elements of an Electric Power System
 - 3.1. Power Generation
 - 3.2. Power Transmission
 - 3.3. Utilization of Electric Energy
- 4. Power System Design, Operation and Control
- 5. Equipment Models
 - 5.1. Generator Modeling
 - 5.2. Excitation System Modeling
 - 5.3. Prime Mover and Governing System Modeling
 - 5.4. Power System Load Modeling
 - 5.5. Transmission Network Modeling
- 6. Modelling and Simulation of Power System Performance
 - 6.1. Power Flow Analysis
 - 6.2. Economic Dispatch
 - 6.3. Fault Analysis
 - 6.4. Power System Stability Analysis
 - 6.5. Electromagnetic Transients Analysis

- 6.6. State Estimation
- 6.7. Real-Time Simulation
- 6.8. Power System Harmonics Analysis

Mathematical Models Of Nuclear Energy **200**

Yuri A. Svistunov, *Department of Applied Mathematics and Control Processes, State University of St.Petersburg, Russia*

- 1. Introduction
- 2. Reactor Background
- 3. Neutron Transport Equation
- 4. General Properties of Transport Equation
- 5. Methods of Solution
 - 5.1. P_1 - Approximation and Diffusion Approximation
 - 5.2. Multi-group Calculation
 - 5.3. S_N - Method
 - 5.4. Monte-Carlo Method
- 6. Optimization Models
 - 6.1. Statement of Optimization Problems
 - 6.2. Boundary conditions
 - 6.3. Principle of Maximum
 - 6.4. Example of Optimization Task (Solution 's Scheme)
- 7. Future: Prospective projects of nuclear power engineering

Mathematical Models in Chemical Physics and Combustion Theory **221**

Valentin I. Korotkov, *Department of Physics, S-Petersburg State University, Russia*

- 1. Introduction
- 2. Chain Reactions
- 3. Link between Energy and Kinetics of Reaction
- 4. Length of Chains
- 5. Breaking of Chains
- 6. Breaking of Chains in a Volume and at the Surface
- 7. Development of Chains with Time
- 8. Combustion
- 9. Detonation Waves
- 10. Modeling the Temporal Evolution of a Reduced Combustion
- 11. A Model for Calculating Heat Release

Mathematical Modeling and Simulation Methods in Energy Systems **241**

Olivier Bahn, *Management Sciences Department, HEC Montréal, Canada*

Alain B. Haurie, *LOGILAB-HEC, University of Geneva, Switzerland*

D.S. Zachary, *Physics Dept., American University of Sharjah (United Arab Emirates)*

- 1. Introduction
- 2. Bottom-up versus Top-down Modeling
- 3. Simulation vs. Optimization
 - 3.1. Simulation and Future Tendency Predictions
 - 3.2. Computable Economic Equilibrium Models
 - 3.2.1. The Supply Curve
 - 3.2.2. The Demand Curve
 - 3.2.3. The Supply-Demand Equilibrium
 - 3.3. Tentative Classification of E3 Models
- 4. Technology Ranking
- 5. Issues in Energy Modeling

- 5.1. The Technological Change Issue
- 5.2. The Uncertainty Issue
 - 5.2.1. Typology of Uncertainty Sources in Energy Modeling
 - 5.2.2. Scenario Analysis
 - 5.2.3. Incorporating Uncertainty Analysis in Stochastic Programming
 - 5.2.4. Maximin / Minimax Approaches
- 5.3. The Discounting Issue
- 5.4. The Environmental Issue
 - 5.4.1. Global, Regional and Local Pollutants
 - 5.4.2. Assessing Environmental Policies
 - 5.4.3. Marginal Environmental Cost and Market based Instruments
- 6. Conclusion

Mathematical Models of Climate **258**
 Gerald R. North, *Department of Atmospheric Sciences, Texas A&M University, College Station, USA*
 Carlos Roberto Mechoso, *Department of Atmospheric Sciences, University of California, Los Angeles, USA*

- 1. Introduction
- 2. Models Based upon Energy Balance
- 3. Atmospheric General Circulation Models
- 4. Oceanic GCMs
- 5. Coupled AOGCMs
- 6. Other Climate Components
- 7. Applications of Climate Models
- 8. Challenges for the Future

Mathematical Models in Meteorology and Weather Forecasting **274**
 Eugenia Kalnay, *University of Maryland, College Park, MD 20742-2425.,USA*

- 1. Introduction
- 2. History of Numerical Weather Prediction
- 3. Numerical Models
 - 3.1. Filtered Models
 - 3.2. Shallow Water Model
 - 3.3. Complete Governing Equations
- 4. Data Assimilation
- 5. Ensemble Forecasting and Predictability
- 6. The Future

Mathematical Models of Human-Induced Global Change **288**
 Alex Hall, *UCLA Department of Atmospheric Sciences, Los Angeles, California, USA 90095*

- 1. Introduction
- 2. Historical Development
 - 2.1. Early Models
 - 2.2. Development of GCMs
 - 2.3. Coupling the Atmosphere to the Ocean, Land, and Biosphere
- 3. Current Methodology
 - 3.1. Design of Climate Change Experiments
 - 3.2. Technical Issues
 - 3.3. Model Validation
- 4. Strengths and Weaknesses of Climate Models
 - 4.1. Simulation of Present-day Climate
 - 4.2. Equilibrium Response to External Forcing

- 4.3. Transient Response
- 5. Future Challenges

Mathematical Models in Air Quality Problems

305

Jean Roux, *Environmental Research and Teaching Institute, École Normale Supérieure, Paris, France.*

- 1. Introduction
- 2. A Fundamental Chemical Kinetics System
- 3. Modeling of Linear Advection
 - 3.1. Modeling in 1D
 - 3.1.1. Generalities and Finite Difference Methods
 - 3.1.2. Simple Finite Difference Schemes
 - 3.2. Finite Volume Method in 1D
 - 3.2.1. Formulation of the Finite Volume Method
 - 3.2.2. Godunov Type Methods: General Principle and Formulation
 - 3.2.3. Examination of usual Godunov Methods
 - 3.3. Modeling of the Advection Problem in 3D
- 4. Modeling of Chemical Ordinary Differential Equations
 - 4.1. Formulation of the Problem and Classical Methods
 - 4.2. Special Methods for Chemistry Modeling
 - 4.2.1. The QSSA Method
 - 4.2.2. The Two-step Method
- 5. One Example of the Modeling of the Air Pollution Problem: the CHIMERE Software
 - 5.1. Chemical Mechanism
 - 5.2. Emissions
 - 5.3. Depositions
 - 5.4. Advection, Diffusion and Boundary Layer Modeling
 - 5.5. The Model in 3D
 - 5.6. An Illustrative Example

Infiltration and Ponding

333

David Andrew Barry, *École Polytechnique Fédérale de Lausanne, Switzerland*
 Marc B. Parlange, *Ecole Polytechnique Fédérale de Lausanne, Switzerland*
 Jean-Yves Parlange, *Cornell University, College of Engineering, Ithaca, NY, USA*
 Meng-Chia Liu, *Cornell University, College of Engineering, USA*
 Tammo S. Steenhuis, *Cornell University, College of Engineering, Ithaca, NY, USA*
 Graham C. Sander, *Loughborough University, Leicestershire, UK*
 David A. Lockington, *University of Queensland, Australia*
 Ling Li, *University of Queensland, Australia*
 Frank Stagnitti, *Deakin University, School of Life & Environmental Science, Warrnambool, Australia*
 Shmuel Assouline, *Volcani Center, Israel*
 John Selker, *Oregon State University, USA*
 Dong-Sheng Jeng, *University of Sydney, Australia*
 Randel Haverkamp, *Universite Joseph Fourier, France*
 William B. Hogarth, *University of Newcastle, Newcastle, Australia*

- 1. Introduction
- 2. The Green and Ampt (1911) Model
 - 2.1. Derivation
- 3. Green and Ampt Model and Richards' Equation
- 4. Richards' Equation and Profile Analysis
 - 4.1. θ_s Constant
 - 4.2. q Constant
- 5. Gravity Effects
- 6. Conclusions

Mathematical Equations of the Spread of Pollution in Soils **358**

Frank Stagnitti, *Deakin University - School of Life & Environmental Science, Australia*
 Jean-Yves Parlange, *Cornell University, College of Engineering, USA*
 Tammo S. Steenhuis, *Cornell University, USA*
 David Andrew Barry, *Ecological Engineering Laboratory, Switzerland*
 David A. Lockington, *University of Queensland, Australia*
 Graham C. Sander, *Loughborough University, UK*

1. Introduction
2. Convective-Diffusive Equation
3. Effects of Boundary Conditions
4. Chemical Reactions
5. Nonlinear Adsorption
6. Two Species Competition
7. Interaction of Surface Water and Chemical Transport in Soils
8. Column Flow
9. Transient Unsaturated Water and Solute Transport
10. Scale Dependent Solutions
11. Transient Solution Profiles
12. Source Solutions
13. Conclusion

Mathematical Soil Erosion Modeling **389**

Graham C. Sander, *Loughborough University, UK*
 C.W. Rose, *Griffith University, Australia*
 W.L. Hogarth, *University of Newcastle, Australia*
 Jean-Yves Parlange, *Cornell University, College of Engineering, USA*
 I.G. Lisle, *University of Canberra, Australia*

1. Introduction
2. Surface Hydrology
 - 2.1. Analytical Solutions
 - 2.2. Field Applications
3. Soil Erosion Processes
 - 3.1. WEPP
 - 3.2. EUROSEM
 - 3.3. Rose - Hairsine Model
4. Steady State Solutions of the Rose - Hairsine Model
 - 4.1. Net Erosion Solutions ($q_s = 0$ at $x = 0$)
 - 4.1.1. Rainfall-driven Erosion
 - 4.1.2. Flow Driven Erosion, $\Omega > \Omega_{cr}$
 - 4.2. Net Deposition Solutions ($q_s \neq 0$ at $x=0$)
 - 4.2.1. Single Size Class Solutions
 - 4.2.2. Multi-Size Class Solutions
 - 4.2.3. Multi-Size Class Solutions with Rainfall Redetachment
5. Dynamic Erosion - Time Dependence
 - 5.1. Solutions for $q = 0$ at $x = 0$
 - 5.2. Solutions for $q \neq 0$ at $x = 0$
 - 5.3. Stochastic Sediment Transport Model
6. Field Scale

Index **439**

About EOLSS **447**

VOLUME III

Mathematical Models of Biology**1**P. Haccou, *Institute of Biology, Leiden University, The Netherlands*

1. About Modeling
 - 1.1. What Are Models?
 - 1.2. Features of Models
 - 1.2.1. Manageability
 - 1.2.2. Generality
 - 1.2.3. Precision
 - 1.2.4. Realism
 - 1.3. Application of Models at Different Stages of a Research Program
 - 1.4. Modeling Philosophies
2. Archetypical Models of Evolution and Ecology
 - 2.1. Population Biology
 - 2.1.1. Non-Spatial Models
 - 2.1.2. Migration and Spatial Structure
 - 2.1.3. Metapopulation Dynamics
 - 2.1.4. Demographic Stochasticity
 - 2.2. Evolutionary Genetics
 - 2.2.1. Population Genetics
 - 2.2.2. Quantitative Genetics
 - 2.3. Evolutionary Ecology
 - 2.3.1. Optimal Foraging Models and the Ideal Free Distribution
 - 2.3.2. Evolutionary Game Theory
 - 2.3.3. Adaptive Dynamics
3. Conclusions

Mathematical Models of Marine Ecosystems**28**Sergei V. Petrovskii, *Shirshov Institute of Oceanology, Russian Academy of Sciences, Moscow 117218, Russia*Horst Malchow, *Institute for Environmental Systems Research Department of Mathematics and Computer Science University of Osnabrück, Germany*

1. Introduction: Purposes of Mathematical Modeling in the Study of Marine Ecosystems
2. Processes and Fluxes in Marine Ecosystems
 - 2.1. Spatially homogeneous and spatially structured models
3. Various Approaches to Marine Ecosystems Modeling
 - 3.1. Individual-based Models
 - 3.2. Population-level Models
 - 3.3. Thermodynamical Models
 - 3.4. Dimensional Analysis and Spectral Models
 - 3.5. Network Analysis
4. More about Population-level Models
 - 4.1. A Model of Red Tides
 - 4.2. A Model of Fish Species Replacement
 - 4.3. Models of Vertical Structure
 - 4.4. A Model of Horizontal Structure: “Biological Turbulence”
 - 4.5. Spatial Models of Plankton-Fish Interactions
5. Parameter Estimation and Verification of Models
6. Some Open Problems

Population Models**52**Michael B. Bonsall, *Department of Zoology, University of Oxford, Oxford, UK*

1. Introduction
2. Continuous-Time Population Models
 - 2.1. Pure Birth Processes
 - 2.2. Pure Death Processes
 - 2.3. Birth-death Processes
 - 2.4. Logistic Model
 - 2.5. Competitive Interactions
 - 2.5.1. Interspecific Competition
 - 2.5.2. Mechanistic Interspecific Competition
 - 2.6. Predator-Prey Interactions
 - 2.7. Age Structure
3. Discrete-Time Population Models
 - 3.1. Single Species Interactions
 - 3.2. Predator-Prey Interactions
 - 3.3. Age Structure
4. Stochastic Population Models
 - 4.1. Birth-death Process
 - 4.2. Predation and Competition
5. Future Developments

Models of Biodiversity

77

Jerome Chave, *Laboratoire Evolution et Diversité Biologique, CNRS/UPS, Toulouse, France*

Christophe Thebaud, *Laboratoire Evolution et Diversité Biologique, CNRS/UPS, Toulouse, France*

1. Introduction
2. Description of the biological diversity
 - 2.1. Access to diversity data
 - 2.2. Measures of biological richness
 - 2.2.1. Indices of diversity
 - 2.2.2. Species-area curves
 - 2.2.3. Abundance models
 - 2.3. Measures of ecosystem complexity
3. Dynamic models of diversity
 - 3.1. Generation of diversity
 - 3.1.1. Speciation
 - 3.1.2. Extinction
 - 3.2. Competitive exclusion and niches
 - 3.2.1. Lotka-Volterra model
 - 3.2.2. Niche models
 - 3.2.3. Tradeoff models
 - 3.2.4. Models of density dependence
 - 3.3. Stochasticity and dispersal
 - 3.3.1. Colonization-extinction dynamics
 - 3.3.2. Metacommunity dynamics and community drift
 - 3.3.3. Disturbances
 - 3.4. Relevance of space – Simulations
4. Synthesis and conclusion

Mathematical Models in Epidemiology

102

M. G. Roberts, *Institute of Information and Mathematical Sciences, Massey University, Auckland, New Zealand*

J. A. P. Heesterbeek, *Faculty of Veterinary Medicine, Utrecht University, the Netherlands*

1. Models for Infectious Diseases
 - 1.1. Historical Introduction
 - 1.2. The Concept of Mass Action

- 1.3. The Size of an Epidemic
- 1.4. Compartmental Models
- 1.5. The Basic Reproduction Ratio
- 1.6. Implications for Control
- 1.7. Conclusion
2. Models for Vector-Born Infections
 - 2.1. Historical Introduction
 - 2.2. Modelling Malaria
3. Models for Parasite Populations
 - 3.1. Historical Introduction
 - 3.2. The Population Dynamics of Macroparasites
 - 3.3. Conclusion
4. Models with Structure
 - 4.1. Historical Introduction
 - 4.2. Adding More Classes
 - 4.3. Continuous Age Structure
 - 4.4. A General Theory
 - 4.5. Conclusion

Mathematical Models of Public Health Policy

122

Tamara Awerbuch-Friedlander, *Harvard School of Public Health, Boston MA, USA*
 Richard Levins, *Harvard School of Public Health, Boston MA, USA*

1. Introduction
2. Posing the Question and Design of the Answer
3. Side Effects
4. Constraints of Actions
5. Alternative Actions
6. Policy Adoption and Implementation
7. Properties of Models
8. Simulations
9. Qualitative Models
10. Tailoring Models for Policy - the Intervener as Part of the System
 - 10.1. Models for Control of an Infectious Disease
 - 10.2. A Model of Regulation
 - 10.3. Public Use of a Service
11. Conclusion

Mathematical Modeling and the Human Genome

142

Hilary S. Booth, *Australian National University, Australia*

1. Introduction: The Human Genome
2. Modeling DNA
 - 2.1. Evolutionary Models
 - 2.2. Models of Nucleotide Substitution
3. Modeling Genes
 - 3.1. Finding Genes using Hidden Markov Models
 - 3.2. Modeling Gene Homology
 - 3.2.1. Substitution Matrices
 - 3.2.2. Measuring sequence similarity
 - 3.2.3. Searching databases for similar sequences
4. Conclusion

Mathematical Models of Society and Development: Dealing with the Complexity of Multiple-Scales and the Semiotic Process Associated with Development **154**

Mario Giampietro, *Istituto Nazionale di Ricerca per gli Alimenti e la Nutrizione, Italy*

Kozo Mayumi, *University of Tokushima, Japan*

David Pimentel, *Cornell University, USA*

1. Introduction and Overview of the Underlying Chapters
2. The Epistemological Predicament Associated with the Analysis of the Evolution of Systems Organized Across Multiple Scales
 - 2.1. Jevons' Paradox: For Adaptive Systems "Ceteris" are never "Paribus"
 - 2.1.1. An Example of Wrong Message: The Myth of Dematerialization of Developed Economies
 - 2.2. Hierarchy Theory: The Unavoidable Existence of Legitimate but Non-equivalent Perceptions/Representations of the Reality
 - 2.2.1. A Case Study: The Inconsistencies in Energy Analysis
 - 2.3. Holons have an Elusive Identity, their Process of Becoming cannot be Simulated by Formal System of Inferences
3. The Epistemological Roots of the Predicament faced when Modeling the Sustainability of Human Societies
 - 3.1. The Complexity of the Concept of Identity: Perceptions and Representation must be associated with a Semiotic Process
 - 3.2. Modeling Relation Theory as developed by R. Rosen
 - 3.3. A Modeling Relation can only be formalized within a Given Narrative
 - 3.4. A Modeling Relation Requires Strong Semiotic Identities: The Difference Between Models and Similes
4. Conclusion

Mathematical Models in Demography and Actuarial Mathematics

198

Robert Schoen, *Hoffman Professor of Family Sociology and Demography, Pennsylvania State University, USA*

1. Introduction
2. Life Table Models
 - 2.1. Life Table Structure and Functions
 - 2.2. Actuarial Science
 - 2.3. Analytical Representations of Mortality
3. Stable Populations
 - 3.1. The Stable Population Model in Continuous Form
 - 3.2. The Stable Population Model in Discrete Form
 - 3.3. Population Momentum
 - 3.4. Analytical Representations of Fertility and Net Maternity
4. Multistate Population Models
 - 4.1. The Structure of Multistate Models
 - 4.2. Analytical Representations of Migration and First Marriage
5. "Two-Sex" Population Models
 - 5.1. The "Two-Sex" Problem
 - 5.2. Analyzing the Marriage Squeeze
6. Dynamic Population Models
 - 6.1. Population Projections
 - 6.2. Modeling Populations with Changing Rates

Mathematical Models in Economics

222

Alfredo Medio, *University of Udine Via Treppo 18, 33100 UDINE, Italy*

1. Introduction
2. Mathematics, General Equilibrium and Dynamical System Theory
3. Equilibrium and Disequilibrium Dynamics

4. Implicit Dynamics, Learning, Evolution
5. Concluding Remarks

Ecological and Socio-ecological Economic Models **237**

Jacek B. Krawczyk, *School of Economics and Finance, Victoria University of Wellington, PO Box 600, Wellington, New Zealand*

Jacques Poot, *School of Economics and Finance, Victoria University of Wellington, PO Box 600, Wellington, New Zealand*

1. Introduction
2. Ecological-economic Interaction Models
3. Dynamic Macro and Micro Simulation Models
4. Optimization and Control in Simulation Models
 - 4.1. Multi-objective Optimization
 - 4.2. Optimal Control
5. Game-theoretic Models
 - 5.1. Environmental Games
 - 5.2. Equilibrium in Games
 - 5.3. Dynamic Game Models
6. Equilibrium and Optimality in Dynamic Games
 - 6.1. Information Structure
 - 6.2. Open Loop Solutions
 - 6.3. Feedback Solutions
 - 6.4. Implications for Environmental Management
7. Conclusions

Mathematical Modeling in Social and Behavioral Sciences **261**

Wei-Bin Zhang, *Ritsumeikan Asia Pacific University, Jumonjibaru, Beppu-Shi, Oita-ken, Japan*

1. Introduction
2. Optimization Theory - Job Amenity and Moonlighting
3. Operations Research - The Job Assignment Problem
4. Game Theory - Political Competition
5. Differential Equations - Economic Consequences of Altruism
6. Chaos Theory - Population Dynamics

Mathematical Models of Management of the Environment and its Natural Resources **286**

Carlos Romero, *Professor, Department of Forest Economics and Management, Technical University of Madrid, Spain*

1. Introduction
2. Positive and Negative Externalities
3. Socially Optimum Provision of Environmental Bads
4. Mechanisms to Achieve the Optimal Level of an Environmental Bad
5. Socially Optimum Provision of Environmental Public Goods
6. A Unified Framework for the Optimal Management of Natural Resources

Mathematical Models of Global Trends and Technological Change **303**

Natali Hritonenko, *Department of Mathematics, Prairie View A&M University, Texas USA*

Yuri Yatsenko, *College of Business and Economics, Houston Baptist University, USA*

1. Global Trends and Global Change
 - 1.1. Global Trends
 - 1.2. Climate Change Trends

- 1.2.1. Global Warming
- 1.2.2. Greenhouse Effect
- 1.2.3. Unexpected Trend: Stratospheric Ozone Hole over Antarctica
- 1.3. Demographic Trends
- 2. Modeling of Global Trends and Global Changes
 - 2.1. Simple Models
 - 2.2. Aggregate Indicators in Global Models
- 3. Models of World Dynamics
 - 3.1. Forrester Model
 - 3.1.1. Structure of the Forrester Model
 - 3.1.2. Simulation Results
 - 3.1.3. Limitations of the Forrester Model
 - 3.2. Meadows Model
 - 3.2.1. Structure of the Model
 - 3.2.2. "Limits to Growth"
 - 3.2.3. "Beyond the Limits"
 - 3.2.4. "Limits to Growth: The 30-year Update"
 - 3.3. Mesarovic-Pestel Model
- 4. Integrated Assessment Global Models
 - 4.1. Structure and Classification of Major IA Models
 - 4.1.1. Dynamic / Regional Integrated Model of Climate and the Economy (DICE and RICE)
 - 4.1.2. Global 2100 and Climate Emissions Trajectory Assessment (CETA)
 - 4.1.3. Model for Evaluating Regional and Global Effects (MERGE)
 - 4.1.4. Integrated Model to Assess the Greenhouse Effect (IMAGE)
 - 4.1.5. Tool to Assess Regional and Global Environmental and Health Targets for Sustainability (TARGETS)
 - 4.1.6. Integrated Climate Assessment Model (ICAM)
 - 4.1.7. Policy Analysis of the Greenhouse Effect (PAGE)
 - 4.2. Limitations of IA Models
 - 4.3. Perspectives of Global Modeling
- 5. Models of Technological Change
 - 5.1. Exogenous TC
 - 5.1.1. Autonomous TC
 - 5.1.2. Embodied TC
 - 5.1.3. Vintage Capital Models
 - 5.1.4. Creative Destruction and Economic Efficiency
 - 5.2. Endogenous TC
 - 5.2.1. Induced TC
 - 5.2.2. TC as Separate Economic Activity

Index **329**

About EOLSS **335**