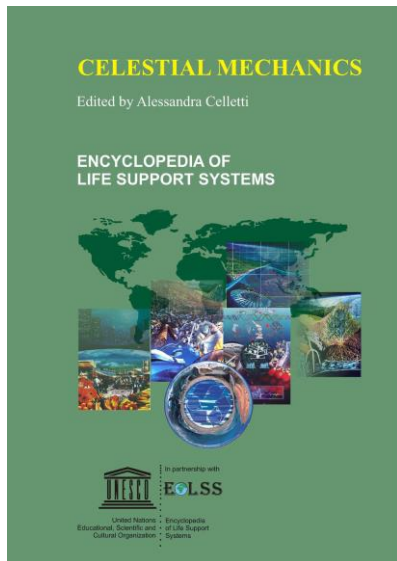


CELESTIAL MECHANICS



Celestial Mechanics

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Alessandra Celletti, *Dipartimento di Matematica, Università di Roma Tor Vergata, Italy*

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