ROTATIONAL DYNAMICS

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Contents

Summary

This chapter provides a short introduction to the main dynamical problems related to the rotational motion of celestial bodies. We start by considering various ways to characterize this motion and to derive the equations of motion. Although the main attention is given to the influence of the gravity torque on the rotational motion, the role of other torques is also briefly discussed. In an elementary way, we establish the key property of the non-resonant, slightly perturbed, rotational motion of a celestial body (under the action of gravity torque only) - the precession of the angular momentum vector around the normal to the orbital plane. The resonant spin-orbit coupling is considered as well.

1. Introduction. Main Assumptions

Since any real celestial body is not a material point, a complete theory of its motion should consider not only the orbital dynamics, but also the rotation of this body around its mass center O. The main properties of the rotational motion are discussed in the next sections. For further reading we can recommend the textbooks by Beletsky (2001), Murray and Dermott (1999) and the reviews from the volume "Dynamics of extended celestial bodies and rings" published in a series "Lecture notes in Physics" under the editorship of Souchay (2006).

The rotational motion of the celestial bodies is usually studied within a "restricted" model, which is based on the assumption that the rotation does not influence the orbital motion. If this model is accepted, the orbital motion (or, more exactly, the motion of the mass center) is supposed to be known – it can be modeled, for example, by considering the celestial body as a point mass.

The "restricted" model is accurate enough when the size of the body is much smaller than the distance to the center of the celestial body (a star or a planet) around which the orbital motion occurs. If the body is orbiting an object of substantially greater mass with more or less spherically symmetric internal structure, then a further simplification is possible: the gravity field of this object is approximated by the gravity field of the attracting center O_*. In this case the "restricted" model is equivalent to the assumption that the body's mass center O moves in a Keplerian orbit around O_*.

Sometimes the assumptions of the "restricted" problem are too restrictive. As an example we can mention the studies on the dynamics of binary asteroids where the analysis of the rotational motion beyond the scopes of the "restricted" problem is needed.

Another important assumption is that we will consider the celestial body as non-deformable (i.e., the distances between any two points of the body keep their values). Quite often the term "rigid body" is used to specify this approximation. Due to the necessity of explaining the tidal phenomena, the rotational dynamics of deformable bodies is actively investigated too. Despite the progress achieved, the theory of the rotation of deformable bodies remains complicated and will not be discussed here.

2. Kinematics of Rotational Motion

2.1. Reference Frames used in Studies of Rotational Motion

To characterize the rotational motion of a body we need two Cartesian reference frames with the origin at the mass center O. One reference frame is fixed in the body – we will denote it as $O\xi\eta\zeta$. The rotational motion leads to a change in the orientation of the fixed reference frame $O\xi\eta\zeta$ with respect to the second reference frame, the choice of which depends on the specific features of the problem under consideration. Quite often it is convenient to introduce the "inertial" reference frame $Oxyz$ with the axes preserving their orientation in the absolute space (the quotation is applied because the translational motion of the origin is not required to be uniform). Since we will usually suppose that the mass center O moves in a non-evolving Keplerian orbit, we can orient the axis Oz of the inertial reference frame along the normal to the orbital plane (in the direction of the angular momentum of the orbital motion with respect to the attracting center O_*) and the axis Ox along the direction to the pericenter from O_*; in that case the axis Oy is tangent to the orbit when the body moves through the pericenter. If the orbit is circular, the axis Ox can be directed along the line passing through the attracting center O_* and the arbitrary point of the orbit.

Sometimes the rotational motion of the body is considered with respect to the so-called orbital reference frame $Ox_{\mathrm{O}}y_{\mathrm{O}}z_{\mathrm{O}}$ defined in the following way: the axis Oz_{O} is oriented along the radius-vector $\mathbf{R}$ of the mass center O ($\mathbf{R}=\overrightarrow{O_*O}$); the axis Oy_{O} is perpendicular to the osculating plane of orbital motion and the axis Ox_{O} forms an acute angle with the direction of the body's motion along its orbit.

2.2. Euler Angles

In the XVIII century the famous mathematician Leonard Euler established that the rigid body with a fixed point can be moved from one position to any other by only one rotation. This statement provides the following opportunity to define the orientation of the body: we specify the rotation which allows us to achieve a current orientation of the fixed reference frame with respect to, for example, the inertial reference frame from a position where the orientations of these reference frames coincide.

The set of all rotations is a group (under the operation of composition) denoted as $SO(3)$. To parameterize this group three parameters are needed. One of the possible parameterizations is to represent an element of $SO(3)$ as a product of three elementary rotations about the axes with pre-defined orientation. In particular such parameterization can be performed by means of the so-called Euler's angles φ, ϑ, ψ (which are called the precession angle, the nutation angle and the proper rotation angle, respectively) corresponding to a sequence of rotations about the axes Oz, ON and $O\zeta$ (Figure 1).

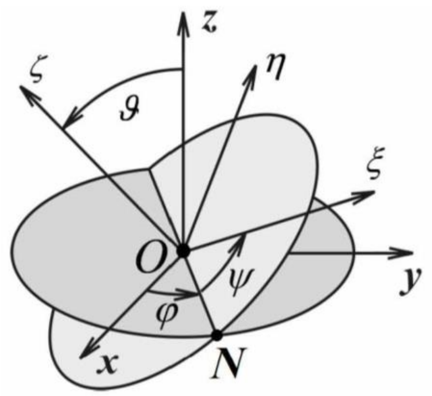

Figure1. Euler's angles used to define the orientation of the body-fixed reference frame with respect to the inertial reference frame.

In studies concerning the rotational dynamics it is frequently necessary to write down the components of a vector in the reference frame under consideration, once they are known in some other frame. To relate the components of the vector in the different reference frames, a transition matrix of the following form is used:

$$\begin{pmatrix} \mathrm{v}_x \\ \mathrm{v}_y \\ \mathrm{v}_z \end{pmatrix} = \begin{pmatrix} a_{x\xi} & a_{x\eta} & a_{x\zeta} \\ a_{y\xi} & a_{y\eta} & a_{y\zeta} \\ a_{z\xi} & a_{z\eta} & a_{z\zeta} \end{pmatrix} \begin{pmatrix} \mathrm{v}_\xi \\ \mathrm{v}_\eta \\ \mathrm{v}_\zeta \end{pmatrix}.$$

Here $\mathrm{v}_x, \mathrm{v}_y, \mathrm{v}_z$ and $\mathrm{v}_\xi, \mathrm{v}_\eta, \mathrm{v}_\zeta$ denote the components of the vector $\mathbf{v}$ in the reference frames $Oxyz$ and $O\xi\eta\zeta$, respectively. To obtain the inverse transformation the transposed matrix should be used.

The elements of the transition matrix are functions of the angles used to define the orientation of the body:

$$\begin{pmatrix} a_{x\xi} & a_{x\eta} & a_{x\zeta} \\ a_{y\xi} & a_{y\eta} & a_{y\zeta} \\ a_{z\xi} & a_{z\eta} & a_{z\zeta} \end{pmatrix} = R_3(\varphi)R_1(\vartheta)R_3(\psi),$$

where $R_1(\cdot)$ and $R_3(\cdot)$ are the matrices defining the elementary rotations around the axis of Cartesian reference frame:

$$R_1(\delta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & -\sin\delta \\ 0 & \sin\delta & \cos\delta \end{pmatrix}, \quad R_3(\delta) = \begin{pmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

By elementary calculations one obtains

$$\begin{aligned}
&a_{x\xi} = \cos\varphi\cos\psi - \sin\varphi\cos\vartheta\sin\psi,\ a_{x\eta} = -\cos\varphi\sin\psi - \sin\varphi\cos\vartheta\cos\psi, \\
&a_{x\zeta} = \sin\varphi\sin\vartheta, \\
&a_{y\xi} = \sin\varphi\cos\psi + \cos\varphi\cos\vartheta\sin\psi,\ a_{y\eta} = -\sin\varphi\sin\psi + \cos\varphi\cos\vartheta\cos\psi, \\
&a_{y\zeta} = -\cos\varphi\sin\vartheta, \\
&a_{z\xi} = \sin\psi\sin\vartheta, \qquad a_{z\eta} = \cos\psi\sin\vartheta, \qquad a_{z\zeta} = \cos\vartheta.
\end{aligned}$$

2.3. Euler's Kinematical Equations

To describe how the body changes its orientation, we introduce a vector quantity known as the "angular velocity". It is a pseudo-vector which specifies the angular speed of the body and the direction of the instantaneous axis of rotation in the motion around the mass center O. Denoting the angular velocity as $\boldsymbol{\omega}$, we write it down as the sum of three terms corresponding to the elementary rotations:

$$\boldsymbol{\omega} = \dot{\varphi}\mathbf{e}_z + \dot{\vartheta}\mathbf{e}_N + \dot{\psi}\mathbf{e}_\zeta \tag{2.1}$$

Here $\mathbf{e}_z$ and $\mathbf{e}_\zeta$ denote the unit vectors of the axis Oz and $O\zeta$ respectively, the unit vector $\mathbf{e}_N$ is directed along the line of nodes ON (Figure 1). In scalar form the relation (2.1) gives us

$$\omega_\xi = \dot{\vartheta}\cos\psi + \dot{\varphi}\sin\vartheta\sin\psi,$$
$$\omega_\eta = -\dot{\vartheta}\sin\psi + \dot{\varphi}\sin\vartheta\cos\psi, \qquad (2.2)$$
$$\omega_\zeta = \dot{\psi} + \dot{\varphi}\cos\vartheta.$$

Resolving (2.2) with respect to $\dot{\varphi}, \dot{\vartheta}, \dot{\psi}$, we obtain the classical Euler's kinematical equations:

$$\dot{\varphi} = \frac{1}{\sin\vartheta}\left(\omega_\xi \sin\psi + \omega_\eta \cos\psi\right),$$
$$\dot{\vartheta} = \omega_\xi \cos\psi - \omega_\eta \sin\psi, \qquad (2.3)$$
$$\dot{\psi} = \omega_\zeta - \operatorname{ctg}\vartheta\left(\omega_\xi \sin\psi + \omega_\eta \cos\psi\right).$$

2.4. Singularities Accompanying the Use of Euler Angles

As one can see, Euler's kinematical equations (2.3) become singular at $\sin\vartheta \approx 0$. This singularity (very unpleasant for numerical studies) is not connected with something special in rotational motion. It is an artifact of the rotation group $SO(3)$ parameterization by means of the Euler angles. To avoid this kind of singularity, the other parameterizations of $SO(3)$ can be applied (for example, by means of quaternions).

3. Rotational Dynamics: Euler's Formalism

3.1. The Relation between Angular Momentum and Angular Velocity

Euler's approach to the rotational dynamics of celestial bodies is based on the angular momentum equation

$$\frac{d\mathbf{G}}{dt} = \mathbf{M} \qquad (3.1)$$

written in the "inertial" reference frame $Oxyz$. In Eq. (3.1) $\mathbf{G}$ denotes the angular momentum of the body with respect to the mass center O, $\mathbf{M}$ is the total torque (with respect to O) of all forces applied to this body.

To compute $\mathbf{G}$ we should sum up the angular momenta of all elements of the body:

$$\mathbf{G} = \int_V \rho(\mathbf{r} \times \mathbf{v})dV, \qquad (3.2)$$

where $\mathbf{r}$ and $\mathbf{v}$ denote the radius vector and the velocity of the infinitesimal volume element dV with respect to the mass center, ρ characterizes the local density of the matter inside the body and the symbol "$\times$" is used to denote the vector product. Taking into account the relation

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

we rewrite (3.2) as follows:

$$\mathbf{G} = \int_V \rho(\mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r}))dV = \int_V \rho[r^2\boldsymbol{\omega} - (\mathbf{r}, \boldsymbol{\omega})\mathbf{r}]dV \,. \qquad (3.3)$$

Here and below the notation $(\cdot,\cdot)$ is applied for the scalar product in R^3.

As one can see from (3.3) $\mathbf{G}$ depends linearly on the angular velocity $\boldsymbol{\omega}$. To write down the relation between these quantities in a more concise way we introduce an operator $\mathbb{J} : R^3 \to R^3$, defined by the formula

$$\mathbb{J} = \int_V \rho[E_3 r^2 - \mathbf{r}\mathbf{r}^{\mathrm{T}}]dV \,. \qquad (3.4)$$

In formula (3.4) E_3 is the 3×3 identity matrix and the dyadic product of vectors is used:

$$\mathbf{a}\mathbf{b}^T = \begin{pmatrix} a_1b_1 & a_1b_2 & \cdots & a_1b_n \\ a_2b_1 & a_2b_2 & \cdots & a_2b_n \\ \cdots & \cdots & \cdots & \cdots \\ a_mb_1 & a_mb_2 & \cdots & a_mb_n \end{pmatrix}, \ \mathbf{a} \in R^m, \mathbf{b} \in R^n.$$

The relation between $\mathbf{G}$ and $\boldsymbol{\omega}$ takes now the remarkable form

$$\mathbf{G} = \mathbb{J}\boldsymbol{\omega}. \qquad (3.5)$$

3.2. Tensor of Inertia and Ellipsoid of Inertia

The formula (3.5) is valid regardless of the reference frame where the components of $\mathbf{G}$ and $\boldsymbol{\omega}$ are provided. For the matrix representation of the operator $\mathbb{J}$ (which depends on the choice of the reference frame) the term “tensor of inertia” is used. We can write down the tensor of inertia both in the reference frame $Oxyz$ (which preserves the orientation) and in the rotating body-fixed reference frame $O\xi\eta\zeta$, but only in the last case the coefficients of the matrix $\mathbb{J}$ do not vary with time.

The eigenvectors of $\mathbb{J}$ give us the directions of the so called principal central axes of inertia: if the body rotates around such an axis, then $\mathbf{G}$ is parallel to $\boldsymbol{\omega}$. In general there exist three mutually perpendicular principal axes of inertia (fixed in the body!). It allows us to introduce the body-fixed reference frame $O\xi\eta\zeta$ in a way which simplifies the structure of the equations of motion – we will suppose below that the axes $O\xi$, $O\eta$, $O\zeta$ are directed along the principal axes of inertia. In this reference frame the tensor of inertia is given by the diagonal matrix:

$$\mathbb{J} = \mathrm{diag}(A, B, C)$$

$$A = \int_V (\eta^2 + \zeta^2)\rho dV, \quad B = \int_V (\zeta^2 + \xi^2)\rho dV,$$

$$C = \int_V (\xi^2 + \eta^2)\rho dV \ .$$

The quantities A, B, C are called the principal central moments of inertia.

The relation $(\mathbf{r}, \mathbb{J}\mathbf{r}) = 1$ defines in R^3 the quadratic surface which is called the ellipsoid of inertia (or, more precisely, the ellipsoid of inertia corresponding to the mass center O). It is easy to prove that the ellipsoid of inertia is rigidly connected to the body: if the body orientation varies in the inertial space, then the orientation of the ellipsoid of inertia varies in the same way. Taking it into account, one can characterize the rotational motion of the body in terms of its inertia ellipsoid motion (See Section 6.2).

Often enough some kind of resemblance exists between the shapes of the body and of its inertia ellipsoid. For example let us consider a homogeneous body bounded by the tri-axial ellipsoid (i.e., by the ellipsoid with the different semi-principal axes). The directions of its longest, intermediate and shortest principal axes coincide with the directions of the corresponding inertia ellipsoid principal axes at the mass center O.

3.3. Euler's Dynamical Equations

In the rotating reference frame $O\xi\eta\zeta$ the angular momentum equation (3.1) takes the form

$$\frac{d'\mathbf{G}}{dt} + \boldsymbol{\omega} \times \mathbf{G} = \mathbf{M} \ . \qquad (3.6)$$

Here the prime indicates that the components of the differentiated vector should be expressed in the frame $O\xi\eta\zeta$:

$$\frac{d'\mathbf{G}}{dt} = \left(\frac{dG_\xi}{dt}, \frac{dG_\eta}{dt}, \frac{dG_\zeta}{dt} \right)^{\mathrm{T}} .$$

Substituting (3.5) into (3.6) and taking into account that in the reference frame $O\xi\eta\zeta$ the matrix $\mathbb{J}$ has constant coefficients, we obtain

$$\mathbb{J}\frac{d'\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times \mathbb{J}\boldsymbol{\omega} = \mathbf{M}$$

or (in scalar form)

$$A\frac{d\omega_\xi}{dt}+(C-B)\omega_\eta\omega_\zeta=M_\xi,$$
$$B\frac{d\omega_\eta}{dt}+(A-C)\omega_\zeta\omega_\xi=M_\eta,$$
$$C\frac{d\omega_\zeta}{dt}+(B-A)\omega_\xi\omega_\eta=M_\zeta. \quad (3.7)$$

Equations (3.7) are called "Euler's dynamical equations".

If the components of the torque **M** are the known functions of the variables $\omega_\xi,\omega_\eta,\omega_\eta,\varphi,\vartheta,\psi$ (and, maybe, of the time t) then Euler's dynamical equations (3.7) and Euler's kinematical equations (2.3) form a closed system of differential equations describing the rotational motion of the celestial body.

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