MAGNETOHYDRODYNAMICS OF THE EARTH'S CORE

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Summary

The main magnetic field of the Earth is created by electric currents flowing in the core, which is iron-rich and electrically conducting. The currents are induced by fluid motions, an energy source which must exist to offset the dissipative processes that act to halt the flow of both fluid and electric currents. Ohmic heating is the main dissipative

loss. Only two sources of energy are potentially strong enough: the luni-solar precession and buoyancy. The main focus is on the latter. The inner core boundary is a freezing front and, as the Earth cools, new material freezes onto the inner core, releasing latent heat and light elements as it does so. The associated thermal and compositional buoyancy drives core convection.

Convective motions mix entropy and composition thoroughly in the fluid core. The resulting adiabatic state is in approximate hydrostatic balance and provides a base from which the gross thermodynamics of the fluid core can be analyzed. This indicates that, in the absence of radioactive sources, the inner core is only 1–2 billion years old. Before that time, convection was thermal, driven by bulk cooling of the fluid core, but the heat flux from core to mantle was then significantly greater than at present.

A simplified (Boussinesq) theory of core convection is used to elucidate the character of core convection. This is dominated by rotational and magnetic forces which are comparable in magnitude. The stability of this state is discussed.

1. Introduction

1.1 General Remarks

"The Earth speaks of its internal movements through the silent voice of the magnetic needle", wrote Christopher Hansteen in 1819. The idea that the Earth is not entirely solid is even older than that; in the late 17th Century, Halley (of comet fame) proposed a differential motion within the Earth to explain the slow, and partially westward, movement of the geomagnetic field relative to the Earth's surface, a phenomenon now called "The westward drift". (See *Magnetic Field of the Earth*.) It was only after the discovery by Oldham in the early 20th Century of the fluid core of the Earth, and the discovery of the inner core by Lehmann nearly three decades later, that Halley's concept was upheld and the extent of the fluidity of the deep Earth became known. In 1919, Joseph Larmor suggested that the magnetic fields of the Sun and Earth are created by self-excited dynamo action, his idea being that the core fluid is electrically conducting and is moving. When a conductor moves in the presence of a magnetic field, it induces differences in electric potential that can drive electric currents. In a self-excited dynamo the currents are precisely those needed to recreate the magnetic field itself. The mechanism therefore operates in much the same way as a generator of electric current in a power station. This origin for the geomagnetic field is by now generally accepted, partly because it provides the only plausible explanation for one very significant fact: it is known from paleomagnetism (the study of magnetic fields trapped in rocks at the time of their formation — see *Paleomagnetism and Rock Magnetism*) that the polarity of the geomagnetic field has reversed many times in the past, and that the field has apparently no preference for one polarity over the other (Section5.3).

The "dynamo hypothesis", as it is called, recognizes that motion can maintain magnetic field but it does not explain why there should be motion at all. The flow of the electric current in the presence of the magnetic field creates a (Lorentz) force that affects, and generally opposes, the fluid motion. Flow will cease unless some agency maintains it. A complete explanation for the main geomagnetic field requires that some "driving mechanism" for core motion be identified. Intuitively, the stronger this mechanism, the more rapid the flow, and the greater the magnetic field it creates. Some kind of balance must be struck that maintains the magnetic field at roughly its present strength. What is the mechanism? How is the balance maintained? To answer these difficult questions, it is clearly necessary to understand better the mutual interaction between magnetic field and fluid flow. This defines the subject of magnetohydrodynamics, or 'MHD' for short. In fact, because the Earth is highly rotating, in a sense that will be described later, only a sub-branch of MHD is relevant to the theory of the geomagnetic field, namely RMHD which stands for 'Rotating Magnetohydrodynamics'. Other acronyms are defined in the Glossary. In particular, CMB, FOC, ICB and SIC are frequently used below. Suffices $_1$ and $_2$ will refer to values on the CMB and ICB, respectively.

1.2 Energy Loss in the Core

The question raised in Section 1.1 above, about the forces needed to keep the core in motion, can be addressed in a different way, by asking how much energy the motions demand to maintain themselves and their associated magnetic field.

The main source of energy dissipation in the core is ohmic heating, which is $q^J \equiv J^2/\sigma$ per unit volume, where $\sigma(\approx 4\times10^5 \text{ S m}^{-1})$ is the electrical conductivity of the core. If $J \approx 0.04 \text{ A m}^{-2}$ is a typical magnitude of the current density $\mathbf{J}$, the net ohmic dissipation,

$$Q^J = \int q^J d^3x, \tag{1.1}$$

is of order 0.7 TW. This estimate, depending as it does on the square of the poorly known J, is rather uncertain, and in what follows we prefer to place bounds on Q^J:

$$0.5\text{TW} < Q^J < 1.5\text{TW}. \tag{1.2}$$

This range may also be typical of the past geomagnetic field; paleomagnetic data indicates that the intensity of the geomagnetic field has not varied by a factor of more than about 3 over the past 3 Gyr. An energy source is needed that continuously makes good not only the ohmic losses that are in the range (1.2) throughout geological time, but also other, though smaller, losses arising from the viscosity of the core fluid. Several possibilities were under discussion until, just over a half-century ago, Bullard convincingly demonstrated that all but two sources are implausible: precession and buoyancy. Which of these is the more important has still not been conclusively decided. We shall focus on the precessional mechanism in Section 2 but subsequently concentrate only on the buoyancy mechanism, which is more widely supported.

2. Precessional Forcing

2.1. Some Definitions

Although the Earth is nearly spherical, centrifugal "forces" associated with the Earth's rotation cause it to be slightly flattened at its poles and to bulge slightly at its equator.

The Sun and Moon exert a couple on this bulge, causing the angular velocity vector, $\mathbf{\Omega}$, of the mantle to rotate slowly about an axis that is almost perpendicular to the ecliptic. This is called the luni-solar *precession*. The cone on which $\mathbf{\Omega}$ lies has a semi-angle of about $23\frac{1}{2}^{\circ}$ and is described in $\tau_{\mathrm{P}} \approx 25,722$ years, i.e., the precessional frequency $\omega_{\mathrm{P}} \equiv 2\pi/\tau_{\mathrm{P}}$ is approximately $7.7\times10^{-12}\ \mathrm{s}^{-1}$.

The precession of the mantle creates core flow in two ways. First, the fluid core is coupled viscously to the mantle and, even if the CMB were completely spherical (which it is not), the precession of the mantle would generate motions in the core. Such motions are called "spherical precessional flow". Second, the CMB is, like the Earth's surface, slightly flattened by the Earth's rotation. Even if core fluid were completely inviscid (which it is not), the pressure differences on the CMB created by the precessional motion of the mantle would generate "non-spherical precessional flow". Both these mechanisms operate in the Earth. The importance of the non-spherical mechanism depends on the degree of flattening of the CMB, which is measured by the dimensionless parameter

$$\varepsilon_{\Omega} \equiv \Omega^2 R_1/g_1 \approx 2\times10^{-3}, \tag{2.1}$$

where g is the acceleration due to gravity and $\mathbf{\Omega}$ is the angular velocity of the Earth.

The form and significance of the flow in the core driven by precession are still incompletely understood. To summarize the state of the subject as it stands today, we initially ignore magnetic effects and the presence of the inner core.

2.2. The Case of no Inner Core and no Magnetic Field

The history of the long search for a precessionally-driven solution started with Poincaré, who published in 1910 an exact nonlinear solution of the equations of motion for non-spherical precession. This is a flow of uniform vorticity about an axis rotated with respect to the symmetry axis of the CMB. It therefore does not obey the "no-slip conditions" on the CMB, i.e., the demand that fluid and solid move together where they are in contact. The precessional Ekman number,

$$E_{\mathrm{P}} = \nu_M/\omega_{\mathrm{P}} R_1^2, \tag{2.2}$$

is however very small, approximately 10^{-8}. Here $\nu_M\,(\approx 10^{-6}\ \mathrm{m}^2\,\mathrm{s}^{-1})$ is the kinematic viscosity of core fluid. The smallness of E_{P} suggests that Poincaré's solution may be valid except in a thin viscous boundary layer abutting the CMB in which $\mathbf{V}$ adjusts to the no-slip condition. This was confirmed by Stewartson and Roberts in the mid 60s; they also presented the first valid solution for spherical precession. It was shown in 1975, however, that such solutions cannot supply more than 10^8 W of power to sustain the core motions and the magnetic fields. They cannot therefore meet the requirement (1.2).

Stewartson and Roberts noted the existence of boundary layer singularities at certain critical latitudes on the CMB, and Busse showed that, added to Poincaré's solution in the FOC, there is a mean flow that has a large shear on the cylinder joining the critical latitudes. Subsequent theoretical work established that these singularities create cylindrical shear layers throughout the FOC, so confirming what had already been seen in experiments on precessionally-driven flow. It was also suggested that the turbulent states of motion seen in some experiments were created by the instability of these shear layers. A rival proposal was that the turbulence reveals a mode of instability in Poincaré’s original solution. Which explanation is correct is currently unknown, but it is possible that, as Malkus originally proposed on the basis of his experiments, the turbulence provides a channel that feeds energy into core motions and maintains the geomagnetic field.

Experimental work suggests that, in the core, the turbulent dissipation by the instabilities could be as much as 10^9 TW; theoretical work has set an upper bound of 10^{12} TW. Such enormous dissipations are ruled out by theories of mantle convection, and would also imply that, in a time of order 10^4 yr, the semi-angle of the precessional cone would diminish significantly, and there is no evidence for this. The large dissipations do, however, indicate the enormous energy reservoir on which the precessional mechanism can draw. Only a tiny part of the available energy would suffice to maintain the geomagnetic field, according to (1.2).

2.3 Generalizations: Future Directions

The investigations just described have ignored the existence of the SIC and have excluded magnetic effects. Some preliminary work has been done to rectify these geophysical defects (see the bibliography). It is clear that much more detailed theoretical work must be done before the importance of the luni-solar precession can be properly assessed, to decide whether it is a viable alternative to the other main possibility, that core motions are buoyantly-driven. From now onwards we consider this possibility exclusively and, since the slight oblateness of the CMB has only a tiny dynamical effect on the convection mechanism, we shall set $\varepsilon_\Omega = 0$; our basic model of the Earth will from now on be spherically symmetric, i.e., its will depend only on $r = |\mathbf{r}|$, where $\mathbf{r}$ is the radius vector from the geocenter. We shall also ignore the dynamical coupling of the mantle to the fluid core, which is weak once precessional effects are ignored and which creates only small (and, for our purposes, negligible) variations in the angular velocity of the mantle. We therefore now assume that $\mathbf{\Omega}$ is constant.

3. Basic State of the Core

3.1. The Cooling Earth; the Adiabatic State

At one time, radioactive heating was thought to be the primary cause of core convection, but most geochemists now agree that the core contains virtually no radioactivity. They believe that convection is a direct result of the slow cooling of the Earth over geological time and that the SIC is the result of the gradual freezing of the FOC. In addition to the buoyancy created by the release of internal energy in the bulk of the core as it cools, the release of the latent heat of crystallization at the ICB as the SIC grows promotes convection.

Heat is not the only factor on which buoyancy depends. It is known from seismology and mineral physics that the density ρ of the core is significantly less than it would be if it were made of pure iron. More likely, it is a combination of many chemical elements, some heavier (such as nickel) but most others lighter (such as sulfur, silicon and oxygen). There is no information about what the predominating alloying elements are. We shall adopt a simple two-component model that contains the essential ingredients of the dynamics with a minimum of complication. We shall call the alloy FeX, where X stands for the 'unknown' light component, the weight-percentage of which will be denoted by ξ.

It appears that core motions are so vigorous that ξ and the specific entropy S are well mixed, i.e., except in thin boundary layers,

$$\xi = \xi_a = \text{constant}, \qquad S = S_a = \text{constant}, \qquad \text{in the FOC} \qquad (3.1,3.2)$$

is an excellent approximation; here the suffix $_a$ stands for 'adiabatic'.

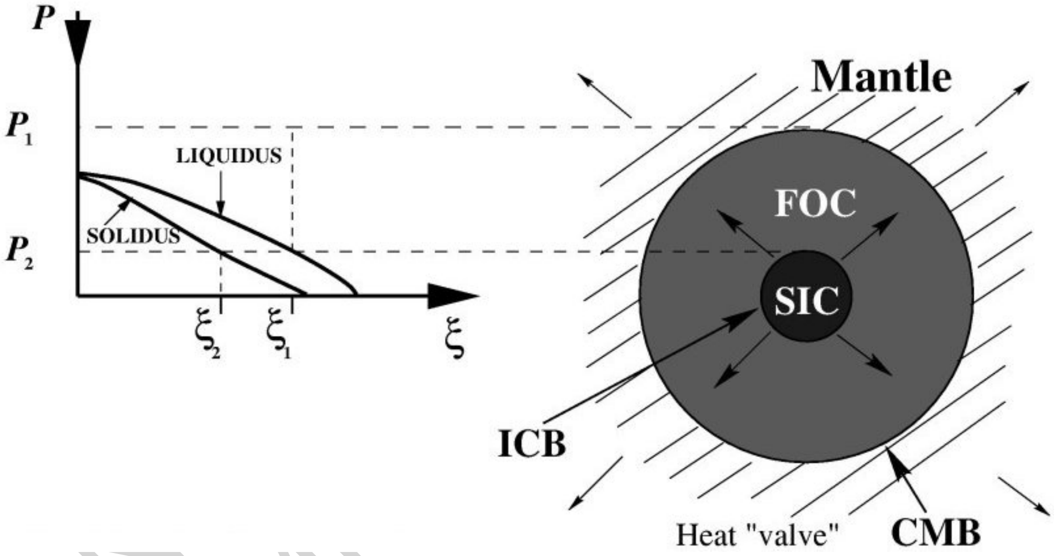

Figure 1. Left: Phase diagram for the fluid core. The liquidus and solidus are shown at constant entropy as a function of composition (ξ) and pressure (P). Right: Structure of the core, showing the solid inner core (SIC) and the fluid outer core (FOC) together with its boundaries, the inner core boundary (ICB) and core-mantle boundary (CMB). Heat flow is indicated by the outwardly-directed arrows and is controlled by mantle convection, as indicated by the term "heat valve"; see Section 3.3. The two diagrams are related as indicated by the horizontal dashed lines.

Figure 1 shows a hypothetical, but not unreasonable, phase diagram for FeX. This is in constant$-S$ projection; see (Section 3.2). The fluid core is represented by points on the line $\xi = \xi_1$ (shown dashed) above the curve labeled 'liquidus'. As P increases with depth in the core, this line meets the liquidus at a point representing phase equilibrium. The horizontal dashed line through this point meets the solidus at $\xi = \xi_2$, which defines the composition of the solid that plates onto the inner core. Since $\Delta\xi \equiv \xi_1 - \xi_2$ is positive this material is richer in iron and therefore denser. This accounts for one part, $\Delta^{\xi}\rho$, of the discontinuity, $\Delta\rho \approx 550 \pm 50\ \text{kg m}^{-3}$, in ρ_a at the ICB revealed by analysis of seismic data. The other part, $\Delta^{s}\rho$, is the result of the contraction experienced by the core fluid as it freezes. The release of lighter material at the ICB provides the FOC with a second (compositional) source of buoyancy which supplements the thermal buoyancy in driving core motion. Its strength is clearly sensitive to $\Delta^{\xi}\rho$, being nonexistent for $\Delta^{\xi}\rho$ = 0 and maximal for $\Delta^{\xi}\rho = \Delta\rho$, i.e., $\Delta^{s}\rho$ = 0. In making estimates below, we shall assume that $\Delta^{s}\rho = 310\ \text{kg m}^{-3}$. We shall also take ξ_a = 0.095 and $\Delta\xi$ = 0.02.

Because the FOC is predominantly a liquid metal, its magnetic diffusivity η_M $(=1/\mu_0\sigma$ $\approx 2\ \text{m}^2\,\text{s}^{-1}$, where μ_0 is the magnetic permeability) greatly exceeds both its kinematic viscosity, ν_M, and its thermal diffusivity, $\kappa_M\,(\approx 5\times10^{-6}\ \text{m}^2\,\text{s}^{-1})$, which in turn greatly exceed the diffusivity, $D_M\,(\approx 10^{-9}\ \text{m}^2\,\text{s}^{-1})$, of one component of the alloy relative to the other. Here the suffix $_M$ indicates that these diffusivities are molecular; we shall briefly meet turbulent diffusivities (to which the suffix $_T$ is assigned) in Section 5.

A simple argument suggests that compositional buoyancy alone may be unable to provide the power required by (1.2). The heavy constituent preferentially plates onto the ICB, so that its motion relative to the light constituent is downwards. Thus, as the Earth cools, it becomes increasingly centrally condensed. The concomitant release of gravitational energy provides the source of compositional buoyancy. Suppose that, at some time in the past, the entire core was fluid and uniform in density. Compare its gravitational energy then (E_0^G) with the gravitational energy now (E^G), assuming that the outer and inner cores have uniform densities $\overline{\rho}_a \approx 1.1\times10^4\ \text{kg m}^{-3}$ and $\overline{\rho}_a + \Delta\rho$. The difference $\Delta E_G = E_{G0} - E_G$ is approximately $8\pi G\,\overline{\rho}_a \Delta\rho R_1^3(R_1^2 - R_2^2)/15 \approx 1.2\times10^{24}$ J. When spread out evenly over the 3 Gyr during which the geomagnetic field is known to have existed at approximately its present strength, this gives only about 0.1 TW of gravitational power. Thermal sources of buoyancy seem to be indispensable.

3.2 Models of the Core

Several models of the internal structure of the Earth exist, based on seismic data, on (3.1) and (3.2), and on an assumed hydrostatic balance. The best known of these is PREM; see *Mantle and Core of the Earth*. We shall use a more recent model called 'ak135' which, under certain reasonable assumptions, gives $T_a(r)$ throughout the core and in particular

$$T_1 = 3960\text{K}, \qquad T_2 = 5100\text{K}, \qquad \overline{T} = 4488\text{K}, \tag{3.3}$$

where $\overline{T}$ is the mass-weighted average of T_a.

The adiabatic gradient, $-dT_a/dr$, increases from 0 at the geocenter to about 0.3 K km^{-1}

at the ICB and to about 0.9 K km $^{-1}$ at the CMB. Associated with this is an outward heat flux, $-K_T dT_a/dr$ where K_T is the thermal conductivity. Integration of this flux over the CMB and ICB corresponds to of the 'adiabatic heat flow' heat across these surfaces of about $H_1 = 5.9$ TW and $H_2 = 0.3$ TW. (See *Terrestrial Heat Flow*.) To maintain the adiabatic gradient in the face of the natural tendency of the system to try to become isothermal, convection has to supply entropy to the core at a rate of

$$\Sigma = \int_{\mathrm{FOC}} K_M (\nabla T_a / T_a)^2 \, dV \approx 170 \mathrm{MW/K}. \qquad (3.4)$$

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Biographical Sketch

Professor Paul H. Roberts, obtained his PhD in mathematics in 1954 and his ScD in 1967 from Cambridge University, England. From 1954 to 1955 he was a Research Associate at Yerkes Observatory at the University of Chicago. From 1955 to 1956 he did his military service as a Scientific Officer at AWRE Aldermaston. From 1956 to 1959 he was an ICI Fellow in Physics at the University of Durham and was a Lecturer in Physics there from 1959 until 1961. From 1961 to 1963 he was an Associate Professor of Astronomy at the University of Chicago. From 1963 to 1985 he was a Professor of Applied Mathematics at the University of Newcastle upon Tyne, England. Since 1986 he has been a Professor of Mathematics and of Geophysical Sciences at the University of California, Los Angeles. He was elected a Fellow of the Royal Society of London in 1979 and a Fellow of the American Academy of Arts and Science in 2001. He is a Fellow of the American Geophysical Union and was their Fleming Medalist in 1999. He is sole author of one book, An Introduction to Magnetohydrodynamics, and has co-edited several others. He is an author or co-author of over 260 other publications.