MATHEMATICAL MODELS OF CIRCULATIONS IN OCEANS AND SEAS

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Contents

Summary

This chapter gives an overview of state-of-the-art methods in computational oceanography, presenting the pertinent theoretical, conceptual and numerical aspects. First, the main areas of application for numerical ocean modeling are highlighted, introducing the reader to oceanic phenomena and processes that are relevant for global and regional ocean climate. After a concise presentation of the system of equations that serves as the basis for geophysical fluid dynamics, and a number of widely used approximations, the various philosophies and approaches currently used in ocean modeling are explained. The main part is devoted to the numerical aspects, like coordinates, grids, resolution, algorithms and parameterizations as well as solution procedures. The utilization of observational data as initial, boundary and validation data is presented. In addition, methods on how to derive diagnostic inverse estimates of ocean circulation and climate from observations are discussed. In the two concluding sections, the need for rigorous model evaluation is stressed and some comments on the future of numerical ocean modeling are offered.

1. Introduction

Over the past decades the need for and interest in understanding the ocean circulation and its variability has greatly increased; as a consequence, large efforts have been directed towards describing, monitoring, forecasting oceanic dynamics.

The ocean is subject to external forces that result from air–sea interaction at the interface between these two media (the *contact forces*: fluxes of heat, fresh water, and momentum), the direct solar radiation (penetrating a few tens of meters into the ocean), tidal forces (as a *body force*, acting on each water parcel), as well as geothermal heating (and the corresponding salt input) from below and from lateral boundaries in form of river runoff and melting and freezing of ice.

As a result, a large variety of processes exists, on scales ranging from global to molecular. Many of the corresponding phenomena are further modified by the complex geometry of the ocean basins. The spectrum is continuous in both time and space, and processes on all scales influence each other. Characteristic length and time scales are depicted in Figure 1.

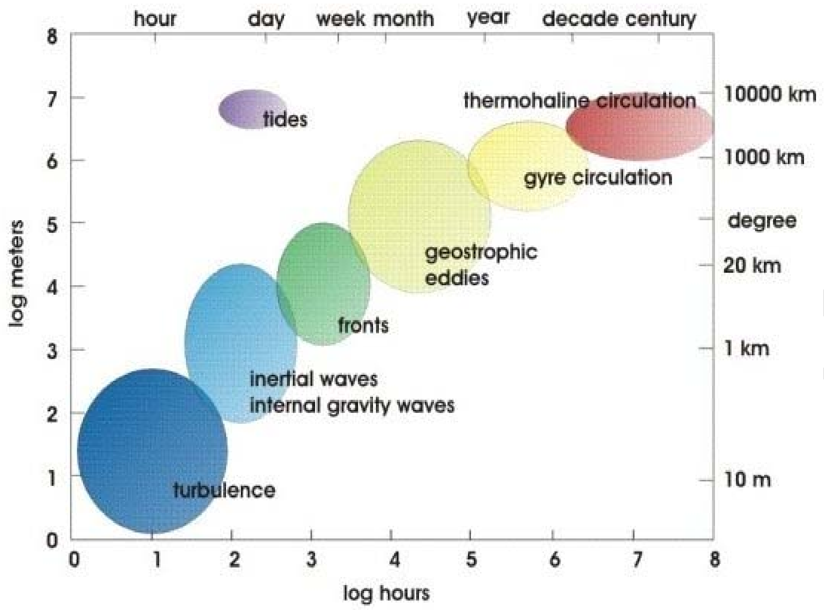

Figure 1: Spatial and temporal scales of oceanic dynamics

Mathematical models of the circulation and water mass distribution in oceans and seas have first been developed in the late 1960s. They are based on the numerical implementation of partial differential equations and offer the possibility to study nonlinear ocean dynamics and thermodynamics in complex geometries and with (temporally and spatially) variable forcing.

The numerical methods are similar to those used in atmospheric models. Still, the ocean requires special measures, as some of the important dynamical aspects are quite different from the atmosphere. In comparison with the atmosphere we note : (a) the existence of *lateral boundaries* on the fluid which introduces unique characteristics; (b) relatively small horizontal scales of highly energetic motions; (c) relatively long time scales associated with important branches of the global circulation; (d) forcing at the surface rather than at the bottom; and (e) a much sparser observational data base.

Today, a number of Ocean General Circulation Models (OGCMs) exist and are available for research within the scientific community. They are used for both forecast/hindcast of the state of (parts of) the ocean in support of atmospheric prediction or the operation of ships (operational tidal prediction models also fall into this category), as well as the investigation of physical parameter sensitivities, the relative importance of atmospheric forcing as well as individual mechanisms of ocean dynamics.

Increasingly, ocean models are used as components for *climate research*, where the interannual and interdecadal variability of the ocean, both natural and anthropogenic, is investigated. Questions like abrupt climate change, past climates (including sea level changes) are addressed, as well as predictions ("scenarios") for the next century. Another focus is the use of ocean models in coupled studies of biogeochemical aspects. Prominent examples are the global carbon cycle (including the biological carbon pump), ecosystem studies (relevant for fisheries), sedimentation and sediment transport as well as studies of contaminant pathways. Many characteristics of the variability, steady states, equilibria, and limits of marine ecosystems can only be understood within an interdisciplinary framework that builds on a mathematical model of ocean physics, coupled to a biological model. Thus, ocean modeling has developed into a field with importance far beyond physical oceanography. Mathematical modeling forms the basis for targeted observations, and at the same time allows decision-makers to develop strategies to restrict anthropogenic impact or react to perturbations to these systems, especially in the coastal oceans.

This chapter presents the theoretical, conceptual and numerical aspects of mathematical ocean modeling, as well as *quantitative model validation and quality of model results*. First, we give a brief overview of the current and future areas of research, followed by a presentation of systems of equation, concepts, numerical and technical aspects.

2. Areas of Model Application

Today's high priority research areas and topics must be seen as a net of multiply-connected subjects. In its center, we find the overarching field of *climate research*, specifically the ocean's role in climate, including aspects of biogeochemistry and marine ecosystems. Modeling studies are carried out on both global and regional scales. Examples are given in the following chapters.

2.1. Elements of the Large-Scale Circulation

2.1.1. Thermohaline Circulation

The most prominent global oceanic phenomenon is the thermohaline circulation, a system of connected basin–scale circulations that fill all major parts of the world's ocean, with associated time scales of centuries to millennia. This circulation is often described in a simplified way as the *oceanic conveyor belt* (see Figure 2). It is characterized by relatively small regions of downward motion in high latitudes (mainly in the Greenland, Labrador and Weddell Seas) and gradual upwelling throughout the ocean but concentrated near continental margins and other regions with large variations in bottom topography. This quasi-continuous circulation is constrained by large scale bottom topography, which divides the global ocean into individual basins. Often only small passages allow for an exchange between these basins – which highlight the critical importance of small scale processes for even the large scale circulation.

This circulation mode features a meridional overturning component that transports heat poleward. It is therefore one of the fundamental components of our global climate system.

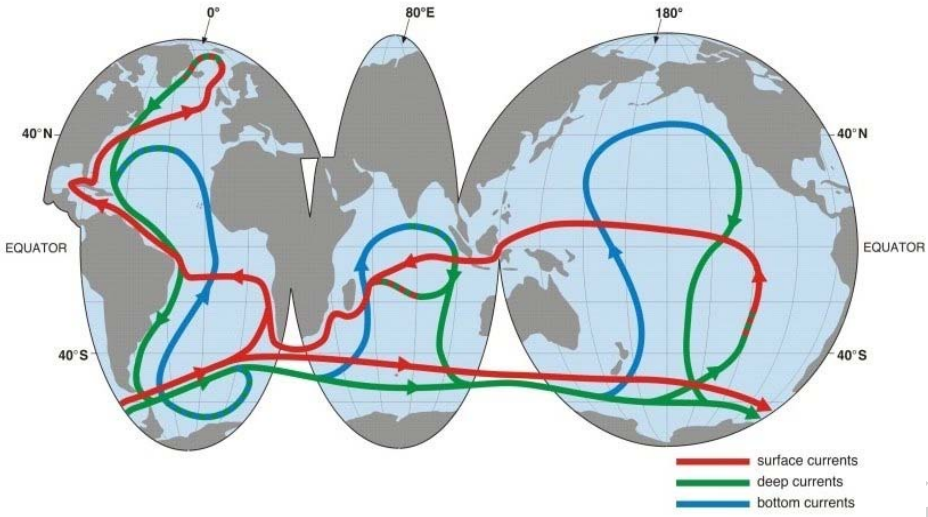

Figure 2: A three-layer, interbasin-scale thermohaline conveyor belt. (After [1])

2.1.2. Wind-driven Circulation

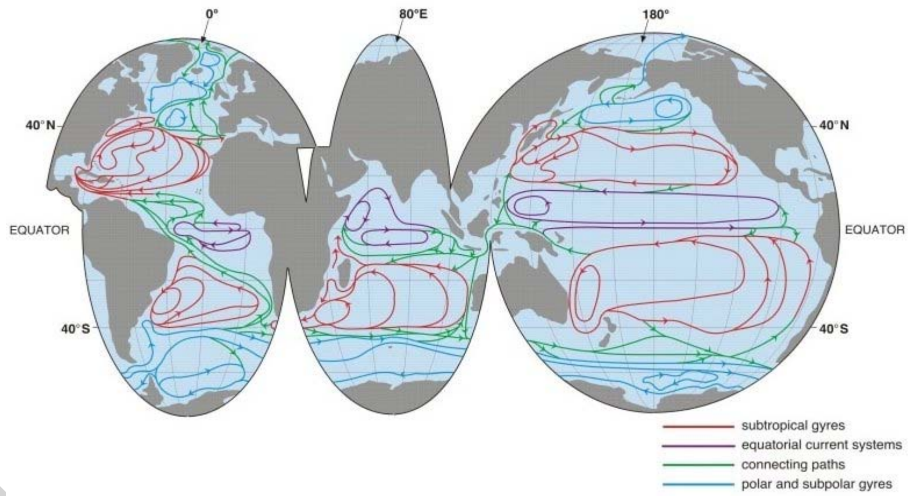

Figure 3: Schematic representation of mean circulation features in the global ocean

The wind–driven part of the ocean circulation is mainly horizontal and extends down to the main thermocline (i.e., the upper 1000 m in mid-latitudes). It reflects the large-scale patterns of the atmospheric wind field (Figure 3). Due to the influence of the Earth's rotation, these wind-driven gyres have intensified western boundary currents. Meridional gyre boundaries are the polar and subpolar fronts which are associated with strong temperature changes, as well as the equatorial current system. The eastern basins are characterized by relatively weak and variable currents, but also by prominent upwelling regions and eastern boundary currents. The upwelling regions are important areas of biological production and intensified exchange between ocean and atmosphere.

2.1.3. Global Fresh-Water Cycle and Sea Ice

Combined with the meridional heat transport by ocean currents, fresh water is redistributed. In high latitudes, part of the transport is achieved by sea ice. The understanding of its growth, movement and decay is important not only scientifically but also logistically, because it affects human activities in these areas (e.g., ship traffic, offshore technology).

Consequently, models of the ocean circulation in high latitudes have to be supplemented by an interactive sea ice component to study the consequences of ice–ocean interaction on regional to global scales. The sea ice model predicts the temporal evolution of the ice cover, thus providing the boundary conditions for the ocean circulation model. The most important interactions between sea ice and ocean occur due to,

- the growth of (low salinity) sea ice through freezing of sea water; the correspondingbrine release leads to an increase in density and may induce oceanic convection;
- the drift of ice in response to wind and ocean currents, the deformation (ridging) due to shear and convergence; the generation of areas of open water (leads, polynyas) with locally increased oceanic heat flux;
- the melting of ice and the corresponding stabilization of the surface stratification; and
- isolation of the ocean from the atmosphere in ice covered areas,

all of which need to be adequately represented (either resolved or parameterized) in coupled sea ice-ocean models.

2.1.4. Patterns and Modes of Climate Fluctuations

The variability and predictability of large–scale patterns is a key question of climate research. One of the widely-known phenomena is *El Niño*, a process of closely linked atmosphere–ocean dynamics in the tropical Pacific. Other examples are the North Atlantic Oscillation (NAO), or the Antarctic Circumpolar Wave (ACW). Oscillations and wavelike phenomena are a fundamental class of variability. The latter is an example of the class of wavelike perturbations; planetary Rossby waves (traveling westward), Kelvin waves and topographic waves are the main mechanisms for transmitting signals across large distances.

2.2. Important Small-Scale Processes

A number of small-scale ocean processes are important for understanding and predicting climate variability and change: Most of them fall into the two categories of *oceanic boundary layers* (including the coastal ocean) and *turbulent mixing* (including vertical convection).

2.2.1. Boundary Layers

While the interior of the ocean can be largely described as geostrophic and hydrostatic, the dynamical balances are different at the boundaries (Figure 4). The *surface mixed layer* (SML) represents the part of the ocean that is directly in contact with the atmosphere. It is characterized by enhanced levels of turbulence, and an almost complete vertical homogeneity. In low-to-mid-latitudes it isolates the deep ocean from diurnal and seasonal variations.

In high latitudes it transmits the atmospheric variability into the abyss. Air-sea coupling often requires a separate model for the surface mixed layer; for high latitudes, the inclusion of sea ice is necessary. The dynamics of the SML is also an important part of pelagic ecosystem studies.

A similar boundary layer exists at the bottom of the sea, where friction removes the kinetic energy supplied by the wind and thermohaline forcing. This *bottom (or benthic) boundary layer* (BBL) connects the shallow seas with the abyssal ocean basins. Detailed understanding and adequate representation of BBL processes in numerical ocean circulation models is necessary for,

- regional coastal studies (BBL dynamics determine how the coastal ocean redistributes physically, biologically, and chemically important tracers and material);
- global thermohaline circulation and climate variability studies (cross-slope transport and entrainment in the BBL control the rate and properties of dense water formation, the ventilation of the abyss, as well as the dynamics of overflow and passage throughflow between ocean basins; and
- sediment transport and paleoceanographic studies (near-bottom flows interact with the uppermost layers of the sediment).

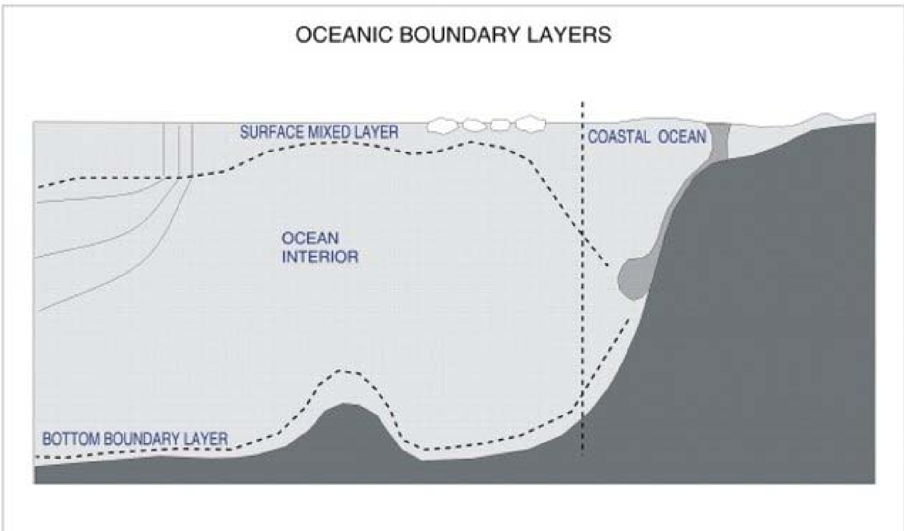

Figure 4: Schematic cartoon of oceanic boundary layers. The thickness of the surface mixed layer varies spatially and temporally in response to surface wind and thermohaline forcing. The bottom boundary layer is thicker in areas of rough and sloping topography due to enhanced turbulence. Both vertical boundary layers converge in the coastal ocean, which can be viewed as the lateral boundary of the deep ocean. The thin dashed lines represent isopycnals.

Important aspects are the role of complex small-scale seafloor topography on the dynamics of down-slope flows (plumes) and along-slope flows. Time-mean flows due to topographic stress (see section 5.5.3.) are generated under a wide variety of circumstances (e.g., tides, eddy motion).

From the ocean's perspective, the *coastal seas* represent the lateral boundaries. Although these areas occupy only about 10% of the global ocean area, they are by far the most important areas for human activities such as fish harvesting, mining, waste disposal, and recreation. The land-ocean interface also represents the primary zone of human habitation in many parts of the world.

Physical and biological processes in the coastal zones are often intimately linked, usually at short time scales and at small spatial scales. The main ingredients are wind-induced up- and down-welling, coastal waves and along-shore currents and tides.

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Biographical sketches

Aike Beckmann, (born in 1958) studied oceanography at the Christian-Albrechts-University in Kiel, Germany, where he received his PhD in 1987 for a thesis on modeling quasi-geostrophic instability processes in the ocean. During his post-doc years at the Chesapeake Bay Institute in Baltimore, Maryland, USA, he began primitive equation modeling with a terrain-following coordinate model, specializing in process studies on topographic effects in the ocean. Back in Germany at the Institute for Marine Research Kiel, he also got involved in large-scale modeling activities. In 1994 he became senior scientist at the Alfred Wegener Institut for Polar and Marine Research, and led a group on modeling of coupled ice-ocean processes in the Antarctic Marginal Seas. In 2000, he attained his Habilitation at the Carl-von-Ossietzky-University in Oldenburg, Germany, for a thesis on flow around isolated seamounts. In 2003 he moved to the University of Helsinki where he currently leads a group working on ice-ocean modeling and marine systems research. His work areas include ocean process studies (topographic effects, seamounts, mesoscale instabilities and eddies, bottom boundary layer dynamics, Mediterranean outflow, tidal effects), ice-ocean climate modeling (North Atlantic, Antarctic Marginal Seas and global), marine ecosystem modeling (dynamics of phytoplankton and interactions with the physical environment), as well as numerical methods and parameterizations (alternate coordinate systems, representation of dense downslope flows, ice shelf melting and mixing in large scale models). He co-authored a book on "Numerical Ocean Circulation Modeling", and has been teaching graduate and undergraduate courses in the areas of physical oceanography, marine geophysics, numerical modeling and marine systems.

Jens Schröter, was born in 1951. He studied meteorology and oceanography at the University Hamburg, Germany. He received his diploma in meteorology in 1978 with a thesis on downscaling of synoptic data for calculating dispersion of trace gases in the atmospheric boundary layer. For his graduate studies he worked experimentally at the Max-Planck-Institut für Meteorologie in Hamburg, Germany, with the task of active microwave remote sensing of the sea surface to derive sea state, wind vector and current structure at the oceans surface. After his Ph.D. degree in 1983 he spent his post doc period at the Massachusetts Institute of Technology, Cambridge, MA, where he began data assimilation in oceanography with linear and nonlinear methods. After his return to Germany he continued that work at the Max-Planck-Institut für Meteorologie in Hamburg with a focus on global modeling. In 1987 he became senior scientist at the Alfred-Wegener-Institut für Polar- und Meeresforschung. He was involved in the World Ocean Circulation Experiment and worked for its Data Products Committee. More and more he was concerned with satellite altimetry and geodesy with the goal of retrieving absolute velocities in the ocean. He served as principle and as co-investigator in several satellite missions. The assimilation studies involved quite diverse methods and led to the development of a suite of inverse models. For a better treatment of oceanic boundaries he became more involved in unstructured mesh methods in oceanography where he co-chairs international efforts on regional and global modeling. Currently he heads the German group on the development of a finite element ocean model (FEOM) and is head of the modeling group in the context of the German Tsunami Early Warning System for the Indian Ocean.