MATHEMATICAL MODELS OF CLIMATE

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Contents

Summary

This chapter presents an overview of the mathematical formulation of climate models, whose development is viewed as one of the great intellectual achievements in the latter half of the Twentieth Century. The presentation begins with the models based on the energy balance principle, describes their conceptual formulation and their application in climate studies. The climate feedbacks triggered by the effects of changing insolation on the extent of polar ice-caps, special radiative behavior of the water vapor, and variations in cloud cover are briefly discussed. Next the focus is on the state of the art complex simulation models for the atmosphere, ocean and the coupled atmosphere-ocean system. The formulation of the models is discussed, including the technique used to incorporate effects of processes with scales smaller than the model's resolution. The need to include components to represent the sea ice, land surface processes and human induced effects is emphasized. A selection of climate model applications is reviewed and outstanding challenges for the future are listed.

1. Introduction

Life on earth as we know it depends upon environmental conditions that are not too extreme. Clearly for familiar life forms to survive there must be air and water and their temperatures must be below the boiling point of water and at least some of the time above freezing. In addition, winds are required to be moderate and the chemical composition of the air to be nontoxic. If the annual cycle of weather at a location is not too severe in all its manifestations, life can be sustained. But weather fluctuates from time to time and even its annual average varies from one year to another. Climate is a statistical summary of weather from the local to the global scale and from days to centuries.

There is ample reason to think that the climate of the planet is governed by the laws of physics as we apply them in all phases of science and engineering. The laws of physics express relations between the variables usually associated with weather, such as the values of wind components, air pressure, water vapor, cloud, and temperature. Based upon many years of experience with similar systems, it is useful to cast these governing equations for the weather variables into a form from which they may be approximated by modern high-speed computers. Our belief that such a system of equations leads to essentially deterministic solutions (at least in an average sense) has led to a massive worldwide effort to implement a program in climate modeling.

Implementation of the program requires many ingredients. First, there must be sufficiently many talented scientists trained in a variety of disciplines ranging from applied mathematics, meteorology, oceanography, biology, etc. Second, the serious modeling of climate requires very fast computers with large storage capacities. Finally, there must be a balanced program of observations featuring a blend of studies of individual processes such as tropical storms involving localized aircraft measurements to a program of satellites looking down on the planet and providing global coverage. Essential to the effort is a climate monitoring program reaching as far as possible into the past and steadily into the future.

This chapter presents an overview of the mathematical formulation of climate models. It begins with the simplest forms of the models as a step toward understanding the state of the art complex simulation models. Realization of a successful program in climate modeling is necessarily a long-term effort because of its very complexity. There are also important issues from the philosophy of science that need to be considered as the program progresses.

2. Models Based upon Energy Balance

The simplest concept in constructing a mathematical climate model is that of the balance of energy entering and leaving the earth/atmosphere system from space over long time periods. Energy flows into and out of the system in the form of electromagnetic radiation in two streams: 1) solar radiation, which warms the earth by the rate of its absorption in the atmosphere and in the surface features, and 2) terrestrial radiation, which flows from the top of the atmosphere out to space, cooling the planet. When averaged over long periods (say decades) these rates of incoming and outgoing energy flows must balance if the climate is to be in steady state, with each component being about 240 watts per square meter (averaged over the planet and through the year). Actually, this statement involves some assumptions such as that the flow of heat from hot interior of the earth does not contribute significantly to the surface heat budget, or storage of large amounts of heat in the interior of the ocean where it may not reappear at the surface for thousands of years. Measurements from earth orbiting artificial satellites suggest that the solar and terrestrial energy streams averaged over the globe and over a few years do indeed balance to a remarkable degree.

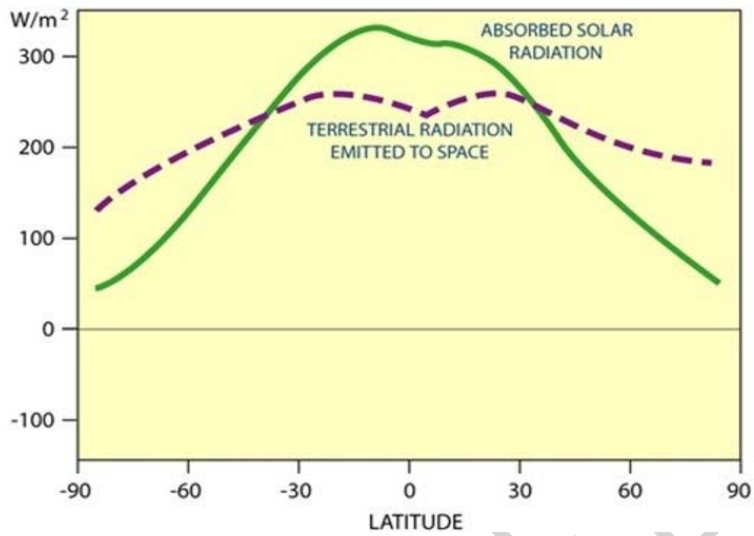

Figure 1. Absorbed solar radiation (solid line) as a function of latitude averaged around the latitude circle and through the year. The dashed line shows the radiation from earth to space in the same units and using the same averaging. Note that the heat absorbed by the earth-atmosphere system is larger than the emitted in the tropics and vice versa in the extratropics. This implies that for a global balance there must be transport of heat from the warmer areas toward the cooler.

Figure 1 shows a graph of the absorbed radiation from the sun along with the radiation of energy from the earth to space based upon satellite measurements as a function of latitude. The excess of absorbed energy in the equatorial regions is balanced by an excess of emitted energy in the polar regions. This indicates that energy must be transported from the warmer tropics towards the poles in order to maintain the global balance of heating and cooling rates. This transport of heat is conveyed by motion in the atmosphere and oceans.

The models based on the energy balance principle employed by the Russian climatologist M. I. Budyko and independently by the American William Sellers in the late 1960s and early 1970s were among the first to have serious impact. These models made use of the balanced energy streams for individual latitude belts. In order to relate the energy streams to climate it was necessary to connect the outgoing radiation to the surface temperature through simple mathematical formulas. In addition there must be provision for heat to be transported from one region to its neighbors. Energy balance modelers applied simple rules based upon flows of heat proportional to the temperature differences between neighboring regions. Using these simple rules the problem could be cast into the form of an equation, and the surface temperature could be solved for as a function of position at the earth's surface. With very little adjustment in the empirical coefficients in the model, the agreement to the observed values of the temperature field was remarkable. Figure 2 is a schematic diagram indicating a single box-shaped region with arrows indicating the flow of heat horizontally out of the box. Modelers took the flows of heat were taken to be proportional to temperature differences across the borders of the box edges. Not shown are the flows vertically due to the radiation heating and cooling.

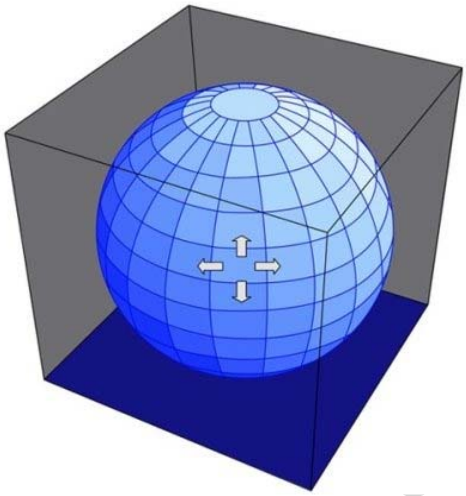

Figure 2. A schematic diagram of the planetary surface divided into rectangular cells. The arrows indicate the net flow of heat from one cell to its four neighbors through transport by winds and ocean currents. In more realistic models the cells are stacked vertically as well with flows of momentum, air, water vapor and radiation as well.

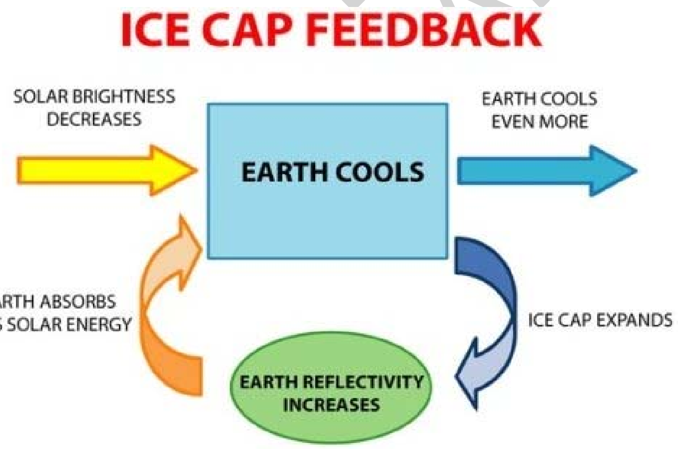

Figure 3. A schematic diagram illustrating the ice cap feedback mechanism. If the solar brightness is lowered slightly, the planetary temperature lowers slightly; this leads to an expansion of the ice caps. This latter causes the planet to be more reflective to sunlight, leading to an enhanced cooling. This is an example of a positive feedback mechanism – the climate response is enhanced by the internal workings of the system.

Both Budyko and Sellers included an idealized polar ice-cap in their models which grew in size if the solar heating decreased. Ice has a very different reflectivity to sunlight than open ocean or land areas. Hence, the larger the ice caps, the more sunlight will be reflected back to space as indicted in Figure 3. This was the first important indication of feedback mechanisms in climate models. A positive feedback mechanism is one in which an internal mechanism in the system responds in such a way as to enhance an externally imposed perturbation. Since those early discoveries simple climate models have helped to identify and elucidate several other important feedback mechanisms.

One of these feedback mechanisms now recognized as very important is the so-called water vapor feedback mechanism. Water vapor absorbs infrared radiation when it is in the air, but it is relatively transparent to incoming solar radiation (i.e., it is a greenhouse gas). When substantial water vapor is present in the air the radiation to space effectively originates from a higher altitude and therefore is emitted at a lower temperature.

The result is when such a greenhouse gas is present there needs to be a higher surface temperature to generate enough infrared radiation to space to balance the heating from the sun. As the temperature of the earth increases, more water vapor tends to be in the atmosphere. This leads us to the powerful water vapor positive feedback. This feedback mechanism may amplify the response by as much as a factor of two. The magnitude of this effect, however, is a matter of current debate.

Other feedback mechanisms were identified in the 1970s including the possible effects of clouds. Clouds play a dual role in the energy balance. They are white and therefore good reflectors (back to space) of solar radiation, but they also absorb infrared radiation then re-emit it from their cold tops. Hence, the reflectivity property tends to cool the planet and the infrared absorption and re-emission tend to warm the planet. In fact, calculations suggest that the two effects nearly cancel. Presently clouds cover about 50% of the planet.

An important question is: if the climate warms, will the amount of cloud cover increase? Moreover, if it does increase, which will win the reflectivity effect or the greenhouse effect? Energy balance models cannot answer these questions because in this class of models such factors as cloud fraction and their properties have to be specified rather than generated internally by the model. One must turn to models that actually include convection and the physics of clouds to find a satisfactory solution.

While energy balance models have become very sophisticated in their mathematical formulation and in the insight they have provided, there are some features of climate they cannot hope to simulate faithfully – the details of the transport of heat and material in the atmosphere and oceans. These features can only be incorporated in the mathematical formulation by including the flow of the winds and the ocean currents. Undertaking to include this level of sophistication in the physics of the models requires several orders of magnitude of computational resources and large teams of scientists.

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Bibliography

Hartmann, D. L. (1994). *Global Physical Climatology*, 411 p. San Diego, USA: Academic Press. [This covers climatology from a physics perspective and addresses critical issues of climatology.]

Mote, P. and O'Neill, A. (1998). *Numerical Modeling of the Global Atmosphere in the Climate System*, 517 p. Boston, USA: Kluwer Academic Publishers. [This provides the use, analysis, design, and additional applications of atmospheric general circulation models.]

Trenberth, K. E. (Editor), (1992). *Climate System Modeling*, 788 p. Cambridge, UK: Cambridge University Press. [This discusses the mathematical, physical, chemical, and biological basis for component models of the Earth System.]

Biographical Sketches

Gerald R. North, has been a Distinguished Professor of Atmospheric Sciences at Texas A&M University since 1986. He is currently Editor in Chief of the *Reviews of Geophysics*. North received his PhD in theoretical physics from the University of Wisconsin in 1966 and has served eight years as a research scientist at NASA/Goddard Space Flight Center. He has published in the fields of paleoclimatology, statistical methods in climate dynamics and in simplified models of climate and climate change. He was the proposer and first US Study Scientist for the Tropical Rainfall Measuring Mission, a satellite jointly supported by the US and Japan launched in 1997. A common theme in his research is the application of mathematical and statistical approaches to the better understanding of mechanisms of climate change both observational and theoretical.

C. Roberto Mechoso, is a Professor of Atmospheric Dynamics in the Department of Atmospheric Sciences at the University of California Los Angeles (UCLA). Mechoso's current research interests are ocean-atmosphere interactions, numerical weather prediction, meteorology of the Southern Hemisphere, and high performance computing. He is author of more that 150 scientific publications in his fields of interest. His papers aim to increase the understanding of climate variability based on the analyses of highly realistic simulations with UCLA numerical models, complemented by studies with observational data. Targeted topics have been El Niño/Southern Oscillation and its impacts, American monsoon systems, stratospheric warnings, and instabilities on atmospheric fronts. He was first in running a coupled atmosphere-ocean model in geographically distant supercomputers.Mechoso was raised in Montevideo, Uruguay, where he obtained an engineering degree from the University of the Republic; his MA and PhD are from Princeton University. He is a fellow of the American Meteorological Society, member of the American Geophysical Union, Honorary Professor at the University of Uruguay, Senior Fellow of the San Diego Supercomputer Center, and Corresponding Member of the National Academy of Engineering of Uruguay. Mechoso was founding Chair of the panel on Variability of the American Monsoon Systems (VAMOS) of the World Climate Research Programme (WCRP).