OPTIMIZATION AND OPERATIONS RESEARCH

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Keywords: decision analysis, decision models, game theory, mathematical programming, modeling process, optimization models, stochastic processes, scientific decision making

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Summary

Designing and controlling complex systems, solving hard problems of efficiently allocating scarce resources using incomplete information, and developing sustainable strategies to master situations of conflict and co-operation have always been necessary and challenging for individuals, organizations, and economies. Optimization and Operations Research are the disciplines that deal with such problems on a scientific basis; that is, they apply scientific methods and information technology to problem solving and provide the problem owner with quantitative approaches to informed and rational decision-making.

Central to any quantitative decision analysis and decision-making are the foundation on mathematical models and the application of mathematical theories and methods. Here the mathematical model should abstract the crux of the decision-making problem. It should give a suitable and well structured view of the underlying real problem, so that the conclusions obtained from analyzing and manipulating the model are valid for the real problem.

Optimization is that discipline within applied mathematics that deals with optimization models, their mathematical properties (optimization theory), and the development and implementation of algorithms (numerical analysis and algorithmic design). Specifying and formulating the problem, constructing a suitable mathematical model and deriving a solution from it, testing and modifying the model, and finally implementing the model solution in the real problem situation are the phases of the Operations Research approach. Thus, it is optimization that supports this process with a powerful tool-box.

In this article we give an introduction to the many faces of Optimization and Operations Research. We discuss conceptual as well as technical aspects, and report examples demonstrating impact and excellence. Discussing implementation issues, we focus on the interface with the human decision-maker, computer science, and information systems.

1. Introduction

In this introduction we want to specify the intention of the two concepts under this theme, “Optimization” and “Operations Research” and their relationship. Both concepts are closely related and often are used as synonyms but we want to distinguish between the two. (Otherwise the title of this theme would be somewhat redundant.)

The attempt to describe precisely what is meant by these terms is not equally simple for both concepts. While optimization has grown out of mathematics, a classical science, and thus rests on a solid theoretical foundation and has many academic aspects and roots, Operations Research is a rather young, purely pragmatic, and practically oriented synthesis of different disciplines, including mathematics. Nevertheless let us give it a try.

1.1. What is “Optimization”?

Optimization is that discipline within applied mathematics that deals with optimization problems, or so-called mathematical programs.

In an optimization problem or mathematical program we seek to minimize or maximize a (real valued) function over a set of (decision) variables subject to constraints.

Example:

Find integer values for the variables x, y, z and k with $k > 2$ such that $x^k + y^k - z^k$ is minimal.

This is the well-known Fermat problem, which remained unsolved for about 350 years and ultimately proved by Andrew Wiles in 1997.

Thus Optimization is problem oriented, yet these problems are purely ideal and formal constructs. Carl Friedrich Gauss, the great German mathematician, claimed problems like the Fermat problem were of no interest for him since the solution would yield no generalizable insight or knowledge. But research on this kind of seemingly purely formal and theoretical problems has been the engine for progress in mathematics leading to the development of many “applicable” theories and methodologies.

Some examples of problem types which come from the meat of optimization/mathematical programming, and which from a certain point of view are more “interesting” and “practical” follow.

In *linear programming* we study optimization problems of the following form:

$$\text{maximize} \quad c_1 x_1 + \cdots + c_n x_n \tag{1}$$

$$\text{subject to} \quad \begin{aligned} a_{1,1}x_1 + \cdots + a_{1,n}x_n &\le b_1 \\ &\vdots \\ a_{m,1}x_1 + \cdots + a_{m,n}x_n &\le b_m \end{aligned} \tag{2}$$

$$x_1 \ge 0, \ldots, x_n \ge 0 \tag{3}$$

where $m, n \in \mathbb{N}$, c_j, b_i and $a_{i,j}$ are constants, and x_j are the (decision) variables ($i=1,...,m$; $j=1,...,n$).

Such problems are called *linear optimization problems* or *linear programs* (*LP*), since all functions involved are linear. The mathematical treatment of this problem has grown out of the area of linear algebra, one of the basic mathematical disciplines, which, because of its fundamental role, is an integral part of every mathematical high school curriculum. A special interest of this problem class stems from the fact that the linear programming paradigm can serve as the formal model for a number of economic resource allocation problems.

Consider for instance the problem of determining the most profitable mix of products for a manufacturer. For each of the n products a decision variable is introduced to represent its production rate. Let c_j be the profit per unit of product j produced. The choice of the product mix is constrained by limited capacities of the production facilities which are necessary for production of these products. Let m be the number of different types of facilities needed, for each type i let b_i be the amount available during the planning period, and let $a_{i,j}$ be the amount of capacity i used by each unit of product j which is produced. Then solving the associated linear program gives the optimal product mix.

In *combinatorial optimization* the mathematical language is not based on real numbers and (in)equalities but on "sets". Let $\boldsymbol{S} \subseteq 2^E$, be a collection on some finite ground set E and c: $\boldsymbol{S} \rightarrow \mathbb{R}$. Then the task is to find a subset $S^* \in \boldsymbol{S}$ that maximizes (or minimizes) c on $\boldsymbol{S}$, i.e. $c(S^*) \geq c(S)$ (or $c(S^*) \leq c(S)$) for all $S \in \boldsymbol{S}$. Set theory, mathematical logic, and graph theory are the mathematical "mother" disciplines, and again the relevance stems from the fact that quite different real world problems of optimal system design can be described using this mathematical formalism and "solved" by applying the mathematical methods developed: for instance, the design of telecommunication networks and distribution systems.

Consider for instance the requirement to establish a communication network among n sites by underground cable. The sites can be interpreted as nodes in a graph, and every connection of two sites constitutes an edge between the two nodes with a specific length or cost. Now we want to determine a subset of connections of minimal total cable length or cost, such that any two sites are connected by a path of connecting cables. This problem can be solved by determining the spanning tree of minimal cost in the associated graph.

In contrast to these areas, which are based on static and deterministic concepts, the field of *stochastic decision processes* focuses on temporal relationships among random variables and decisions. Here we study dynamic non-deterministic systems where at certain decision time points t a controller/decision-maker chooses an action $a(t)$ from a set of available actions A based on the observed state $S(t)$ of the system. The optimization problem is to choose a sequence of actions $(a(t))_{t=0,1,2,...}$, also called a policy, that will maximize the performance of the system over a given time horizon. Here, the fields of probability theory, stochastic processes, differential equations, and several others are the mathematical prerequisites, and the special interest stems from the applicability of this concept to describing and controlling complex real world processes like queuing, inventorying, and investment.

Consider for instance the following production–inventory problem. At the beginning of each period t, a firm must decide how many units $x(t)$ should be produced during the current period at a per unit variable production cost of c units. For each period t, the period's random demand $d(t)$ is observed and met out of current production and inventory, and at the end of every period the firm holds a certain end of period inventory $i(t)$ for which a per unit holding cost of h units occurs. Given a certain initial inventory $i(0)$ and a probability distribution for each period's demand, the problem is to determine a production policy that minimizes the expected net cost incurred during a specific number of periods.

The term "mathematical programming", which is used as a synonym for optimization, refers to the study of these kind of problems, i.e.:

- Their mathematical properties (optimization theory)
- The development and implementation of algorithms (numerical analysis and algorithmic design), and also
- The application of these programs and algorithms to real world problems.

To clarify a common misinterpretation, mathematical programming does not specifically refer to computer programming. Here programming refers to the development of a plan or procedure for dealing with a problem.

1.2. What is "Operations Research"?

Since there is no "official" definition of Operations Research (OR), the introduction of this concept has to be more descriptive. In what follows, we try to summarize the central focus of OR as a "problem and system oriented management approach" and illustrate the "nature" of this concept, describing several facets and characteristic features such as quantification, model based optimization, and interdisciplinarity.

OR is problem oriented. Operations research is a problem solving approach. It is called "operational research" in the United Kingdom, where the term was coined during the Second World War when scientists were asked to analyze and support the solution of a range of military problems in connection with the deployment of radar, the configuration of convoys, and so on. The term was invented to distinguish "research on operations" from technical research. Thus, in contrast to mathematical programming, the problems with which OR manages to cope are not abstract and formal but specific and real.

OR is a management approach. Today the term "Operations Research" means a scientific approach to the solution of problems in the management of complex systems arising in industry, government, the military, and other areas. Under this aspect a frequent substitute for OR is the term "management science" (MS), and often both terms are combined and the discipline is referred as "MS/OR." MS/OR aims to provide a rational basis for decision-making by seeking to understand and structure complex situations and to use this understanding.

OR is system oriented. Operations research adopts an organizational viewpoint and attempts to:

- Search for causes of a problem indicated through system failure, malfunction, or under-performance
- Identify potentials and possibilities for improving system performance
- Evaluate the effect of changes in any part of a system on the performance of the system as a whole, and
- Redesign structures and processes appropriately and implement these changes in the system.

Here OR is usually concerned with systems in which human behavior plays an important role, and it attempts to resolve the conflicts of interest among the components of the organization in a way that is best for the organization as a whole. This distinguishes OR from systems engineering (SE), which concentrates more on systems in which human behavior is of minor importance.

OR is a decision approach. Operations research is not a science itself, but rather the application of science to the solution of managerial problems. As can be seen from the examples above, the subject matter of OR consists of decisions that control the operation of systems. In OR we study not how managerial decisions are made but how they should be made. OR is a prescriptive and normative, not a descriptive, approach to decision-making. Yet Operations Research is not "decision-making", it is only a decision aid or decision support for the (human) decision-maker, the firm's management say. However, the results of an OR project or OR study must have direct and unambiguous implications for executive actions.

Since the emphasis on making decisions and taking actions is central to all OR applications, another term related to OR is "decision analysis" (DA). A decision analysis process may take two basic forms: qualitative or quantitative. While qualitative analysis is based primarily on the manager's experience and judgment, in a quantitative approach an analyst will concentrate on the "hard", quantitative facts or the data he or she thinks to be relevant to the problem. The motivation for applying a quantitative approach stems from the complexity, importance, and novelty of many managerial problems, which should prevent the management from relying on purely "soft" information and intuition. If the problem is repetitive, management gains efficiency by creating and applying routine decision recommendations based on quantitative analysis.

From the point of view given so far, one may use the terms Operations Research, management science, and quantitative decision analysis almost interchangeably.

A common definition of Operations Research is the following:

> *Operations research is concerned with scientifically deciding how to best design and operate man–machine systems, usually under conditions requiring the allocation of scarce resources.*

A few examples to help illustrate the scope of Operations Research were given by the Operational Research Society of Great Britain:

- "*There are too many lorries on the road*." A common cry but something can be done! A bakery used OR to devise an efficient scheduling system for its delivery vehicles. The new system reduced lorry mileage, road congestion, and pollution as well as saving money for the bakery.
- "*I had to wait all morning in hospital*." Great pressure on consultants' time, coupled with some patients who do not always keep their appointment times, can cause real problems for hospitals. But by using OR, appointment systems have been designed that substantially reduce waiting times whilst keeping highly qualified medical staff fully occupied.
- "*We've just got to increase our sales*." Easier said than done, but OR proved equal to the task for a mail order firm. The OR model helped boost catalog sales by designing an ideal mix of discounts, special promotions, and customer incentives.
- "*Bottlenecks*: *the bane of my life*." How many production managers say that? We know of some that don't, because they used OR! One manager wanted to ensure the efficient operations of his new automated warehouse by simulating the operation of alternative material handling equipment. This meant that a selection could be made which eliminated any bottlenecks and delivered the required output.

The following two aspects of Operations Research establish the link to optimization.

OR and modeling. Central for any quantitative decision analysis are the foundation on mathematical models and the application of mathematical theories and methods such as mathematical programming/optimization, simulation, and graph theory. In this respect, model building can be viewed as the essence of the OR approach to problem solving, and mathematical methods as OR's powerful tool-box. Here the (mathematical) decision model should abstract the crux of the decision-making problem. It should give a suitable and well structured view of the underlying real situation, that is, a representation of the structure, function, and behavior of the system, the fundamental properties of system operations, and the organizational objective, such that the conclusions obtained from analyzing and manipulating the model are valid for the real problem.

OR and optimization. Another characteristic is that OR attempts to find the best possible solution to the problem/model and to identify the course of action which creates optimal performance of the real system. In other words, OR aims to recommend optimal decisions. This "search for optimality" is an important driver which, although not always realizable in practice, is inherent in the OR paradigm and distinguishes the goal-oriented OR approach from decision-making on the basis of "ground rules" representing expertise and (best) practice.

In the following paragraphs we want to focus on two other important features of Operations Research.

OR is an interdisciplinary team approach. It is evident that no individual should be expected to have expertise on all aspects of the OR work: the problem domain, mathematical modeling, algorithmic implementation, and the design and technical implementation of information systems. Thus applying OR requires a team of individuals with diverse backgrounds and skills. This interdisciplinary viewpoint contributes to the charm as well as to the significance of OR. It distinguishes OR from other disciplines, and from optimization, and it has been a central element of OR from the beginning of its development in the middle of the last century. Its versatility – with respect to applications as well as methodologies – has always been the strength and the demand of Operations Research, but more than this, it has been its charm.

OR is knowledge and information management and processing. Optimization/Operations Research was developed in the middle of the last century, and it has evolved together with disciplines like computer science (CS) and information systems (IS). There has been a link between OR and CS/IS throughout this time. Solving OR models representing, for instance, gasoline blending problems in the petroleum industry, was among the first business applications for computers, and OR problems have constantly been a challenge for computer technology. On the other hand, the implementation of OR models and methods in a firm depends on the availability of high power computers and requires the integration into a computer (data) based information system. Operations research has been influenced significantly over the last fifty years by developments in the design of user interfaces, as well as in data/knowledge management for information systems, in telecommunications, and in distributed information processing, allowing OR models to access databases worldwide and in real time. These developments have changed the environment and the requirements as well as the impact, but they have not changed the basic paradigm and intention of OR.

The close linkage between these fields and the change in the focus on information is best represented in the modern definition given by the Institute for Operations Research and the Management Sciences (INFORMS), the society which has been the result of the merger between the American Society of Operations Research (ORSA) and the Institute for Management Science (TIMS):

Operations Research and Management Sciences are the professional disciplines that deal with the application of information technology for informed decision-making.

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Biographical Sketch

Ulrich Derigs is Director of the Department of Information Systems and Operations Research (WINFORS) at the University of Cologne, Germany. He received a Master's degree (1975) and a Doctoral degree (1979) in Mathematics and a Doctoral degree (1981) in Economics from the University of Cologne. In 1985 he completed his habilitation at the University of Bonn. He was Assistant Professor at the Mathematical Institute (1976 to 1979) and at the Seminar for Industrial Engineering (1979 to 1981) at the University of Cologne. From 1981 to 1985 he was Assistant Professor at the Institute for Econometrics and Operations Research at the University of Bonn and member of a Sonderforschungsbereich (DFG). From 1985 to 1990 he was Professor of Information Systems and Operations Research at the University of Bayreuth, and since 1990 he has been Professor at the University of Cologne.

He has done extensive research in combinatorial optimization with an emphasis on the design, analysis, and evaluation of efficient algorithms and industrial applications. Today his interest lies in the interface between Operations Research and information systems. His focus is the design and implementation of model-based decision support concepts and systems in different application areas such as routing and scheduling, production planning, logistics, finance, and telecommunication.

Ulrich Derigs is on the editorial board of several journals. From 1988 to 1992 he was editor in chief of *OR-Spektrum*, the journal of DGOR, the German OR society. From 1992 to 1998 he was a member of the board of DGOR and from 1996 to 1998 President of the DGOR.